Competition Among Mechanism Designers in a Common Value Environment

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April 17, 1998

Abstract

A competitive economy is studied in which sellers offer alternative direct mechanisms to buyers who have private information about their own private use value for the commodity being traded. In addition the commodity has a common value to all buyers, perhaps represented by the future resale value of the commodity. A competitive equilibrium in mechanisms is described. In every such equilibrium it is shown that sellers must offer mechanisms that are allocationally equivalent to English ascending price auctions. The reservation prices that sellers set are shown to be below their ex post cost of trading the commodity.

1. Introduction

One of the variables that auction designers can use to extract buyer surplus is the reserve price. Increasing the reserve price raises the price that the winning bidder will pay on average at the cost of reducing the number of low valuation bidders who participate. Observed reserve prices are often very different from what the theory predicts that they should be. For example [6] report that reserve prices for real estate auctions ran as low as 50% of the appraised value of houses being sold. More often than not, reserve prices are kept secret. For example [3] study first price sealed bid timber auctions in France where reserve prices are secret.

Explanations for these phenomena usually have something to do with buyer participation (for example [9]). For instance, low or undeclared reserve prices encourage participation by low valuation buyers whose behavior might convey valuable information to higher valuation bidders worried about the winner's curse. The trade-off between the reserve price and buyer participation is also central to the argument in [1] who show that in a common value environment similar to the one discussed in this paper, a seller should always be willing to switch from the optimal reserve price to a reserve price equal to his cost, if this deviation is sure to bring the seller one additional bidder.

The idea that reserve prices might affect participation suggests that bidders have alternatives, or that the auction environment is at least partly competitive. In the real estate auctions discussed in [6], for example, buyers always have the option of buying on the usual real estate market instead of participating in the Federal Deposit Insurance Corporation Auctions. In the takeover procedure discussed by [1], potential buyers for a firm have many alternative ways to invest their money. Certainly participants in timber auctions can procure alternative lumber supplies in nearby regions.

These considerations suggest that the appropriate way to model the trade-off between seller reserve prices and buyer participation might be in a competitive model. Recently ([5, 8]) a competitive model of competition among mechanism designers has been developed for the case where buyer valuations are independently distributed. The difficulty that arises in applying these results is that the real estate and share auctions discussed above both have important common value aspects that have not been addressed in the literature.

The purpose of this paper is to apply the competitive approach suggested in the literature to a problem where objects traded have an important common value component. We allow sellers to use any negotiation scheme that they find profitable. Provided that the seller's valuation for the object being traded is not too sensitive to buyers' information, all sellers use mechanisms that are allocationally equivalent to English auctions with reserve prices. Attention is focussed on the reserve prices that prevail in this equilibrium. They are set equal to the seller's expected opportunity cost of sale conditional on the event that only a single 'marginal' buyer[1] participates in the seller's mechanism. This reserve price

[1] A 'marginal' buyer is a buyer who is just indifferent about whether or not he trades at the reserve price once he learns that he is the only buyer participating.

is considerably lower than the one that is optimal when the seller's demand is independent of his mechanism (as is typically assumed in the theory of mechanism design).

The qualifier 'marginal' is key to this result. A buyer is 'marginal' if he is just indifferent about whether or not to trade at the reserve price (equal to the seller's opportunity cost). This has two interesting implications. First, if the seller finds only a single potential buyer, then that buyer's valuation will on average be considerably higher than the marginal buyer's valuation. That is because the marginal buyer has the lowest possible valuation that is consistent with trade. Thus the seller will regret in a very strong sense having to trade with the buyer. He regrets it because the price he actually receives will be below his ex post opportunity cost of trade. This finding may explain the tendency for assets (like buildings) to be sold to a single bidder then subsequently resold at much higher prices. The original seller in such circumstances held the reserve price low to attract bidders, but was then unlucky enough to be contacted by only a single bidder.

Secondly, the reserve price is set equal to the seller's opportunity cost conditional on having only a single bidder who is then indifferent about trading at that reserve price. For the bidder to be indifferent, he must, in fact, have a quite a low valuation. This outcome is much worse news than the fact that there are no bidders at all, since it is quite likely that there are many bidders with high valuations who simply chose other alternatives. So suppose sellers' opportunity cost is simply the discounted value of the price at which they can expect to sell in the future. Suppose further that the seller's estimate of this resale price conditional on having no buyers is an unbiased estimate of the objects true resale value. Then the seller's equilibrium reserve price will be below the average resale price of an unsold object. This helps explain the [6] finding that reserve prices in the FDIC auctions ended up so far below the average resale price of unsold houses.

Independent private values is a special case of the model considered here. Competition in this environment has been considered by [5] who shows that second price auctions are equilibrium mechanisms. One contribution of the current paper is to extend this result to a common value environment. However, the description of a competitive equilibrium is more transparent here. This makes the trade-off between demand and the mechanism that the seller uses easier to understand. Furthermore, the proof of the main theorem presented here, when applied to the independent private value case is quite different from the proof in McAfee. So this paper should be viewed as an illustration of how powerful the competitive model of contract markets is when applied to informational environments more complicated than the one considered in [5].

2. The Model

2.1. Basics

There is a finite number J of sellers, each of whom owns a single unit of a homogenous and indivisible commodity. Seller j has a type represented by a real number s_j. The seller's type affects his valuation for the homogenous commodity that sellers wish to trade. It will be assumed that the sellers' types are all common knowledge though the model could be extended to allow this to be private information.

