Auction Design for the Colombian Electricity Market

Ennio Stacchetti
University of Michigan

Version: June 28, 1999

This work originated at Fundesarrollo with the inquiry of Alfredo Garcia and Guido Nule. Alfredo Garcia was an invaluable source of information and suggested numerous comments and references. The members of the Colombian Regulatory Commission (CREG), Jaime Saldarriaga, Carmenza Chahin and Eduardo Afanador, discussed with me an earlier draft of this paper and provided useful suggestions. I would also like to thank Luis Camargo and Jorge Alberto Valencia of ISA for a detailed presentation of the Colombian electricity market and ISA's operations, and for providing me with the data for this study.

1. Introduction

The Colombian electricity market, "La Bolsa", began to operate in July 1995. Colombia is the first Latin American country to design an electricity market. There are only a few other countries in the world that currently operate similar markets, and many countries are now assessing the potential benefits and risks. Thus, the Colombian market offers a special opportunity to understand how well these markets work. In this paper I study the incentives at play in La Bolsa and how they are determined by the current market mechanism. One goal is that this will inform policy discussions in Colombia and other countries considering a privatization and/or deregulation of the electricity sector. In particular, the study will shed some light on the opportunity that firms (may) have to exercise market power and the interplay between industry concentration and idle capacity.

The public company ISA administers La Bolsa and the CND (Centro Nacional de Despacho). Each day the firms submit a bid schedule consisting of a bid price and an available capacity for each hour of the following day. Every hour the CND dispatches the firms up to their available capacity in increasing order of their bids, until the sum of their capacities exceeds demand. The last firm dispatched is called the marginal firm and it is only dispatched for the residual demand not covered by the other dispatched firms. The spot market price during that hour is set equal to the marginal firm's bid. All the energy sold by the producers (and bought by the distributors) is traded at the spot market price. The spot market then operates every hour like a first-price multiunit auction. These rules describe the ideal "economic dispatch" of the CND. However, this dispatch does not take into account the system constraints, and the CND needs to make various adjustments in order to satisfy them. For example, the ideal dispatch may require that the transmission lines between two points carry more than their capacity. The CND may also need to make adjustments in response to unexpected surges in demand and/or unexpected changes in production capacities. When it rains, for example, run-of-river hydroelectric plants[1] may experience an unexpected increase in their production capacity. A full description of the rules that apply to these adjustments and the definition of the corresponding transaction prices is complex. However, normally the adjustments are relatively small and they do not affect very much the ideal dispatch or the incentives of the firms.[2]

The English Spot Market started to operate in March 1990 and has been the focus of numerous studies. Up until 1996, only three firms produced electricity in the English market: Nuclear Electric, National Power, and PowerGen. The National Grid Company (NGC) operates and maintains the transmission system, and its operations are regulated by the government. Nuclear Electric is a public company that, due to its technology, supplies base load at a price that is normally below that of its competitors. Nuclear Electric covers about 25% of demand. Thus, in effect, National Power and PowerGen are a duopoly that

[1] A run-of-river plant only has storage capacity, or pondage, sufficient for daily or weekly storage of the total river flow. Therefore, such a plant must use the streamflow shortly after it arrives.

[2] An interesting monopolistic opportunity that we do not study in our models is caused by the limited capacity of the transmission lines that connect the coast region (Cartagena and Barranquilla) with the rest of the system. Peak demand in this region is substantially more than the capacity of the transmission lines. Thus, the local thermoelectric producers effectively enjoy a captive demand for part of the day.

shares the 75% of residual demand. National Power and PowerGen operate coal, oil and gas powered plants.

Every day each generator submits a bid schedule specifying the prices at which it would be willing to supply power the following day. However, unlike the Colombian market, the schedule describes a “supply curve”, which includes parameters like marginal costs for different levels of utilization, as well as ramp-up times. The central dispatcher calculates the operating schedule that minimizes the costs of meeting demand and identifies the marginal plant in each half hour. The price that the marginal plant bid is paid for all the energy generated in that half hour.

Contrary to the claims that its market design should lead to a highly competitive outcome, various authors conclude that the English Spot Market equilibrium generates a high markup on marginal cost and substantial deadweight losses (see Green and Newberry (1992) and Wolfram (1998, 1999)). However, this literature[3] is not very relevant for the Colombian experience for two reasons: first, La Bolsa adopted different auction rules than those used in the English Spot Market; and second, the characteristics of the electric sectors of the two countries are very different. While in the English market, nuclear energy provides base load, the mayority of the Colombian energy is produced by hydroelectric plants.

A common feature of the British and Colombian spot markets is that the price is determined by the bid of the marginal firm. It has been observed that in a uniform price multiunit auction, the bidders have incentives to shade their bids (see Ausubel and Cramton (1996)). This may result in an inefficient dispatch because plants that are allowed to produce may have higher marginal costs than idle plants (see Wolfram (1998)). Perhaps this effect could be partially corrected by setting the spot price equal to the lowest bid among the plants that are not dispatched. While this change may be inappropriate in the British market with only two competitors, the Colombian market has a larger number of firms. This is an interesting modification that deserves some consideration, especially because the unattractive features of the equilibrium we study are directly related to the current definition of the spot price.

The classic model of Green and Newbery (1992) for the British spot market uses the supply function equilibrium framework developed by Klemperer and Meyer (1989). The model assumes that demand is *elastic* and that the firms submit *smooth* supply functions. Fehr and Harbord (1993) observe that demand is actually inelastic and that the firms can only bid *step-supply schedules*, where essentially each plant’s capacity is offered at a unique price. Fehr and Harbord (1993) propose instead a modified oligopoly model of capacity-constrained price competition with inelastic and random demand. I find these two features of their model relevant for the Colombian market and adopt a similar model. However, since the largest producers in Colombia operate hydroelectric plants, I concentrate on the dynamic incentives of the firms, and assume that each dominant firm must offer a unique price for all its plants (or, equivalently, that each dominant firm has only one plant). As demonstrated by the recent drought of 1997-1998, storage constraints play an important role in the strategic behavior of the Colombian firms.

In the next section, I describe the main characteristics of the Colombian electricity

[3] See also Vickers and Yarrow (1991) and Wolak and Patrick (1997).

industry. I present a static model in Section 3. This model captures most of the strategic concerns of the firms in any given day, and is a building block for the fully dynamic model in Section 4. Futures contracts, which are not incorporated in my models, are discussed in Section 5. In 1997, the meteorologic phenomenon "El Niño" caused a severe drought that intensified the strategic behavior of the producers and pushed prices up sharply, raising the concerns of the regulatory comission. In Section 6, I discuss these events and some of the weaknesses of the market design they exposed. Section 7 presents conclusions.

2. The Colombian Electricity Sector

Four types of agents participate in the electricity market: producers, distributors, transmission companies, and commercial companies. La Bolsa is administered by the public company Interconección Eléctrica S. A. (ISA), and its activities are monitored by the regulatory commission (Comisión de Regulación de Energía y Gas, CREG). The only "active" participants in La Bolsa are the producers and commercial companies; the transmission and distribution companies do not participate in the bidding proccess. Although the commercial companies may play an important role in the futures market, all the producers are also commercial companies and collectively control the vast majority of the futures market. Moreover, this study is confined to the spot market (where only the producers participate), and in the model I focus exclusively on the producers. The detailed ISA report "Informe de Operación 1996" gives a full description of La Bolsa and its participants (see also the ISA report "Análisis del Mercado Mayorista de la Electricidad Colombiana", 1997). Here, I will only give a partial description of the Colombian electricity market that is relevant for the analysis.

With the addition of new plants and ongoing restructuring, many industry relevant parameters are constantly changing. As of 1997, five producers (EEB, EEPPM, Isagen, CORELCA, and Chivor) owned 76% of the installed capacity, while the remaining 24% was divided among 17 small producers. EEB, EEPPM and Chivor owned the four largest water reservoirs, whose combined capacities represented 79% of the country's energy storage capacity (EEB's and EEPPM's water reservoirs alone represented 72% of the storage capacity). EEB and Isagen are mainly hydroelectric producers but also own thermoelectric plants.[4] CORELCA, the fourth largest producer with an 8% of the production capacity, only operates thermoelectric plants. In 1997, 69% of demand was covered by hydroelectric production and 31% by thermoelectric production.[5] While the total demand for 1996 was 40253 GWh, the total storage capacity was only 14283 GWH (or about 35.5%). Thus, the Colombian electric system is especially vulnerable to climate conditions: supply depends heavily on hydroelectric production, but actual available capacity is constrained by storage, which is relatively small.

In 1997 the total production capacity installed was 10601 MW, divided into 8017 MW (or 76%) of hydroelectric capacity and 2584 MW (or 24%) of thermoelectric capacity.

[4] Colombia has thermoelectric plants as well as combined cycle gas turbine power plants. For brevity, I will designate all these plants as thermoelectric.