There are $I = kJ$ buyers participating in the market, where k is a fixed finite ratio of buyers to sellers. Buyers all have private information characterized by their *type*. The type of buyer i is $b_i \in [0, 1]$. It is assumed that buyers' types are identically and independently distributed according to some distribution F. This distribution is assumed to have a support equal to $[0, 1]$ and to possess a well defined density function f. Let $b = \{b_1, \ldots, b_{kJ}\}$ denote the vector of buyer types for all the buyers in the population. Types might represent buyers valuations for the commodity as in a private value model. Alternatively, these types might be information that buyers have about the common value of the commodity.

Buyers and sellers' valuations for the unit of output being traded depend on the private information of all the other buyers and sellers. Since seller types are assumed to be common knowledge, no generality is lost by suppressing the notation that describes this relationship. So let $V(b_i, b_{-i})$ and $U(b, s_j)$ represent the ex post valuations[2] of a buyer i with valuation b_i and a seller j with valuation s_j respectively.

To get a precise characterization of the equilibrium below it will be assumed that the valuation functions are *additively separable* in the sense that

$$V(b_i, b_{-i}) = v(b_i) + \gamma(b_{-i})$$

and

$$U(b_i, b_{-i}, s_j) = s_j + \psi(b_i) + \gamma(b_{-i})$$

[2]The fact that buyers' and sellers' valuations depend on the types of all of the sellers in the market is simply embedded in the functions V and U.

where $v(\cdot)$, $\psi(\cdot)$ and $\gamma(\cdot)$ are all non-decreasing functions that vanish at 0..

For example, the goods being traded might be treasury bills held by different brokers. The bills are all close substitutes and each buyer gets an idiosyncratic payoff associated with his own diversification requirement plus the future resale value of the bill which is given by the average valuation of each of the investors, i.e., $V(b_i, b_{-i}) = \bar{v}(b_i) + \sum_{j=1}^{I} b_j/I$. The seller gets only the market value of the treasury bill so that $U(b_i, b_{-i}, s_j) = \sum_{j=1}^{I} b_j/I - s_j$. In this case $\psi(b_i) = b_i/I$ and $\gamma(b_{-i}) = \sum_{j \neq i} b_j/I$ and $v(b_i) = \bar{v}(b_i) + b_i/I$. Alternatively, imagine $v(b_i)$ to be the current flow of utility buyer i gets from using the services of some durable good (like a house) for one period. Let γ denote the expected payment that *any* trader can extract from a market consisting of some random subset of the $I-1$ buyers who fail to trade in the current period assuming that current buyers valuations are given by b_{-i}. Then for the seller $\psi(b_i)$ is the expected payoff that the seller can get if he convinces i to deal with him in the following period. If this event does not occur, then the seller gets the same expected payment $\gamma(b_{-i})$ from the other buyers as buyer i would.

The valuation functions are assumed to be differentiable everywhere and non-decreasing in each of their arguments. Furthermore, it is assumed throughout that the difference between these functions is monotonically increasing in it's first argument, i.e.,

$$\frac{\partial V(b_i, b_{-i})}{\partial b_i} - \frac{\partial U(b_i, b_{-i}, s_j)}{\partial b_i} > 0$$

Not only is the value of the commodity increasing in the signal observed by the buyer with whom trade occurs, but in addition, the gain to exchange is also an increasing function of this signal[3].

Buyers and sellers are assumed to be risk neutral, so if a buyer with type b_i trades with a seller having information s_j at a price p, the expected payoff each receives is

$$\mathbf{E}V(b_i, b_{-i}) - p$$

for the buyer and

$$p - \mathbf{E}U(b_i, b_{-i}, s_j)$$

for the seller. The expectation is taken conditional on all of the information that the traders have available at the time that they trade.

[3]This is trivially true for the case usually discussed in the literature in which the seller's use value for the commodity is zero independent of the buyers' information.

2.2. The market process

The market process[4] that determines all trades now proceeds. At the beginning of the game, each seller offers a direct mechanism to buyers. A mechanism is described in more detail below, but basically it describes how the seller will determine a price and trading partner (if there is one) among the buyers who visit him. The outcome will depend on messages that the buyers send to the sellers. For example, the seller could promise to hold an auction with a fixed reserve price. Alternatively he could simply offer his unit of output at a fixed price

Once the buyers see the mechanisms that are being offered by sellers, they select *one and only one* seller as a potential trading partner. Thus, sellers can try to attract buyers by modifying the mechanisms they offer. In the real estate example, sellers might consider lowering the list price, or reducing the probability that a buyer will face competition from other buyers.

After buyers select sellers, they communicate with the sellers they have chosen as specified by the seller's announced mechanism. Trades and payments occur, then the game ends. The problem is similar to any static mechanism design problem except for the fact that buyers have many alternative suppliers to whom they might turn, and sellers recognize this[5].

2.3. Direct Mechanisms

It is assumed here that sellers offers buyers direct mechanisms in which buyers are asked to report the signals that they have observed to the seller.[6] The only complication involved with this arises from the fact that the seller does not know which buyers will ultimately communicate with him. Here the convention used is that there is a signal $b_\emptyset$ that buyers can use to report to a seller that they

[4]It would be appropriate to call this process a game form, but as will be seen, the solution concept will not be Nash in the usual sense. Rather competitive ideas will be employed. For this reason it seems preferable to refer to the market process.