[5] In 1996, 82.3% of demand was covered by hydroelectric production, 17% by thermoelectric production, and 0.3% by imports; 0.4% of demand was not served.

However. on average only 86.4% of the hydroelectric capacity and 64.1% of the thermoelectric capacity was available. Table 1 below lists the installed capacity of the six largest producers together with the capacities of their largest plants. Except for CORELCA, all these firms are mainly hydroelectric producers, and of all the plants in the table, only Barranquilla, Flores and Guajira are thermoelectric plants. In the case of EEB, for example, the sum of the capacities for its listed plants (2030 MW) is not equal to its total capacity (2320 MW) because the table does not include some of its smaller plants.

Capacity of the Colombian Electricity System				
Company:	Plant	MW		%
EEB		2320		21.9
	Guavío		1150	
	La Guaca		310	
	Paraíso		270	
	Colegio		300	
EEPPM		1710		16.1
	Guatapé		560	
	Playas		300	
	Guadalupe		472	
	La Tasajera		306	
ISAGEN		1790		16.9
	San Carlos		1240	
CORELCA		1350		12.7
	Barranquilla		530	
	Flores		243	
	Guajira		302	
CHIVOR		1000		9.4
BETANIA		500		4.7
TOTAL		8670		81.8

Table 1

Figure 1 shows the demand profile for a typical day in 1997. Demand is minimal – about 3000 MW – between 3:00 am and 4:00 am, and reaches a peak of about 7000 MW around 8:00 pm. Demand remains above 6000 MW between 6:00 pm and 10:00 pm. The daily demand profile is relatively stable, though there are a few fluctuations. In May, for example, the minimal daily demand was only about 2600 MW, and in October, the peak demand reached about 7400 MW.

Figure 2 shows the evolution of available capacity and the range for the daily demand in 1997. As I noted above, daily demand does not fluctuate very much. However, available capacity fluctuates substantially. For example, the *average* daily capacity in February was about 8400 MW, while that in December reached about 9400 MW (the table reports monthly averages, thus the actual variance of the daily capacity is obviously larger). As I explain below, this observation exposes a second weakness of the Colombian market: on a typical day, demand peaks at a level just about 1000 to 2000 MW below available capacity, while the total capacity of each of the two largest producers is about 2000 MW.

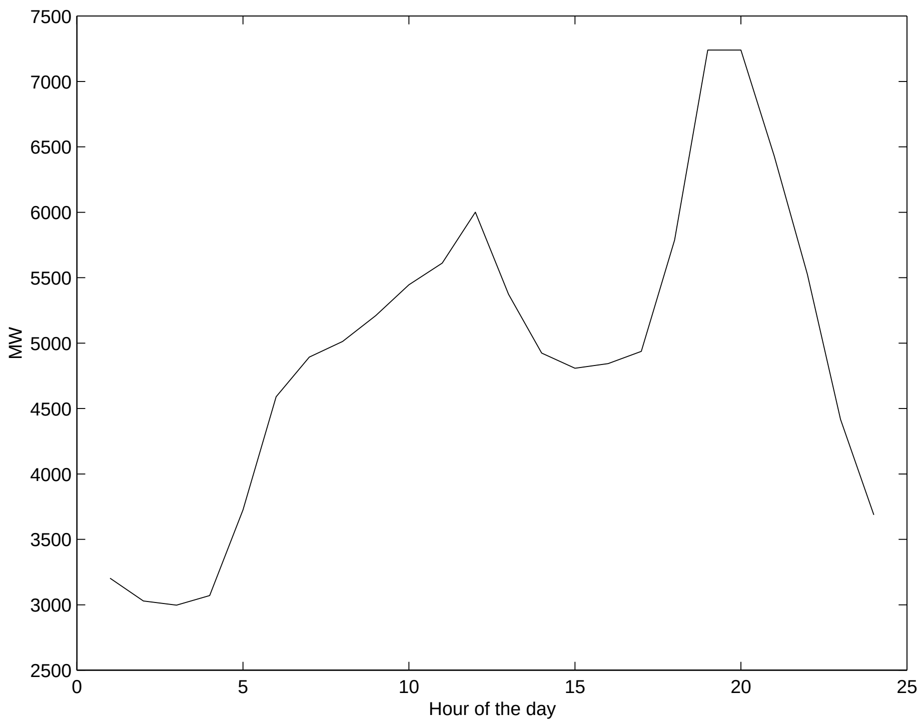

Figure 1: Demand profile for a typical day.

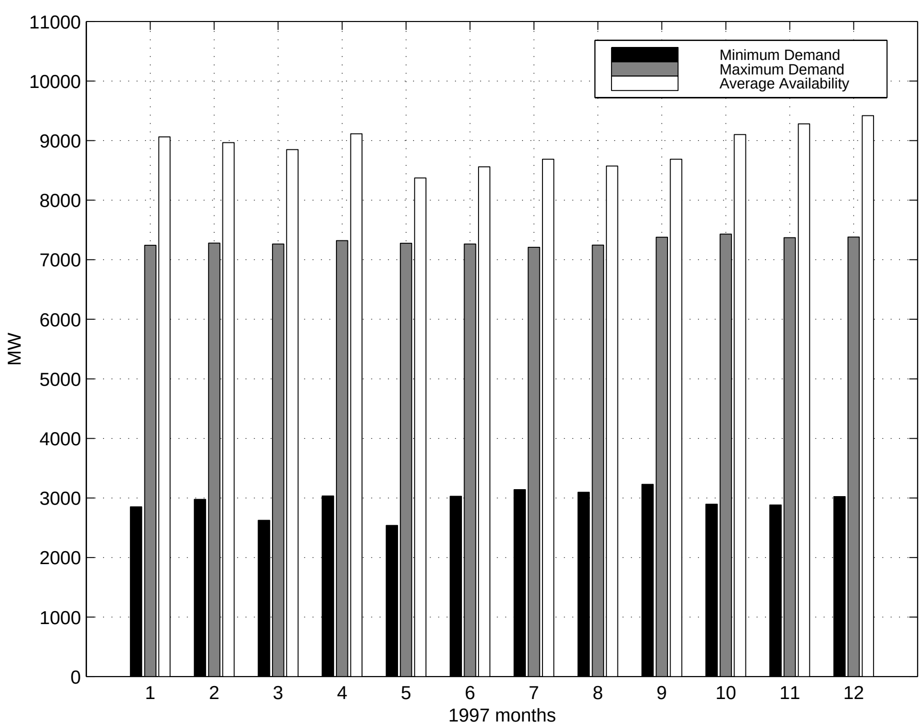

Figure 2: Evolution of demand and available capacity for 1997.

Prior to 1995, when La Bolsa began operating, production and prices were determined by a centralized system operated by the public company ISA. ISA also owned many of the power plants, and with the exception of the small firm Proelectrica (with a 100 MW plant), all the power plants were owned by the state (though they were controlled by regional public companies). Thus, both ISA and CREG possess detailed information about the operations of many power plants, including the production costs of thermoelectric plants. In addition, the prices of their main inputs – coal, oil and gas – are publicly available. The current regulation states that the thermoelectric producers' bids must reflect their "variable costs", and that the hydroelectric producers' bids must reflect their "water opportunity costs" (resolution CREG-55 of 1994). In practice, CREG has monitored the thermoelectric plants' bids closely, questioning any price that diverges substantially from CREG's estimates of the corresponding variable costs. On the other hand, interpreting the "water opportunity costs" has proven to be more difficult and controversial. Thus, the thermoelectric producers are constrained to bid in a relatively narrow interval, while the hydroelectric producers feel that their bids are only constrained by the rationing marginal cost $p^* = 267$ pesos/KWh.[6] The parameter p^* is roughly defined as the social cost for each KWh of unsatisfied demand. Obviously, it is not possible to reduce to a single parameter the social losses that depend on the magnitude and duration of a rationing; p^* is computed for a specific set of circumstances. CREG imposes a "rationing state" if in five consecutive days the spot price exceeds p^*. In this case, the Ministerio de Minas y Energía assumes control of all production and distribution. The rules that apply during a rationing state are financially harmful to the producers. In practice, the agents view p^* as a "ceiling price".

Three of the largest firms, EEB, EEPPM and Chivor, operate mainly hydroelectric plants and are privately owned. The other two largest firms are ISAGEN and CORELCA. CORELCA operates thermoelectric plants only, and ISAGEN is publicly owned. Though ISAGEN should seek to maximize profits as any other private company, it may also have political incentives to practice a less aggressive bidding strategy. Hence, in the models of the next two sections, I focus on the case where only two or three dominant firms exercise their market power while the other firms behave "competitively". A competitive firm restricts its bids to a narrow range, close to its marginal cost.

The models do not deal with future contracts. I briefly discuss how these contracts affect the firms' incentives in Section 5. Until 1997, 97% of the energy was traded in future contracts, and only 3% was traded in the spot market. During a transition period, the regulation has set minimum fractions of their demand that distributor and commercial companies must trade in future contracts.

3. Stationary Model

In this section I try to capture many of the features of the Colombian market with a simple static model. Besides making numerous simplifying assumptions, I introduce an artificial agent who represents La Bolsa in "the future". Although (production) capacity constraints

[6] As I discuss in Section 6, during the drought of 1997-1998 the hydroelectric producers ventured to bid prices in the range 225 to 250 pesos/KWh without provoking any reaction from CREG.

are included, stock constraints are not. The artificial agent offers the firms differentiated energy prices to reflect indirectly these scarcity constraints in the future. However, the differentiated prices cannot signal the scarcity constraints in the current period. In the next section, I explicitly study a dynamic game, and deal with the situation where storage is constrained and it is possible to exhaust the water in the reservoirs during the dry season.

Given the discussion of the previous section, we find most relevant the cases where there are two or three dominant firms. Consider first the situation with two dominant firms. This could correspond, for example, to the case where EEB and Chivor are the dominant firms, and where all other firms, including EEPPM, behave competitively. I assume that the competitive firms bid their marginal cost c_T (which I assume for simplicity to be the same for all competitive firms) for every hour of the day, and concentrate in the strategic behavior of the dominant firms during the peak hours. The total capacity of EEB and Chivor is 3320 MW, but if we assume that their average available capacity is the industry average (86.4 %), [7] then their total (average) available capacity is about 2900 MW. That means that the total available capacity of all other producers fluctuates between 8400 - 2900 = 5300 MW and 9400 - 2900 = 6300 MW. As we noted before, demand is between 6000 MW and 7000 MW from 6:00 pm until 10:00 pm. It is also above 6300 MW at *peak hours*, between 6:30 pm and 9:30 pm. Thus, if the two dominant firms bid prices above the competitive price of the other firms, they would face a "residual demand" between 6300 - 6300 = 0 MW and 7000 - 5300 = 1700 MW. Given the behavior of the competitive firms, each dominant firm can effectively choose the amount of energy it sells during the off-peak hours by bidding a price just below c_T.[8] Therefore, to simplify, in the model I reduce each dominant firm's strategic variables to a vector (p, q), where p is a bid price for the peak hours (unique for all peak hours) and q is an amount of energy to sell during the off-peak hours. For the hydroelectric producers, energy is storable. To capture the dynamic nature of their strategic problems, I assume that each dominant firm i produces up to capacity during the peak hours and that the energy it does not dispatch through La Bolsa is bought by an artificial agent at a given price s_i. The parameter s_i represents the average (discounted) price at which firm i will sell in the future the (turbined) water that it was prepared to sell today during peak hours, but that La Bolsa did not dispatch. We call s_i the *peak shadow price of water*.

To keep numbers smaller (and integer), in the model we make the unit of account for power equal to 0.5 GW = 500 MW, and correspondingly, the unit of account for energy equal to 0.5 GWh = 500 MWh.

THE MODEL: Two hydroelectric firms with capacities (K_1, K_2) and 0 marginal costs, simultaneously choose $(p_i, q_i) \in [0, p^*] \times [0, 21K_i]$, $i = 1, 2$. Demand d is stochastic for each of the three peak hours. The quantities demanded at different peak hours are

[7] Their average available capacity is likely to be higher as there are several hydroelectric producers that operate run-of-river plants, whose production capacity fluctuates with the rain fall.

[8] Even the minimal demand during the day, about 3000 MW, is more than the joint total capacity of the dominant firms (about 2900 MW). Therefore, each dominant firm can select off-peak hours to sell energy up to its capacity at a price (just below) c_T. Bidding prices strictly above c_T during off-peak hours ensures that the dominant firm is not dispatched during those hours.

independent random variables, and

$$d = \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ 3 & \text{with probability } \frac{1}{2}. \end{cases}$$

The firm that bids the smallest price is dispatched first by La Bolsa up to the minimum of d and its capacity during each peak hour. When both firms submit the same bid, La Bolsa chooses randomly which firm to dispatch first, each with probability 1/2. For every peak hour, if the demand is not fully covered by the first firm, the other firm is also dispatched for the residual demand. The spot price during the peak hours is the bid of the *marginal firm* (the last firm dispatched). Each firm i produces up to capacity during peak hours; the excess energy produced and not dispatched by La Bolsa, is sold at a given price s_i, where $0 \leq s_i \leq p^*$. During the off-peak hours, each firm i sells q_i at price c_T.

Below, for $i = 1, 2$, I always use $j = 3 - i$ to denote firm i's opponent.

THEOREM 1: *Assume the capacities* (K_1, K_2) *are in the L-shaped region* $(0, 3) \times (0, \infty) \cup (0, \infty) \times (0, 3)$. *Then, the static game has a unique equilibrium. In this equilibrium, the firms choose their prices randomly in a common interval* $[\ell, p^*]$, *where* $\ell \geq \max\{s_1, s_2\}$, *and choose* $(q_1, q_2) = 21(K_1, K_2)$.

PROOF: Clearly, the strategic situation during peak hours is independent of the quantities q_1 and q_2. Moreover, since in this model the firms have no stock constraints, each firm will optimally choose to produce up to capacity during the off-peak hours, that is $q_i = 21K_i$.

Since firm i is guaranteed the price s_i during peak hours, it will never bid prices below s_i. Suppose that there is a pure strategy price equilibrium (p_1, p_2). It cannot be that $p_1 < p_2$, for example, because in this case, bidding $x \in (p_1, p_2)$ gives firm 1 a strictly higher expected profit than bidding p_1. Any bid $x < p_2$ ensures that firm 1 is dispatched first, but when demand is low $(d = 1)$, firm 1 is marginal and the spot price is set equal to x. Thus, $p_1 = p_2$. If $p_i = s_i$, firm i would prefer to bid p^* because doing so, firm i will still sell its energy at $s_i = p_i$ when demand is low, but will sell $\min\{K_i, 3 - K_j\}$ units at price p^* and the rest at price s_i when demand is high $(d = 3)$. Thus, we must have $p_1 = p_2$ and $p_i > s_i$, $i = 1, 2$. But then, La Bolsa is choosing randomly which firm to dispatch first. In this case, firm 1 would prefer to bid 1 cent less than p_1 and make sure to be dispatch first all the time. Therefore, the price equilibrium must be in mixed strategies, and firm i must choose its price p_i in a subinterval $[\ell_i, u_i]$ of $[s_i, p^*]$ randomly according with the distribution F_i.

We must have that $\ell_1 = \ell_2 = \ell$ and $u_1 = u_2 = u$. Suppose to the contrary that $\ell_1 < \ell_2$, for example. Consider any pair of bids $x, y \in [\ell_1, \ell_2)$ for firm 1, with $x < y$. Both x and y guarantee firm 1 that its bid is lower than that of firm 2. In either case, when demand is low, firm 1 is marginal and its bid becomes the spot price. Hence, firm 1 strictly prefers y to x. This is a contradiction. A similar argument shows that $u_1 = u_2 = u$, and that there cannot be any gaps: the support of each F_i is the whole interval $[\ell, u]$.

The distribution F_i cannot have point masses in $[\ell, u)$. By contradiction, suppose that F_2 has a point mass at $y \in [\ell, u)$. That is,

$$\mu = F_2(y) - \lim_{x \uparrow y} F_2(x) > 0.$$

For any $\epsilon > 0$ small, firm 1 must is willing to bid $x = y + \epsilon$. But clearly, the bid x gives firm 1 a strictly lower expected profit than y because it ensures that firm 1 is never dispatched first when firm 2 bids y, which happens with strictly positive probability.

Although F_i does not have point masses in $[\ell, u)$, F_i is not always absolutely continuous because it may have a point mass at u. The "density" of its absolutely continuous part is denoted by f_i, and the mass at u is denoted by μ_i. That is, for each $x \in [\ell, u)$,

$$F_i(x) = \int_\ell^x f_i(t)dt, \quad \text{and} \quad \mu_i = 1 - \int_\ell^u f_i(t)dt.$$

However, at least one of the two masses μ_1 and μ_2 must be 0. If $\mu_i > 0$ for $i = 1, 2$, then, for some $\epsilon > 0$, firm 1 would strictly prefer bidding $u - \epsilon$ than bidding u, which is a contradiction. Finally, we argue that $u = p^*$. Again by contradiction, assume that $u < p^*$ and $\mu_2 = 0$, for example. Then, bidding u or p^* guarantees firm 1 that its bid is the highest, and when demand is high, firm 1 will be marginal. But then, firm 1 strictly prefers bidding p^* than bidding u, another contradiction.