[5]It is not difficult to convert this problem to a dynamic one in which buyers and sellers who fail to trade right away, can try to do so again in the following period. This extension is straightforward provided it is reasonable to assume that buyers and sellers believe that their current actions have no impact on future payoffs (which is reasonable in the large game environment that we study here).

[6]The revelation principle requires that buyers report their 'types' to sellers, not just their signals. There may be a difference if buyers have better information about the mechanisms on offer in the market than sellers do.

have or are planning to communicate with someone else.[7] In this formulation, the seller designs a mechanism that is offered to all of the buyers in the market. All buyers in the market will report to the seller, though some will report the type $b_\emptyset$, which means that they have no interest in participating in the seller's mechanism. The space from which buyer $i's$ valuation is drawn can be simply thought of as $[0,1] \cup b_\emptyset \equiv B_i$. The space of types for all the buyers is $B = \prod_{i=1}^{I} B_i$. The decision to participate is endogenous, so that the probability distribution over B perceived by the seller is endogenous, as will be described momentarily. It will also be convenient to assume that $b_\emptyset$ is a number that is smaller than any number in $[0,1]$ so that a buyer who decides not to participate in the seller's mechanism is treated as if he has a signal that is lower than the signals of any buyer who does decide to participate.

A mechanism μ is a mapping from the set B into the set of $I+1$ vectors $\{p_j, q_j\}$. The mechanism μ specifies the probability with which each buyer will trade with the seller, and the transfer that each buyer will make to the seller. So for example, $q_{ij}(b)$ is the probability that buyer i will trade with seller j when the vector of reports by the buyers is given by b. The vectors lie in $\mathbf{R}^{I+1}$ because the seller might decide not to trade with any of the buyers who have currently contacted him.

With this latter convention, the mechanism must require that the sum of the assignment probabilities is one, and that the trading probability and transfer must both be zero for any buyer who 'reports' $b_\emptyset$ to the seller. This means that $b_i = b_\emptyset \Rightarrow p_{ij}(b) = q_{ij}(b) = 0$ and $\sum_{n=0}^{I} q_{nj}(b) = 1 \; \forall b \in T$.

Attention will also be restricted to anonymous mechanisms having the property that if b and b' differ only in that the i^{th} and m^{th} component have been interchanged, then $q_{ij}(b) = q_{mj}(b')$. Anonymity is a restrictive assumption when sellers can identify individual buyers. For example if the competition among sellers is modelled as a Bayesian game, then sellers are assumed not only to know who their potential customers are, but also the strategies that these customers will use to choose among sellers in equilibrium. In the competitive environment considered here, there is no need to assume that sellers can identify individual buyers. Since beliefs about buyers are assumed to be symmetric in the competi-

[7] The fact that a particular buyer does not choose to participate in a particular seller's auction may be informative to the seller when there are small numbers. Even when there are large numbers, seller may be able to make inferences about the distribution of buyer types in the population from the number of buyers who actually choose to bid. These calculations are demonstrated below.

tive environment considered here, anonymity is not restrictive. Let , denote the set of anonymous mechanisms.

2.4. Beliefs

The key problem in the competitive environment is to describe how the seller's beliefs about the buyers who select him will vary with the mechanism that he offers. It will be assumed that buyers choose symmetrically among sellers using the strategy $\pi(x) = (\pi_1(x), \ldots \pi_J(x))$ where $\pi_j(x)$ is the probability with which a buyer of type x chooses seller j. In equilibrium, this strategy will chosen as a continuation equilibrium for the subgame that ensues following the sellers mechanism announcements.

As will be seen shortly, our solution concept does not require that sellers understand any of this. Instead, it is assumed that each seller j knows the number of potential buyers I and believes that each potential buyer has a valuation (independently) distributed according to a common distribution z_j. These beliefs will be called *admissible* if there is a function $\pi(\cdot)$ such that

$$z_j(x) = 1 - \int_x^1 \pi_j(s)\, dF(s) \tag{2.1}$$

Notice that if $\pi_j(x) > 0$ on a set of positive Lebesque measure, then $z_j(0) > 0$, so this distribution will typically have a mass point at 0. Otherwise, it's density is well defined almost everywhere and is given by $z_j'(x) = \pi_j(x) f(x)$. $z_j^I(x)$ is the probability with which all buyers either have valuations below x or visit some seller other than seller j.

In equilibrium, sellers' beliefs will be correct and so will coincide with buyer beliefs. Hence the probability with which any buyer believes that all the other buyers will either have valuations below x or will choose some seller other than seller j is given by $z_j^{I-1}(x)$.

2.5. Payoffs

Let $\mu_j = (p_j, q_j)$ denote seller $j's$ mechanism. The expected payoff that a buyer of type x gets from choosing to participate in the mechanism μ_j offered by seller j, conditional on the selection strategy π, is given by

$$\beta_j(x; \mu_j, z_j) = \mathbf{E}_{b_{-1}|x} q_{1j}(x, b_{-1}) V(x, b_{-1})$$
$$-\mathbf{E}_{b_{-1}|x} p_{1j}(x, b_{-1})$$

where the expectation is taken using the distribution z_j as determined by (2.1).