As we will see below, the condition that each firm must be indifferent among all bids in the interval $[\ell, p^*)$ leads to a pair of differential equations for F_1 and F_2. These differential equations with the terminal conditions $F_1(p^*) = F_2(p^*) = 1$ have unique solutions. ∎

The next theorem deals with the complementary case when $(K_1, K_2) \geq (3, 3)$, and its proof is omitted. In this case each firm can always cover all the demand. For technical reasons, in the theorem we also assume that there is a minimal unit of account, say one cent.

THEOREM 2: *Suppose that $(K_1, K_2) \geq (3, 3)$ and $s_1 < s_2$. Then, the unique equilibrium requires that firm 2 bids s_2 and firm 1 bid one cent less, and firm 2 is never dispatched. If $s_1 = s_2$, in equilibrium both firms bid their peak shadow prices of water, and each is dispatched with probability 1/2.*

Theorem 1 establishes that for $(K_1, K_2) \in (0, 3) \times (0, \infty) \cup (0, \infty) \times (0, 3)$, the model has a unique equilibrium and the firms must use random strategies. We now compute the equilibrium explicitly for any (K_1, K_2) in the region

$$R = \{\, (K_1, K_2) \in (1, 3) \times (1, \infty) \cup (1, \infty) \times (1, 3) \mid K_1 + K_2 \geq 3 \,\},$$

where we can express the expected profit of each firm in a single formula. The equilibrium for (K_1, K_2) in the complement of R (in the L-shaped region) is similar, but it involves a different formula for the firms' expected profits. In R, the capacity of each firm is larger than low demand, and the joint capacity is larger than high demand. By symmetry, without loss of generality, we assume that $(K_1, K_2) \in (1, \infty) \times (1, 3)$. We need to consider two cases: $K_1 < 3$ and $K_1 \geq 3$. In the former case, when demand is high and firm 1 bids less than firm 2, firm 1 is not able to cover all the demand. Since $K_2 < 3$ by assumption, firm 2 is never able to cover all the demand when demand is high.

Suppose $K_1 < 3$. When firm i bids the price $x \in [\ell, p^*)$ for the peak hours, it expects a profit for each of the peak hours equal to

$$\begin{aligned}\Pi_i(x) &= \frac{1}{2}\big[K_i s_i F_j(x) + \big((K_i - 1)s_i + x\big)(1 - F_j(x))\big] \\ &+ \ \frac{1}{2}\big[\big((K_1 + K_2 - 3)s_i + (3 - K_j)x\big)F_j(x) + K_i(p^*\mu_j + \int_x^{p^*} y f_j(y)dy)\big] \\ &= \frac{1}{2}\Big[x + K_i\Big(s_i F_j(x) + p^*\mu_j + \int_x^{p^*} y f_j(y)dy\Big) \\ &\quad + (K_j - 2)(s_i - x)F_j(x) + (K_i - 1)s_i\Big]. \end{aligned} \tag{1}$$

The term in the first line corresponds to the case the demand is low ($d = 1$), and the term in the second line to the case the demand is high ($d = 3$). In each of these two lines, there are two terms, corresponding respectively to the cases in which firm j's bid is lower and higher than that of firm i.

For firm i to be willing to randomize, it must be that its expected profit $\Pi_i(x)$ is constant in x. That is,

$$2\Pi_i'(x) = 1 + (2 - K_j)F_j(x) + [K_1 + K_2 - 2](s_i - x)f_j(x) = 0.$$

Assume $K_j \neq 2$, and let $\beta_j = (K_j - 2)/(K_1 + K_2 - 2)$. Then, the solution to the previous differential equation is

$$F_j(x; \alpha) = \frac{\alpha}{(x - s_i)^{\beta_j}} + \frac{1}{K_j - 2},$$

where α is a constant of integration. When $K_j = 2$, the differential equation becomes

$$1 + K_i(s_i - x)f_j(x) = 0 \quad \text{or} \quad f_j(x) = \frac{1}{K_i(x - s_i)}.$$

In this case,

$$F_j(x; \alpha) = \alpha + \frac{1}{K_i}\log(x - s_i),$$

where α is a constant of integration.

Now suppose that $K_1 \geq 3$. In this case, when firm 1 is the marginal firm, firm 2 is never dispatched by La Bolsa because firm 1 can cover all demand even when this is high. Hence

$$\begin{aligned}\Pi_2(x) &= \frac{1}{2}\big[K_2 s_2 F_1(x) + \big((K_2 - 1)s_2 + x\big)(1 - F_1(x))\big] \\ &+ \ \frac{1}{2}\big[K_2 s_2 F_1(x) + K_2(p^*\mu_1 + \int_x^{p^*} y f_1(y)dy)\big].\end{aligned}$$

Thus

$$2\Pi_2'(x) = 1 + (K_2 + 1)(s_2 - x)f_1(x) - F_1(x) = 0.$$

Let $\beta_1 = 1/(K_2+1)$. Then, the solution to this differential equation is

$$F_1(x;\alpha) = 1 + \frac{\alpha}{(x-s_2)^{\beta_1}} \,.$$

Let $R_0 = R \cap (1,3) \times (1,3)$, $R_1 = [3,\infty) \times (1,3)$, and $R_2 = (1,3) \times [3,\infty)$. The parameters ℓ and α for each distribution are determined as follows. For each i, let α_i be the constant of integration such that $F_i(p^*;\alpha_i) = 1$, and ℓ_i be such that $F_i(\ell_i;\alpha_i) = 0$. That is,

(K_1, K_2) in	α_i	β_i	ℓ_i
R_0 and $K_i \neq 2$	$-\frac{3-K_i}{K_i-2}(p^*-s_j)^{\beta_i}$	$\frac{K_i-2}{K_1+K_2-2}$	$(3-K_i)^{\frac{1}{\beta_i}}p^* + (1-(3-K_i)^{\frac{1}{\beta_i}})s_j$
R_0 and $K_i = 2$	$1-\frac{1}{K_j}\log(p^*-s_j)$		$e^{-K_j}p^* + (1-e^{-K_j})s_j$
R_i	0	$\frac{1}{K_j+1}$	$-\infty$

If $\ell_1 = \ell_2$, then $\ell = \ell_1 = \ell_2$, $\mu_1 = \mu_2 = 0$, and $F_i(x) = F_i(x;\alpha_i)$ for all $x \in [\ell, p^*]$, $i = 1,2$ (in this case, both F_i are absolutely continuous). If $\ell_i < \ell_j$, then $\ell = \ell_j$, $\mu_j = 0$, and $F_j(x) = F_j(x;\alpha_j)$ for all $x \in [\ell, p^*]$ (here F_j is absolutely continuous but F_i is not). Now, the parameter α_i needs to be changed so that $F_i(\ell;\alpha_i) = 0$. Then, $F_i(x) = F_i(x;\alpha_i)$ for all $x \in [\ell, p^*)$, and

$$\mu_i = 1 - \int_{\ell}^{p^*} f_i(x)dx,$$

where $f_n(x) = F_n'(x)$, $n = 1,2$.

EXAMPLE 1: Suppose $K_1 = K_2 = 2$ and $0 \le s_1 \le s_2 < p^*$. Then

$$\ell_i = e^{-2}p^* + (1-e^{-2})s_j \in (s_i, p^*),$$

and clearly $\ell_2 < \ell_1 = \ell$.

$$F_1(x) = 1 - \frac{1}{2}\log\left[\frac{p^*-s_2}{x-s_2}\right], \quad f_1(x) = \frac{1}{2(x-s_2)} \quad \text{and} \quad \mu_1 = 0,$$
$$F_2(x) = \frac{1}{2}\log\left[\frac{x-s_1}{\ell-s_1}\right], \quad f_2(x) = \frac{1}{2(x-s_1)} \quad \text{and} \quad \mu_1 = 1 - \frac{1}{2}\log\left[\frac{p^*-s_1}{\ell-s_1}\right].$$

Since $\Pi_i(x)$ is constant in $[\ell, p^*]$,

$$\Pi_i(x) = \Pi_i(p^*) = \frac{1}{2}[(1+2\mu_j)p^* + 3s_i].$$

Now, $q_i = 2 \times 21 = 42$ implies that firm i's total expected profit is $\Pi_i^* = 3\Pi_i(p^*) + 42c_T$.

Example 2: Suppose $K_1 = 4$ and $K_2 = 5/2$. Then $\beta_1 = 2/7$, $\beta_2 = 1/9$, $\ell_1 = -\infty$, and

$$\ell = \ell_2 = \frac{1}{512}p^* + \frac{511}{512}s_1 \in (s_1, p^*).$$

Thus,

$$F_2(x) = 2 - \left[\frac{p^* - s_1}{x - s_1}\right]^{\frac{1}{9}}, \quad f_2(x) = \frac{1}{9}\left[\frac{p^* - s_1}{(x - s_1)^{10}}\right]^{\frac{1}{9}} \quad \text{and} \quad \mu_2 = 0,$$

$$F_1(x) = 1 - \left[\frac{\ell - s_2}{x - s_2}\right]^{\frac{2}{7}}, \quad f_1(x) = \frac{2}{7}\left[\frac{(\ell - s_2)^2}{(x - s_2)^9}\right]^{\frac{1}{7}} \quad \text{and} \quad \mu_1 = \left[\frac{\ell - s_2}{p^* - s_2}\right]^{\frac{2}{7}}.$$

We now consider a model with three dominant firms. We have in mind here the case in which EEB, Chivor and EEPPM are the dominant firms and all other firms behave competitively. The total capacity of these three firms is 5030 MW. Assuming again that only about 86.4% of their installed capacity is available in average, their total (average) available capacity is about 4350 MW. This means that the total (average) available capacity for all the other firms fluctuates between 8400 - 4350 = 4050 MW and 9400 - 4350 = 5050 MW. We now define the peak hours to be the period between 6:00 pm and 10:00 pm, when demand ranges between 6000 MW and 7000 MW. If the competitive firms bid c_T throughout the day and the dominant firms bid above c_T during the peak hours, they enjoy a residual demand that fluctuates between 6000 - 5050 = 950 MW and 7000 - 4050 = 2950 MW. Although now the total available capacity of the dominant firms (4350 MW) is larger than the minimal demand during the day (3000 MW), I will keep the structure of the previous model and assume that the dominant firms can choose the amount of energy they sell during off-peak hours. To avoid excesive complexities, I will only work here with a symmetric model in which all dominant firms have the same capacity and the same shadow price for water.

The Model: Three hydroelectric firms with capacity K each and 0 marginal costs, simultaneously choose vectors $(p_i, q_i) \in [0, p^*] \times [0, 20K]$, $i = 1, 2, 3$. Demand is random during peak hours:

$$d = \begin{cases} 3 & \text{with probability } \frac{1}{2} \\ 5 & \text{with probability } \frac{1}{2}. \end{cases}$$

During peak hours, the firms are dispatched by La Bolsa in the order of their bids, starting with the lowest bid firm, until demand is satisfied. The spot price is the bid of the marginal firm (the last firm dispatched). The firms produce up to capacity during the peak hours and the production that is not dispatched by La Bolsa is bought at a given price s by an artificial agent.

We want to study how the nature of the equilibrium changes as the excess capacity increases. In our model, this is equivalent to increasing K. We consider two cases: $K \in [1.5, 2.5)$, $K \in [2.5, \infty)$. In the first case, one firm can never cover demand alone, and two firms can cover demand when this is low but not when this is high. In the second case, two firms can always cover demand.

When $K \in [1.5, 2.5)$, there are no equilibria in pure strategy for the bidding game. In the Appendix we construct a symmetric mixed strategy equilibrium in which each firm draws a bid price in $[s, p^*]$ randomly with the (absolutely continuous) distribution F with density f. This is in fact the only equilibrium in this case. Thus, the equilibrium exhibits the same characteristics as that for the two firm model. Since the equilibrium for the two firm model has a simpler expression (which we can compute in closed form), in the next section we restrict attention to a two firm model.

However, the nature of the equilibrium changes when $K \geq 2.5$. In this case, in the unique equilibrium of the model, each firm bids s. Therefore, the equilibrium is in pure strategies. By contradiction, suppose that there exists a mixed strategy equilibrium (not necessarily symmetric) where the firms randomize their bids in an interval $[\ell, u]$ with $u > \ell$. For a similar argument to that in the proof of Theorem 1, the support of the firms' bidding distributions must coincide. Suppose that the distributions have no point masses at u; the argument when they do is similar. Then, bidding u assures firm i that its bid is the largest, and that it will never be dispatched since the other two firms can always cover demand. Hence, its expected profit is Ks. But then, offering the price $u - \epsilon$ for some small $\epsilon > 0$ is strictly better. In this case there is a positive probability that the firm will be dispatch at prices greater or equal to $u - \epsilon$, and all the energy that the firm does not dispatch, it will still sell at the price s.

It is interesting to note that the Colombian situation corresponds approximately to the boundary case between the regions where the pure and the mixed strategy equilibria attains. Recall that we estimated that the residual demand for EEB, EEPPM and Chivor fluctuates in the interval $[950, 2950]$. If we assume that the totality of their installed capacity is always available, then EEB with either EEPPM or Chivor can always cover demand, but Chivor with EEPPM cannot. If only 86.4% of installed capacity is available, then EEB with Chivor cannot always cover demand. The situation is not symmetric, as we have assumed in the model. With the estimated parameters, probably EEB should not bid s at peak hours in equilibrium. If both EEPPM and Chivor bid s and have available all of their installed capacity (2710 MW), EEB must be dispatched for 2950 - 2710 = 240 MW when demand is high, even if it bids p^*. Assuming that EEB can guarantee the dispatch of all its capacity if it bids c_T, EEB may still prefer to bid p^* if $240p^* > 2320c_T$ (recall that 2320 MW is EEB's installed capacity). Finally, when peak demand is higher, around 7400 MW (as it was in October 1997), then the residual demand fluctuates in $[950, 3350]$, and then EEB must be dispatched for 3350 - 2710 = 640 MW. In this case EEB may prefer to bid p^* if $640p^* > 2320c_T$.

The equilibrium with four or more dominant firms is not qualitatively different from that for the model with three firms. Depending on the firm's capacities, the pure strategy or the mixed strategy equilibria may attain.

4. Dynamic Model with Limited Storage Capacity

We now extend the two firm model of the previous section to a dynamic setting in which the dominant firms have limited storage capacity, and the expected rainfall may be insufficient to allow each dominant firm to operate up to capacity in every hour and every day. We confine ourselves to the case where each firm has capacity of 1 GW. In the static model, the dominant firms together dispatch through La Bolsa an average of $0.5 \times 1 + 0.5 \times 3 = 2$ units (recall that 1 unit = 0.5 GW) during each peak hour, and 2 GWh during each off-peak hour. If we assume that there are m peak hours in every day, this means that each firm dispatches in average $m/2$ GWh during peak hours and $24 - m$ GWh during off-peak hours (per day).

Suppose that after the Colombian winter (the wet season), each dominant firm i has stored $S_{i,1}$ GWh of water in their reservoirs, and during the Colombian summer (the dry season), there is no rain fall. The summer lasts T days. The interesting case is when

$$\frac{m}{2}T < S_{i,1} < \left(24 - \frac{m}{2}\right)T, \quad i = 1, 2.$$

In this case, each dominant firm can cover half the demand during peak hours for every day, but cannot do this and always produce up to capacity during off-peak hours.

In the dynamic model, the firms compete in La Bolsa in a similar fashion as that described in the stationary model, with two exceptions:

(i) There is no artificial agent that buys during peak hours the excess energy that the firms produce and do not dispatch with La Bolsa.

(ii) The day starts with the m peak hours and continues with the $24 - m$ hours of "low demand". Each day, the firms choose bid prices and quantities *sequentially.* The firms choose the quantities they sell during off-peak hours *after* they learn the quantities they dispatch during peak hours.

Modification (ii) represents an additional simplification that facilitates the analysis of the model substantially.

As we did before, in this model we make again the units of account equal to 500 MW for power and 500 MWh for energy.

The Model: At the beginning of each day $t = 1, \ldots, T$, the firms simultaneously bid prices $(p_{1,t}, p_{2,t})$ for the peak hours. Demand for each of the m peak hours is random and equal to 1 or 3 with probability 1/2 each. The quantities demanded in each peak hour are independent random variables. During peak hours, La Bolsa dispatches the dominant firms in the order of their price bids, starting with the lowest bidder, until production satisfies the demand. At all times, the firms produce exactly the amount they dispatch through La Bolsa.[9] Every peak hour, the marginal firm is the last to be dispatched, and the spot price is equal to its bid. After observing the amounts of energy $(q^1_{1,t}, q^1_{2,t})$ dispatched by the firms during the peak hours, the firms simultaneously choose quantities $(q^2_{1,t}, q^2_{2,t})$ to produce during the off-peak hours. The price of electricity is constant and equal to c_T during off-peak hours. Firm i's quantity choice $q^2_{i,t}$ is constrained by production capacity and reserves. Therefore, $q^2_{i,t} \in [0, \min\{48 - 2m, S_{i,t} - q^1_{i,t}\}]$. At the end of each day, firm i's reserves drop by the amount of energy it sold during the day: $S_{i,t+1} = S_{i,t} - q^1_{i,t} - q^2_{i,t}$. Firms discount future profits by the daily discount factor $\delta \in (0, 1)$.[10]

[9] If at some point a firm's reserves drop to 0, the firm cannot dispatch energy in subsequent hours, even if its price bid is the lowest.

[10] We may think that $\delta = (1 + r)^{-1}$, where r is the daily interest rate.

In this model we abstract from the intervention of the reservoirs problem,[11] and assume that the firms control their bid prices and the energy they offer at all times, as long as they have water in their reservoirs.