A mechanism is incentive compatible for buyer strategy π if

$$\beta_j(x;\mu_j,z_j)$$
$$\geq \mathbf{E}_{b_{-1}|x}\, q_{1j}(x',b_{-1})\, V(x,b_{-1})$$
$$-\mathbf{E}_{b_{-1}|x}\, p_{1j}(x',b_{-1})$$

for all $x' \in [0,1]$.

If μ_j is incentive compatible for the selection strategy π_j, then by standard arguments $\beta_j(x;\mu_j,z_j)$ is continuous and increasing. Since $\beta_j(x;\mu_j,z_j)$ is monotonic, it is differentiable almost everywhere. If (and only if) the signals that buyers receive are unconditionally independent, then $\mathbf{E}_{b_{-1}|x}\, p_{1j}(x',b_{-1})$ is independent of x (it depends only on the buyer's reported x'). In this case the important properties of (incentive compatible) mechanisms are determined entirely by the trading probabilities q. So, for example, the derivative of $\beta_j(x;\mu_j,z_j)$ with respect to x (when it exists), is given by

$$\mathbf{E}_{b_{-1}|x}\, q_{1j}(x,b_{-1})\,\frac{\partial V(x,b_{-1})}{\partial x} \tag{2.2}$$

by the envelope theorem. In particular this implies

$$\beta_j(x;\mu_j,z_j) = \bar{\beta}_j(x;\mu_j,z_j) + \int_0^x \beta_j'(y)\,dy$$

where $\bar{\beta}_j(x)$ is almost everywhere constant.

Then using the formula for the derivative, this gives

$$\beta_j(x;\mu_j,z_j) = \bar{\beta}_j(x;\mu_j,z_j) + \mathbf{E}_{b_{-1}} \int_0^x q_{1j}(y,b_{-1})\,\frac{\partial V(y,b_{-1})}{\partial y} dy \tag{2.3}$$

Thus with independence, any mechanism that gives the good to the buyer with the highest signal, for example, will yield buyers the same expected payoff modulo a constant. The function $\bar{\beta}_j(x;\mu_j,z_j)$ can have a countable number of discontinuous jumps in it. For example, if $\pi_j(x) = 0$ over some interval, then in equilibrium, the payoff available with seller j may be smaller than the *market* payoff. At the end of this interval, the payoff could jump up discontinuously to equal the market payoff. If $\pi_j(x)$ is strictly positive for x above some lower bound r, then $\beta_j(x;\mu_j,z_j)$ will contain only a single jump at this lower bound.

The lower bound r is just the greatest lower bound on the set of types who choose to participate in the seller's mechanism with strictly positive probability.

The surplus that seller j enjoys from the incentive compatible mechanism μ_j and the selection rule π_j is equal to the expected surplus generated by trade with each buyer, less the surplus that the seller must offer the buyer to get him to come in the first place. This is

$$\sigma_j(\mu_j, z_j) = kJ\left\{\mathbf{E}_b\left\{U(b_1, b_{-1}, s_j) + [V(b_1, b_{-1}) - U(b_1, b_{-1}, s_j)]\, q_{1j}(b)\right\}\right.$$
$$\left. -\beta_j(b_1; \mu_j, z_j)\right\}$$

where the expectation is taken here using the distribution z_j.

Since the first term in the expectation is independent of the mechanism that the seller offers, the seller's objective can be reduced to maximizing the expected gain

$$kJ\mathbf{E}_b\left\{[V(b_1, b_{-1}) - U(b_1, b_{-1}, s_j)]\, q_{1j}(b) - \beta_j(b_1; \mu_j, z_j)\right\}$$

3. Equilibrium Mechanisms

One of the advantages often attributed to auctions is that they are likely to work pretty well even when the seller is very poorly informed about the buyers he is dealing with. This may be further complicated in that sellers might not be precisely sure who they are competing against. In the FDIC auctions discusses by [6], buyers alternatives include rental markets, and alternative asset markets as well as the local real estate market. To model the FDIC in the usual way, searching for a best reply to a known set of actions on the part of a known set of competitors, is probably not the best way to proceed.

Nonetheless, sellers will need to formulate conjectures about the distribution of types that they face, and about the relationship between this distribution and the mechanism that they offer. This paper uses a rational expectations solution to this problem. Sellers formulate conjectures about this relationship, and on the equilibrium path these conjectures are required to be correct. The solution concept is *competitive* in the sense that sellers take the payoff that they need to offer different buyers in order to attract them to be fixed.

Formally a *competitive equilibrium in mechanisms* is a market payoff function $\beta^*(\cdot)$, a selection strategy π^* for buyers, and an array of mechanisms $\left\{\mu_1^*, \ldots, \mu_J^*\right\}$ such that

1. for each x, and each $j = 1, \ldots J$; $z_j^*(x) = 1 - \int_x^1 \pi_j^*(s) f(s)\, ds$

2. for each $j = 1, \ldots J$, the mechanism μ_j^* and the distribution $z_j^*(\cdot)$ maximize

$$\sigma_j(\mu_j, z_j) =$$

$$kJ \int \left\{ \int \ldots \int \{[V(b_1, b_2, \ldots b_I) - U(b_1, b_2, \ldots b_I, s_j)] q_{1j}(b)\} dz_j(b_2) \ldots dz_j(b_I) - \beta_j(b_1; \mu_j, z_j) \right\} dz_j(b_1)$$

   subject to the constraint that $\beta_j(x; \mu_j, z_j) \geq \beta^*(x)$ for each x for which $z_j'(x) > 0$.

3. for all $x \in [0,1]$ and each $j = 1, \ldots J$,

$$\beta^*(x) = \max\left[0, \max_j \left[\beta_j\left(\mu_j^*; \mu_j, z_j\right)\right]\right]$$

The first condition simply forces each seller's belief to be admissible. The second condition expresses the trade-off that the seller perceives between the mechanism that he offers and the probability with which he will be able to attract buyers of different types. The 'graph' consisting of all $\{\mu_j, z_j\}$ pairs satisfying the market payoff constraint is essentially the 'demand curve' that the seller thinks that he faces given the market payoff function β^* which the seller believes to be fixed and beyond his control. The final condition is the rational expectation condition that requires that the seller's beliefs about the distribution of types that he faces is equal to the true distribution of types that occurs in the symmetric continuation equilibrium of the buyers' subgame.

The advantage of this approach is that the seller's best reply to the market payoff function is found by solving a relatively simple programming problem. The anonymity restriction guarantees that given any signal x, the probability that the seller trades with the buyer having this signal depends only on the distribution of valuations of the other buyers. The function q_{1j} will then completely characterize the seller's mechanism provided it satisfies the anonymity restriction and the summing up constraint that requires that the probability be less than or equal to one that the seller trade with some buyer.

This latter restriction can be expressed in a convenient way following [4]. The function q_{1j} will satisfy the summing up constraint if and only if for each x

$$kJ \int_x^1 \mathbf{E}_{b_{-1}} q_j(s, b_{-1})\, dz_j(s) \leq 1 - z_j(x)^{kJ} \tag{3.1}$$

It is straightforward that every mechanism must satisfy this condition. It simply says that, uniformly in x, the probability that the seller trades with a buyer whose valuation is in the interval $[x,1]$ can be no larger than the probability that there is a buyer with valuation in this interval. Matthews [4] shows the converse.

Dropping the various subscripts for simplicity and using (2.3), the seller's problem can then be expressed as the program maximize

$$kJ\int_0^1 \mathbf{E}_{b_{-1}}\left\{\left[V\left(b_1,b_{-1}\right)-U\left(b_1,b_{-1},s_j\right)\right]q\left(b_1,b_{-1}\right)-\beta^*\left(b_1\right)\right\}dz\left(b_1\right) \tag{3.2}$$

by choosing the assignment rule $q(\cdot,\cdot)$ and the probability distribution $z(\cdot)$ subject to the constraints

$$q\left(x,b_{-1}\right)\in[0,1] \text{ for all } \left(x,b_{-1}\right)\in[0,1]^I$$

$$\beta^*(x)\leq\bar{\beta}(r;q,z)+\int_0^x \mathbf{E}_{b_{-1}}q\left(y,b_{-1}\right)\frac{\partial V\left(y,b_{-1}\right)}{\partial y}dy \tag{3.3}$$

for every x in the support of z; and (3.1).

An important aspect of the seller's decision problem is the shape of the function

$$S_j(x)\frac{\beta'(x)}{v'(x)}-\beta(x)$$

where $S_j(x)=V\left(x,b_{-1}\right)-U\left(x,b_{-1},s_j\right)=v(x)-\psi(x)-s_j$. The valuations of the buyers other than buyer i disappear because of the separability assumption. As will be seen momentarily, this is the expected surplus generated by trade with a buyer of type x less the payoff that the seller needs to offer to attract such a buyer. If this function is not monotonic, the seller will want to choose a mechanism that awards the good to the buyer who generates the largest surplus, and this will not necessarily be the buyer with the highest willingness to pay for the good. In this case the seller will not want to use an auction as an allocation mechanism. To avoid the intricacies of this problem we will focus on problems where equilibrium generates surplus functions and market payoff functions that are monotonic as described above. In this case we have the following theorem.

Theorem 3.1. *Suppose that buyers signals are independent and payoff functions are additively separable. Let the market payoff function $\beta^*(x)$ be a non-decreasing function satisfying $0<\beta^{*\prime}(x)\leq v'(x)$ for all $x\geq\underline{r}\geq 0$, with $\frac{\beta^{*\prime}(x)}{v'(x)}$ increasing in* *x, and suppose that the function* $S_j(x)\frac{\beta^{*\prime}(x)}{v'(x)} - \beta^*(x)$ *is monotonically increasing. Define* r^* *as the solution to*

$$S_j(r)\frac{\beta^{*\prime}(r)}{v'(r)} = \beta^*(r) \tag{3.4}$$

Then the functions $q(\cdot,\cdot)$ *and* $z(\cdot)$ *and* $\beta(\cdot)$ *will constitute best replies to the market payoff* β^* *if and only if*

$$\beta(r^*) = \beta^*(r^*) \tag{3.5}$$

$$\beta'(1) = v'(1) \tag{3.6}$$

$$z(x) = \begin{cases} \left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}} & \text{if } x \geq r^* \\ 1 - \left(\frac{\beta'(r^*)}{v'(r^*)}\right)^{\frac{1}{kJ-1}} & \text{otherwise} \end{cases} \tag{3.7}$$

and

$$\mathbf{E}_{b_{-1}} q(x, b_{-1}) = \begin{cases} z^{kJ-1}(x) & \text{if } x \geq r^* \\ 0 & \text{otherwise} \end{cases} \tag{3.8}$$

The proof of the theorem is tedious and is relegated to an appendix. The theorem says that provided the excess surplus function has the right monotonicity properties, the best reply to the market payoff is an auction in which the seller sets a reserve price low enough that he trades with every buyer type who generates a positive surplus.