Obviously, in this dynamic game, each firm's shadow price of water depends on the reserves of both firms and the number of days left before the wet season. Let's denote by $V_{i,t}^1(k,\ell)$ the total discounted profit that firm i expects in equilibrium from day t through day T when the initial energy stocks are given by the vector $(S_{1,t}, S_{2,t}) = (k,\ell)$. Similarly, let $V_{i,t}^2(k,\ell)$ denote the corresponding expected value at the beginning of the off-peak hours, when $(S_{1,t} - q_{1,t}^1, S_{2,t} - q_{2,t}^1) = (k,\ell)$.

Assume that at the beginning of day t, $(S_{1,t}, S_{2,t}) = (k,\ell) \geq (2m, 2m)$.[12] In equilibrium, each firm chooses its price $p_{i,t} \in [\ell_t, p^*]$ randomly according with the distribution $F_{i,t}$ (with density $f_{i,t}$ and point mass $\mu_{i,t}$). When firm i chooses the price x, it expects a total profit from t through T equal to

$$\begin{aligned} m\Pi_{1,t}(x) = &\frac{1}{2^m} F_{2,t}(x) \sum_{n=0}^{m} \binom{m}{n} \big[nx + V_{1,t}^2(k-n, \ell-(m+n))\big] \\ &+ \frac{1}{2^m}(1 - F_{2,t}(x)) \sum_{n=0}^{m} \binom{m}{n} \big[(m-n)x + V_{1,t}^2(k-(m+n), \ell-n)\big] \\ &+ \frac{1}{2^m} \sum_{n=0}^{m} \binom{m}{n} 2n \big(p^* \mu_{2,t} + \int_x^{p^*} y f_{2,t}(y)dy\big). \end{aligned}$$

The first line has the following interpretation. Firm 2 chooses a price below x with probability $F_{2,t}(x)$. The probability that $d = 3$ in n out of the m peak hours (and that $d = 1$ in the other peak hours) is

$$\binom{m}{n} \frac{1}{2^m}.$$

When $d = 3$ in n of the m peak hours, firm 1 dispatches n units at a price x, and firm 2 dispatches $2n$ at price x and $(m-n)$ at its own bid price. The second and third line correspond to the case in which firm 2's bid is higher than x. In the last line, for example, when $d = 3$ in n of the m peak hours, firm 1 dispatches $2n$ units at firm 2's bid price.

Define

$$\begin{aligned} v_{1,t}^H &= \frac{1}{2^m} \sum_{n=0}^{m} \binom{m}{n} V_{1,t}^2(k-n, \ell-(m+n)) \\ v_{1,t}^L &= \frac{1}{2^m} \sum_{n=0}^{m} \binom{m}{n} V_{1,t}^2(k-(m+n), \ell-n) \\ s_{1,t} &= \frac{1}{m}(v_{1,t}^H - v_{1,t}^L). \end{aligned}$$

[11] See Section 6 below for a discussion of this issue.

[12] If $S_{i,t} < 2m$, firm i does not have enough stock to produce up to capacity during all the peak hours of day t. This affects the expected profits of both firms, and the expressions for $\Pi_{1,t}$ and $\Pi_{2,t}$ below must be modified accordingly.

Since

$$\frac{1}{2^m}\sum_{n=0}^{m}\binom{m}{n} = 1 \qquad \text{and} \qquad \frac{1}{2^m}\sum_{n=0}^{m}\binom{m}{n}n = \frac{m}{2},$$

we can simplify the expression for $\Pi_{1,t}(x)$ as follows:

$$\Pi_{1,t}(x) = \frac{1}{2}\left[x + 2\Big(s_{1,t}F_{2,t}(x) + p^*\mu_{2,t} + \int_x^{p^*} y f_{2,t}(y)dy\Big) + \frac{2}{m}v_{1,t}^L\right]. \tag{2}$$

$\Pi_{2,t}(x)$, $v_{2,t}^H$, $v_{2,t}^L$, and $s_{2,t}$ are defined analogously. The definition of $s_{i,t}$ has been made so equation (2) parallels equation (1) (with $K_j = 2$), and $s_{i,t}$ is the *peak shadow price of water* for firm i in day t.

Since firm i is willing to choose prices randomly, we have that

$$2\Pi'_{i,t}(x) = 1 + 2(s_{i,t} - x)f_{j,t}(x) = 0 \quad \text{or} \quad f_{j,t}(x) = \frac{1}{2(x - s_{i,t})}.$$

By Example 1 of the previous section, if $s_{i,t} \le s_{j,t}$, we have that

$$\ell_t = e^{-2}p^* + (1 - e^{-2})s_{j,t}$$
$$F_{i,t}(x) = 1 - \frac{1}{2}\log\left[\frac{p^* - s_{j,t}}{x - s_{j,t}}\right] \quad \text{and} \quad \mu_{i,t} = 0$$
$$F_{j,t}(x) = \frac{1}{2}\log\left[\frac{x - s_{i,t}}{\ell_t - s_{i,t}}\right] \quad \text{and} \quad \mu_{j,t} = 1 - \frac{1}{2}\log\left[\frac{p^* - s_{i,t}}{\ell_t - s_{i,t}}\right].$$

Obviously, ℓ_t, $s_{i,t}$, $f_{i,t}$ and $F_{i,t}$ are all functions of the initial stocks (k, ℓ), but to keep the notation simple, we have omitted an explicit reference to these parameters.

In equilibrium, $\Pi_{i,t}(x)$ is constant in $[\ell_t, p^*]$. Therefore

$$V_{i,t}^1(k,\ell) = m\Pi_{i,t}(p^*) = \frac{m}{2}(1 + 2\mu_{j,t})p^* + v_{i,t}^H. \tag{3}$$

That is,

$$V_{1,t}^1(k,\ell) = \frac{m}{2}(1 + 2\mu_{2,t})p^* + \frac{1}{2^m}\sum_{n=0}^{m}\binom{m}{n}V_{1,t}^2(k - n, \ell - (m + n)) \tag{4}$$

$$V_{2,t}^1(k,\ell) = \frac{m}{2}(1 + 2\mu_{1,t})p^* + \frac{1}{2^m}\sum_{n=0}^{m}\binom{m}{n}V_{2,t}^2(k - (m + n), \ell - n). \tag{5}$$

For any $(k,\ell) \ge 0$, consider the simultaneous moves game G where player 1 chooses $q_1 \in [0, \min\{48 - 2m, k\}$ and player 2 chooses $q_2 \in [0, \min\{48 - 2m, \ell\}$, and their payoffs are given by

$$W_i(q_1, q_2) = c_T q_i + \delta V_{i,t+1}^1(k - q_1, \ell - q_2), \quad i = 1, 2. \tag{6}$$

If $(q^2_{1t}, q^2_{2,t})$ is a Nash equilibrium of this game, then $V^2_{i,t}(k,\ell) = W_i(q^2_{1,t}, q^2_{i,t})$, $i = 1,2$. Obviously, $q^2_{1,t}$ and $q^2_{2,t}$ are also functions of (k,ℓ).

Equations (4)–(6) define a system of difference equations which can be solved recursively for each day $t = T, T-1, \ldots, 1$ to obtain the functions $V^1_{i,t}$ and $V^2_{i,t}$, beginning with

$$V^2_{1,T}(k,\ell) = c_T \cdot \min\{48 - 2m, k\} \quad \text{and} \quad V^2_{2,T}(k,\ell) = c_T \cdot \min\{48 - 2m, \ell\}.$$

Although the procedure to compute these functions is straightforward, they do not exhibit any stationarity which would allow us to find a simple functional representation for them. However, when (k,ℓ) is large enough so both firms can produce up to capacity in every hour from the current day through day T, we can find these functions explicitly.