Consider the restrictions in reverse order. The final property (3.8) says that the probability with which a buyer of type x trades must be equal to the probability that buyer has the highest valuation among all the buyers who are expected to choose the seller. In other words, a necessary condition for the seller's mechanism to be a best reply is that it implement some kind of auction.

This result goes a long way toward determining the outcome, but recall that the seller's primary interest is the trade-off between participation and surplus extraction. His best reply must also determine the likelihood with which buyers will choose to participate in his mechanism. This trade-off is embodied in (3.7). Rewriting the condition gives

$$z(x) = 1 - \int_x^1 \pi(s) f(s)\, ds = \left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}}$$

Since this condition must hold uniformly, this becomes an expression for the choice strategy that the seller expects buyers to use.

Finally (3.5) is the key to the reserve price results. It says that the seller should offer exactly the market payoff to the buyer whose type is such that excess surplus is zero.

3.1. Existence

Before turning to interpretation, the existence issue should be addressed. There are two problems to be dealt with here. The first is simply whether there is any competitive equilibrium at all. The second is whether there is an equilibrium satisfying the requirements of Theorem 3.1. These issues can be addressed together in the following constructive manner. Each seller is first assigned an auction with an appropriately chosen reserve price. The continuation equilibrium in buyer choice strategies can be calculated explicitly for these auctions. From this continuation equilibrium it is possible to find the market payoff function. It is then possible to provide a class of payoff functions $\psi(\cdot)$ such that the surplus function generated by this market payoff function is monotonic in the manner required by Theorem 3.1. Lastly, it is easy to show that the auctions that sellers have been assigned in the first place are best replies to the market payoffs that they generate. To simplify the argument attention is focussed on the symmetric case.

Theorem 3.2. *Suppose that all sellers have the same cost $s_j = 0$ and that for all x,*

$$\left(\frac{kJ-1}{J}\right)\frac{v(x)-\psi(x)}{\psi'(x)} > 1 - \frac{1-F(x)}{J} \tag{3.9}$$

Then there exists a competitive equilibrium in mechanisms in which all sellers hold english auctions with reserve price equal to

$$\int\cdots\int \gamma(b_2,\dots b_{kJ})\, f(b_1)\dots f(b_{kJ})\, db_1\dots db_{kJ}$$

The restriction (3.9) guarantees that the excess surplus function is strictly increasing in the equilibrium that is being constructed. The reserve price is the value that a buyer with valuation 0 assigns to the good to be purchased conditional on being the only buyer who chooses to participate in the seller's mechanism.[8]

[8]The fact that this is independent of the buyers' strategies is a special property of the equilibrium constructed here. The calculation is done explicitly in the proof.

4. Reserve Prices

Consider the interpretation in which the opportunity cost to the seller is equal to the future resale value of the object being traded and restrict attention to the symmetric equilibrium described in Theorem 3.2. In this case, $V(b_1, b_{-1}, s) = \psi(b_1) + \gamma(b_{-1})$ is the potential future resale value conditional on the information held by existing buyers. The reserve price is equal to the expectation of this value conditional on the fact that only a single buyer chooses the seller's auction *and* conditional on that buyer having a valuation equal to zero. This stems from the requirement that the seller wants to set a reserve price low enough so that it will be worthwhile for all buyer types to trade with him. His opportunity cost under the same circumstances is equal to the expected resale value of the object conditional on the fact that he was selected by a single buyer. This is

$$\int \psi(b_1) f(b_1)\, db_1 + \int \cdots \int \gamma(b_2, \ldots b_{kJ}) f(b_1) \ldots f(b_{kJ})\, db_1 \ldots db_{kJ}$$

which is clearly larger than the equilibrium reserve price.

In this sense the seller regrets trading ex post. Of course, the seller will always regret trading ex post in an auction in the sense that he knows that since the buyer was willing to trade, he would also have been willing to trade at a higher price (at least with high probability). The regret is stronger here, since ex post the seller would have preferred holding on to the good and trying to resell it in the following period.

If no buyers at all choose to participate in the seller's mechanism, so that he is forced to trade it in the following period, then the value of the good to the seller is the expected resale price conditional on no buyers coming, i.e.,

$$\frac{1}{J-1} \int \psi(b_1) f(b_1)\, db_1 + \int \cdots \int \gamma(b_2, \ldots b_{kJ}) f(b_1) \ldots f(b_{kJ})\, db_1 \ldots db_{kJ}$$

again, strictly larger than the reserve price that the seller sets.

To test such a theory, one might proceed as in [6] to collect data on the sale prices of goods that failed to sell in their initial auction. If sellers correctly anticipate these future resale prices, then the *equilibrium* reserve prices that they set should be *lower* than the expected resale prices. This may explain the finding in [6] that reserve prices tended to be below observed resale prices.

There is some relationship between the result presented in Theorem 3.2 and the result in [1]. They show that a seller would prefer to hold an absolute English auction (that is, an English auction with no reserve price) involving $N+1$ bidders, to using the optimal selling mechanism against N bidders *provided that the $N+1st$ bidder is serious*. In other words, the simple English auction will be preferable to the optimal mechanism provided that it attracts one more bidder like the existing bidders. The attraction of this low reserve price to the additional bidder is imposed exogenously in [1]. Here the response of bidders to the lower reserve price is endogenous, and in this case the result suggests that [1] did not go far enough. Interpreting the absolute english auction as one in which the seller sets a reserve price equal to his opportunity cost, the reserve price that sellers set to attract buyers in equilibrium is even lower than the one predicted by [1], since reserve prices are driven below opportunity costs.

4.1. Alternative Informational Assumptions

It is not clear whether these results can be extended to permit more general assumptions about the buyers' and sellers' information. One of the key arguments in the proof arises from the fact that the derivative of the payoff function $\beta'(x)$ is related to the trading rule in a relatively simple way, i.e.,

$$\beta'(x) = \mathbf{E}_{b_{-1}} q(x, b_{-1}) \frac{\partial V(x, b_{-1})}{\partial x} \tag{4.1}$$

When the payoff function V is additively separable, this immediately yields a formula for the trading probability for each buyer type who participates in the seller's mechanism. The substitution of this result reduces the control problem to one of choosing the real function z'. This simplification is immediately lost without separability and the control problem requires determination of the functions z' and q jointly. Alternative methods to the ones described here are needed to resolve this problem.

Within the separable framework, an difficult problem arises when the excess surplus function is non-monotonic. This might occur if a house seller, for example, gets information about the rate at which his house is appreciating in value. Buyers with high willingness to pay might convince the seller that his house is appreciating in value so fast that he should never have tried to sell it in the first place. This is comparable to the problem that occurs when the virtual valuation function is non-monotonic in the independent private value monopoly framework. In this case the seller may not want to use an efficient mechanism that awards trade to the buyer with the highest valuation. Non-auction mechanisms will then be needed, and it is difficult to get much insight into what these might be.

In the affiliated private value framework, separability holds in a trivial sense, but the simplifying equation (4.1) no longer holds because of the fact that the trading probability $\mathbf{E}_{b_{-1}|x} q(x, b_{-1})$ depends on the true value of x as well as on the buyer's report about x. This environment is of particular interest since the competitive environment gives an immediate explanation for why sellers do not extract all the buyer's surplus in the manner of [7]. It remains to see whether competition will explain why sellers do not use explicit entry fees in the fashion described in that paper.

5. Conclusions

The equilibrium mechanism is allocationally equivalent to an English auction with a reserve price equal to the ex post value of a marginal unsold unit of output. Allocation schemes resembling English auctions are relatively common. Private real estate sales typically involve a dynamic procedure of bid and counterbid that resemble English auctions. Distress sales by the Federal Deposit Insurance Corporation and the Resolution Trust Corporation in the United States use explicit English auctions, and involve sales in the billions of US dollars [6]. English auctions of houses and used cars are common in Australia and New Zealand.

Thia paper provides and explanation for why auctions are used in the first place, and perhaps more important, why equilibrium reserve prices in these auctions should appear to be quite low. The conditions under which these results hold require that the seller's payoff be relatively insensitive to buyers information. When this condition fails, equilibrium mechanisms will not involve auctions. The properties of equilibrium mechanisms in this latter case are unknown.

6. Appendix-Proof of the Main Theorem

Proof. The proof proceeds in two parts. In the first part, it is shown that (3.7) and (3.7) are necessary for any firm to profitably implement any payoff function β. In the second part, it is shown that full maximization requires that the firm choose a β satisfying (3.5), (3.7), and (3.4).

6.0.1. Part 1: Most profitable way to implement β

Let β be any payoff function satisfying $S_j(x)\frac{\beta'(x)}{v'(x)} - \beta(x)$ is strictly increasing. By the Lebesque decomposition theorem ([2], p130), β can be decomposed uniquely into a singular (almost everywhere constant) function $\bar{\beta}$ and a function β' such that for all $x \in [0,1]$, $\beta(x) = \bar{\beta}(x) + \int_0^x \beta'(s)\,ds$. Using this, we can define the most profitable way of implementing the payoff β as

$$\psi(\beta) = \max_{q(\cdot,\cdot),z} kJ \int_0^1 \mathbf{E}_{b_{-1}} \left\{ \left[V(b_1, b_{-1}) - U(b_1, b_{-1}, s_j) \right] q(b_1, b_{-1}) - \beta(b_1) \right\} dz(b_1)$$

subject to

$$\beta(x) = \bar{\beta}(r; q, z) + \int_0^x \mathbf{E}_{b_{-1}} q(y, b_{-1}) \frac{\partial V(y, b_{-1})}{\partial y} dy \text{ for all } x$$

$$kJ \int_x^1 \mathbf{E}_{b_{-1}} q_j(s, b_{-1})\, dz_j(s) \leq 1 - z_j(x)^{kJ} \text{ for all } x$$

Notice that this definition ignores the constraint that $q(x, b_{-1}) \in [0,1]$ and the constraint that z be a probability distribution. The payoff β that is being implemented has to be associated with some incentive compatible mechanism, so we can assume without loss of generality that it is continuous, non-decreasing and has $\frac{\beta'(x)}{v'(x)}$ non decreasing. From the second constraint

$$\beta'(x) = \mathbf{E}_{b_{-1}} q(x, b_{-1}) \frac{\partial V(x, b_{-1})}{\partial x}$$

for z-almost all[9] x.