THEOREM 3: *Suppose that t and (k,ℓ) are such that $(k,\ell) \geq 48(T-t+1)(1,1)$. Then, the peak shadow price of water for both firms in day t is 0, though the average price at which they sell each unit of energy during peak hours is $p^*/2$ for every day $t+1,\ldots,T$.*

PROOF: Suppose $(k,\ell) \geq 48(T-t+1)(1,1)$. Since both firms have the same production capacities and their stock constraints are not binding, $s_{1,t} = s_{2,t}$. Therefore,

$$\mu_{1,t} = \mu_{2,t} = 0, \quad f_{1,t}(x) = f_{2,t}(x) = \frac{1}{2x},$$
$$F_{1,t}(x) = F_{2,t}(x) = 1 - \frac{1}{2}\log\left[\frac{p^*}{x}\right], \quad \text{and} \quad \ell = e^{-2}p^*.$$

By symmetry, both firms expect to sell their energy at the same average price $\hat{p}$ every day (this average price includes the energy sold during off-peak hours). Also, every day, each firm expects to sell m units of energy during peak hours, and since its stock constraint is not binding, will optimally sell $48 - 2m$ units of energy during the off-peak hours. Thus, in the regions $(k,\ell) \geq (T-t+1)(48-2m)(1,1)$ and $(k',\ell') \geq (T-t)(48-2m)(1,1)$, $V^1_{i,t}(k,\ell)$ and $V^2_{i,t}(k',\ell')$ are constant, and omitting the arguments, we get

$$V^1_{i,t} = \hat{p}(48-m)(1+\delta+\cdots+\delta^{T-t}) = \hat{p}(48-m)\left[\frac{1-\delta^{T-t+1}}{1-\delta}\right] \tag{7}$$
$$V^2_{i,t} = (48-2m)c_T + \delta V^1_{i,t+1} = (48-2m)c_T + \delta\hat{p}(48-m)\left[\frac{1-\delta^{T-t}}{1-\delta}\right]$$
$$v^H_{i,t} = v^L_{i,t} = V^2_{i,t} \quad \text{and} \quad s_{i,t} = 0.$$

By (3), we have that

$$V^1_{i,t} = \frac{m}{2}p^* + (48-2m)c_T + \delta\hat{p}(48-m)\left[\frac{1-\delta^{T-t}}{1-\delta}\right],$$

and comparing this expression with (7), we obtain

$$\hat{p}(48-m) = \frac{m}{2}p^* + (48-2m)c_T \quad \text{or} \quad \hat{p} = \frac{(m/2)p^* + (48-2m)c_T}{48-m}.$$

Since firm i sells on average m units during peak hours and $48 - 2m$ units during off-peak hours (at price c_T), this implies that firm i sells its output during peak hours at an average price of $p^*/2$. ∎

When $(k, \ell) \geq 48(T - t + 1)(1, 1)$, every day $t, t + 1, \ldots, T$, the firms produce up to capacity during off-peak hours, and cover, on average, m units of demand during peak hours. Thus, those units that a firm was prepared to dispatch during peak hours, but did not dispatch, cannot be sold at any time in the future. Accordingly, the peak shadow price of water is 0.

Observe that when the firms are not constrained by their stocks, the average spot price during peak hours is not $p^*/2$. The reason is that when demand is high, the price is (in average) higher than when demand is low. So prices and quantities are correlated: more energy is sold when prices are higher. Therefore, the average spot price during peak hours is lower than $p^*/2$. Let p_t denote the spot price during peak hours in day t. Then, since $F_{1,t} = F_{2,t}$,

$$P[p_t \leq x] = \frac{1}{2}F_{1,t}(x)[2 - F_{1,t}(x)] + \frac{1}{2}[F_{1,t}(x)]^2 = F_{1,t}(x),$$

and we have that

$$E[p_t] = \int_{\ell}^{p^*} x f_{1,t}(x)dx = \frac{1}{2}[p^* - \ell] = \frac{p^*}{2}(1 - e^{-2}).$$

Another extreme case we can solve explicitly corresponds to the situation where the total water in the reservoirs is never enough to cover the total demand during peak hours for the remaining days.

THEOREM 4: *Assume that $S_{1,t} + S_{2,t} \leq (T - t + 1)m$. Then, the peak shadow price of water for both firms is p^* and the spot market price remains equal to p^* through the peak hours for each day $t, t + 1, \ldots, T$.*

When $S_{1,t} + S_{2,t} \leq (T - t + 1)m$ both firms are guaranteed to dispatch all their energy during peak hours. Indeed, when the above inequality is strict, even if every peak hour demand is low, there will be rationing. Therefore, the firms should bid p^* all the time and dispatch no energy during off-peak hours.

Intuitively, we expect that for $\tau \in \{1, 2\}$, $k < k'$ and $\ell < \ell'$,

(i) $V^{\tau}_{1,t}(k, \ell) < V^{\tau}_{1,t+1}(k, \ell)$, $V^{\tau}_{1,t}(k, \ell) \leq V^{\tau}_{1,t}(k', \ell)$ and $V^{\tau}_{1,t}(k, \ell) \geq V^{\tau}_{1,t}(k, \ell')$.

(ii) $V^{\tau}_{2,t}(k, \ell) > V^{\tau}_{2,t+1}(k, \ell)$, $V^{\tau}_{2,t}(k, \ell) \geq V^{\tau}_{2,t}(k', \ell)$ and $V^{\tau}_{2,t}(k, \ell) \leq V^{\tau}_{2,t}(k, \ell')$.

We also expect that, at the same stocks, the peak shadow price of water decreases with time because having fewer days left, there are fewer opportunities to place production during peak hours. On the other hand, if the stock of either firm is increased, the peak shadow prices of water for both firms should decrease because the additional water in the reservoirs increases the energy the firms can produce, and hence lowers the average spot market price for the remaining days. These two effects tend to offset each other: in equilibrium, as the game moves from one period to the next, stocks decrease by the firms' productions and the time horizon decreases by one day.

In the dynamic model we have made two assumptions that depart from the way in which La Bolsa really operates: (1) the firms offer a *unique* bid for all the peak hours; and (2) the firms make their strategic choices for the off-peak hours *after* they learn the outcome of the peak hours. Fortunetely, these divergences tend to partially cancel each other. In La Bolsa, each firm places a bid for each of the 24 hours of the following day, and thus does not have any information about the outcome of the peak hours at the time it makes its bids for the off-peak hours. However, by choosing every hour's bid separatedly, the firm can control more precisely the amount of energy it sells during the day.

5. Futures Contracts

Futures contract positions modify the firms' incentives we have studied in the previous model. Consider an unrealistic situation in which a producer can predict precisely the amount of energy it dispatches each day. Suppose the firm has sold 100% of its capacity in futures contracts at a price π (which we assume constant for simplicity), strictly higher than its marginal cost c. This firm is not affected by the spot market prices and its total profit is constant as long as it is dispatched to capacity all the time. To ensure this will happen, the firm must bid a price always below the spot price. If the firm bids above the spot price p at some point, the firm is not dispatched and must buy energy in La Bolsa to meet its obligation in the futures contracts. Assuming that the spot price is higher than its marginal cost ($p > c$), the firm gets a markup of $\pi - p$ instead of the markup $\pi - c$ it would get if it were dispatched. Since $\pi - c > \pi - p$, the firm prefers to produce the energy to meet its obligation. Therefore, the firm should always bid its marginal cost c. If all the firms have sold in futures contracts 100% of the energy they will dispatch, all the firms should be bidding their marginal costs and we would expect a stable, relatively low spot price.

Now, consider a firm that has oversold its capacity in futures contracts. This firm will have to buy energy in the spot market to meet its obligation in futures contracts. Therefore, this firm would like the spot prices to be low, and therefore it should bid 0. For the energy the firm sold in futures contracts in excess of the amount it produces, the firm makes a profit if the spot price is lower than its futures contract's price, and a loss otherwise. Therefore, assuming that the spot price remains above its marginal cost, the firm has now two incentives to offer low bids: it wants to be dispatched up to capacity, and it wants to lower the spot price to buy more cheaply the energy it needs to cover its position in futures contracts.

Our model, that has ignored the incentives induced by the futures contracts, concludes that the dominant firms must constantly bid during peak hours prices above their peak shadow price of water. Many believe that EEPPM underestimated the extent of the 1997-1998 drought who would have taken a short position in the futures market. This may explain its passive bidding strategy during the 1997-1998 drought.

6. Anecdotal Evidence

In 1997, El Niño produced a severe drought in Colombia. In December 1997, 15 water reservoirs were controlled by CREG, involving 73% of the production capacity. The current regulation (resolutions CREG-025 of 1995 and CREG-215 of 1997) stipulates two critical levels for each water reservoir: the inferior and superior operative levels. When the level of a water reservoir drops to the superior operative level (the higher of the two), CREG assumes control of the reservoir and the bid prices of the corresponding power plant(s) are set equal to the maximal bid among all the power plants of other firms that are not currently controlled by CREG.[13] This mechanism is supposed to guarantee that plants whose water reservoirs are below the superior operative level are dispatched last, and only if they are necessary to satisfy demand. When the level of the water reservoir drops to the inferior operative level, the corresponding power plant is effectively shut down; water is only released to satisfy other needs, like irrigation downstream. In practice, this regulation exacerbates the tendency to offer prices close to p^* because it reduces competition.

At the beginning, the firms still competing in prices continued the bidding strategies they were using prior to CREG's intervention. But, the firms that were not controlled by CREG, indirectly determined the bidding of the controlled firms, making it even more attractive to increase their own bidding prices. With such an important fraction of the reservoirs being controlled, smaller firms that normally behave competitively, began to enjoy market power. As we have mentioned earlier, the bigger firms are mainly hydroelectric producers. Since many hydroelectric plants were unable to bid, the thermoelectric producers were now controlling the spot price. From January 22, two thermoelectric plants (Flores 2 and Ballenas 2) increased their prices sharply. Flores, that previously was bidding prices around 25 pesos/KWh, started to bid prices as high as 200 pesos/KWh. Meanwhile, Ballenas increased its prices from about 25 pesos/KWh to 90 pesos/KWh. CREG reacted, issued a new resolution (CREG-018), and started legal action against Flores and Ballenas for violating various CREG resolutions. As we discussed in Section 2, the current regulation states that the thermoelectric producers' bids must reflect their variable costs, and those of the hydroelectric producers must reflect their water opportunity costs. The hydroelectric producers have bid prices close to 250 pesos/KWh (recall that $p^* = 267$ pesos/KWh) during the drought months without provoking any reaction from CREG. Thus, the prices bid by Flores and Ballenas were not exaggerated compared to those bid by the hydroelectric producers. The events of January and February 1998 further corroborated CREG's determination to apply the regulation, and the difficulty of imposing any meaningful bound on the hydroelectric producers' bids. These events also exposed the market presures in the Colombian electricity market and raised questions about the fitness of the system.

7. Conclusions

The paper investigates the nature and the extent of the producers' market power in the Colombian electricity sector. Our model suggests that the firms' market power originates with the small slack in production capacity as well as with large shares of firms' capacity. Thus, for example, had ten firms shared the production capacity equally and had peak

[13] In an attempt to curve down prices, in February 1998, CREG passed a new resolution (CREG-018) that modified the rules for setting the prices of the controlled plants.

demand been equal to total available capacity, then each firm would have enjoyed *full* market power. That is, each firm would be guaranteed to produce up to capacity (during peak hours) regardless of the price it bids, and in equilibrium all the firms would bid p^*. On the opposite extreme, if there are only two firms, each with enough capacity to satisfy demand by itself, the firms would have no market power, and in equilibrium would bid their marginal costs (assuming they are the same). The proposition of building excess capacity to reduce the firms' market power is, of course, very unattractive.

As we discussed before, in the transition period, La Bolsa has been shackled with special constraints. Distributors and commercial companies have been required to cover a fraction of their demand with future contracts. Accordingly, the producers have not been very aggressive in their bidding strategies. But their behavior will most likely change once this requirement is relaxed. With the privatization of the industry and the expansion of installed capacity to match the demand growth, the distribution of market share will also change.

The resolution CREG-128 of 1996 establishes a 25% limit to the fraction of total installed capacity that any single firm can own. In 1997, EEB, EEPPM and Chivor owned respectively 21.9%, 16.1% and 9.4%. However, critical combinations of capacities already seemed to be present with those market shares. Therefore, the regulatory comission should assess carefully any capacity expansions by or transfer of public assets to the largest producers.

In the model, what drives the aggressive bidding and the high spot prices is the fact that during peak hours, dispatching *all* the dominant firms may be required to satisfy demand. The aggressive bidding strategies are not sustainable anymore when any $N-1$ of the N dominant firms are sufficient to cover peak demand. This provides a point of reference for future decisions and regulation.

Appendix

Equilibrium for the Three Firm Model: For $K \in [1.5, 2.5)$, we construct a symmetric mixed strategy equilibrium in which each firm draws a bid in $[s, p^*]$ randomly with the (absolutely continuous) distribution F with density f. This is in fact the only equilibrium when $K \in [1.5, 2.5)$. For simplicity, suppose that $K = 2$. Then, when firm 1 bids a price $x \in [s, p^*]$, its expected profit for each peak hour is

$$\begin{aligned}\Pi(x) &= \frac{1}{2}\int_x^{p^*} 4y(1-F(y))f(y)dy + \frac{1}{2}\int_x^{p^*} 4y(F(y)-F(x))f(y)dy \\ &+ \ \frac{1}{2}\big[x+s\big]2F(x)(1-F(x)) + \frac{1}{2}\int_x^{p^*} 4yF(x)f(y)dy \\ &+ \ \big[\frac{1}{2}2s + \frac{1}{2}(x+s)\big]F(x)^2 \\ &= 2\int_x^{p^*} yf(y)dy + \big[x+s\big]F(x)(1-F(x)) + \frac{1}{2}\big[x+3s\big]F(x)^2.\end{aligned}$$

The first three lines of the expression above correspond respectively to the cases in which x is the lowest, the middle, and the largest bid of the three firms. In the first line, for example, the integrals corresponds respectively to the cases when demand is low ($d = 3$) and demand is high ($d = 5$). The spot price in La Bolsa is given by the lowest bid b between firms 2 and 3 when demand is low, and by the highest bid B between firms 2 and 3 when demand is high. For $y > x$, we have

$$\begin{aligned}P[b \le y \mid b > x] &= 1 - P[b > y \mid b > x] = 1 - \frac{(1-F(y))^2}{(1-F(x))^2} \\ P[B \le y \mid b > x] &= \frac{(F(y)-F(x))^2}{(1-F(x))^2}\ .\end{aligned}$$

For the second term in the second line, we note that

$$P[B \le y \mid b < x < B] = \frac{2F(x)(F(y)-F(x))}{2F(x)(1-F(x))}\ .$$

Since firm 1 is willing to randomize, $\Pi(x)$ must be constant in the interval $[s, p^*]$. Therefore

$$2\Pi'(x) = F(x)(2-F(x)) - 2(x-s)(1+F(x))f(x) = 0.$$

The solution of this differential equation is implicitly given by

$$(2-F(x))^3 = \frac{\alpha}{x-s}F(x),$$

where α is a constant of integration. Since $F(p^*) = 1$, $\alpha = p^* - s$, and the previous equation can be written as

$$\left[\frac{x-s}{p^*-s}\right](2-F(x))^3 = F(x).$$

That is, $F(x)$ is a root of a cubic polynomial. The roots of this polynomial can be computed explicitly. Since two of its roots are complex, $F(x)$ is the only real root. Let

$$z = \frac{x-s}{p^*-s}, \quad D = 324z^3 - 27z^2(1+12z), \quad \text{and} \quad E = \left[D + \sqrt{27z^3 + D^2}\right]^{\frac{1}{3}}.$$

The variable z represents a linear rescaling of the bids; since the bids range in the interval $[s, p^*]$, z takes values in the interval $[0, 1]$. The real root of the polynomial is then

$$F(x) = 2 + \frac{E}{3z} - \frac{1}{E}.$$

F and f are plotted below as functions of z.

We have that

$$\int_0^1 F(z)dz = \frac{3}{4} \quad \text{y} \quad \int_0^1 2zF(z)dz = 0.870833.$$

Therefore, the mean and variance of z are

$$E[z] = \int_0^1 zf(z)dz = 1 - \int_0^1 F(z)dz = \frac{1}{4}$$
$$V[z] = E[z^2] - (E[z])^2 = \int_0^1 z^2 f(z)dz - \frac{1}{16} = 1 - \int_0^1 2zF(z)dz - \frac{1}{16} = \frac{1}{15},$$

which implies that

$$E[x] = \frac{1}{4}\left[p^* + 3s\right] \quad \text{y} \quad V[x] = \frac{1}{15}\left[p^* - s\right]^2.$$

Since $\Pi(x)$ is constant in $[s, p^*]$,

$$\Pi(x) = \Pi(p^*) = \frac{1}{2}[p^* + 3s].$$

Each firm i will optimally choose $q_i = 40$, and therefore its total expected profit (for a day) is

$$\Pi^* = 4\Pi(p^*) + 40c_T.$$

References

Ausubel, L. M. y P. C. Cramton (1996): "Demand Reduction and Inefficiency in Multi-Unit Auctions," mimeo, University of Maryland.

Green, R. J. y D. M. Newbery (1992): "Competition in the British Electricity Spot Market," *Journal of Political Economy*, **100**, 929–953.

ISA (1996): "Informe de Operación 1996," Colombia.

ISA (1997): "Análisis del Mercado Mayorista de la Electricidad en Colombia," Colombia.

Klemperer, P. D. y M. A. Meyer (1989): "Supply Function Equilibria in Oligopoly under Uncertainty," *Econometrica*, **57**, 1243–1277.

Vickers, J. S. y G. Yarrow (1991): "The British Electricity Experiment," *Economic Policy*, **12**, 188-232.

von der Fehr, N-H. M. y D. Harbord (1993): "Spot Market Competition in the UK Electricity Industry," *Economic Journal*, **103**, 531–546.

Wolak, F. A. y R. H. Patrick (1997): "The Impact of Market Rules and Market Structure on the Price Determination Process in England and Wales Electricity Market," mimeo, Stanford University.

Wolfram, C. D. (1998): "Strategic Bidding in a Multi-Unit Auction: An Empirical Analysis of Bids to Supply Electricity in England and Wales," *RAND Journal of Economics*, **29**, 703–725.

Wolfram, C. D. (1999): "Measuring Duopoly Power in the British Electricity Spot Market," forthcoming, *American Economic Review*.