The separability assumption implies that $\frac{\partial V(x, b_{-1})}{\partial x}$ is independent of b_{-1} so that this can be written

$$\mathbf{E}_{b_{-1}} q(x, b_{-1}) = \frac{\beta'(x)}{v'(x)} \tag{6.1}$$

for z-almost all x.

Using this, and making use of separability, the objective can be rewritten

$$kJ \int_0^1 \mathbf{E}_{b_{-1}} \left\{ \left[V(x, b_{-1}) - U(x, b_{-1}, s_j) \right] q(x, b_{-1}) - \beta(x) \right\} dz(x)$$

$$= kJ \int_0^1 \left\{ \left[v(x) - \psi(x) - s_j \right] \mathbf{E}_{b_{-1}} q(x, b_{-1}) - \beta(x) \right\} dz(x)$$

$$= kJ \int_0^1 \left\{ S_j(x) \frac{\beta'(x)}{v'(x)} - \beta(x) \right\} dz(x)$$

[9]This means for all x for which $z'(x) > 0$. This is an unusual use of this terminology since formally, z does not have to be a distribution function in this argument.

where $S_j(x) \equiv [v(x) - \psi(x) - s_j]$. The last step makes use of (6.1).

Continuing with this argument, we need to choose $z'(x) \geq 0$ to maximize

$$kJ \int_0^1 \left\{ S_j(x) \frac{\beta'(x)}{v'(x)} - \beta(x) \right\} z'(x)\, dx$$

subject to the constraint

$$kJ \int_x^1 \frac{\beta'(s)}{v'(s)} z'(s)\, ds + \left[1 - \int_x^1 z'(s)\, ds\right]^{kJ} \leq 1$$

for all x.

This problem can be solved using standard Lagrangian methods. Let $\lambda(x)$ denote the multiplier associated with the constraint. The resulting Lagrangian expression is strictly concave (pointwise) in the $z'(x)$. Thus a necessary and sufficient condition for an optimum is that

$$kJ \left\{ S_j(x) \frac{\beta'(x)}{v'(x)} - \beta(x) \right\}$$
$$- \int_0^x \lambda(s) \left\{ kJ \frac{\beta'(x)}{v'(x)} - kJ \left[1 - \int_s^1 z'(y)\, dy\right]^{kJ-1} \right\} ds = 0 \qquad (6.2)$$

for all x such that $S_j(x) \frac{\beta'(x)}{v'(x)} - \beta(x)$.

As $S_j(x) \frac{\beta'(x)}{v'(x)} - \beta(x)$ is assumed monotonically increasing, if there is a segment $[0, r]$ along which it is negative, then $z'(x)$ can be set equal to zero along this segment without violating the constraint. Since this would raise expected profits, we can assume that there is some r^* such that $z'(x) = 0$ for all $x \leq r^*$.

Now suppose that there is some interval $[a, b]$ above r^* along which the multiplier $\lambda(x) = 0$. Since the second term in (6.2) will be constant along this interval while the first term is strictly increasing, there must be some point along the interval at which the necessary condition is violated. We conclude that $\lambda(x) > 0$ for almost every $x > r^*$, and consequently that the constraint (3.1) holds with equality for almost all $x \geq r^*$. Differentiating the constraint (3.1) gives $\frac{\beta'(x)}{v'(x)} = z(x)^{kJ-1}$ for almost all $x \geq r^*$

This is not quite sufficient to give a complete result since most profitable implementation of β ignores the fact that z must be a probability distribution. The choice for z made by this program will fail to be a probability distribution whenever β has the property that $\beta'(1)/v'(1) < 1$. For in this case $z(1) < 1$. To deal with this, we now suppose that β satisfies the market constraint and show that if $\beta'(1)/v'(1) < 1$ then there is an alternative payoff function $\tilde{\beta}(\cdot) \geq \beta(\cdot)$ satisfying $\tilde{\beta}'(1)/v'(1) = 1$ which yields the seller strictly higher profits than β.

6.0.2. The most profitable β

The fact that $z(x) = \left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}}$ gives

$$\psi(\beta) = \int_r^1 \left\{ S(x) \frac{\beta'(x)}{v'(x)} - \beta(x) \right\} d\left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}} \tag{6.3}$$

Suppose that the seller has chosen a feasible payoff function for which $\frac{\beta'(1)}{v'(1)} = \delta < 1$. Then $\psi(\beta)$ will strictly exceed the maximum profit the seller can attain from β when he adjusts z to satisfy the constraint that $z(1) = 1$. We will construct a payoff $\tilde{\beta} \geq \beta$ satisfying $\frac{\tilde{\beta}'(1)}{v'(1)} = 1$ which has the property that $\psi\left(\tilde{\beta}\right)$ is arbitrarily close to $\psi(\beta)$. Since any such payoff function must satisfy the market constrain if β does, this implies that payoff functions that do not satisfy the terminal condition $\beta'(1)/v'(1) = 1$ cannot be optimal.

Let $y < 1$. On the region $[y, 1]$ define a new $\tilde{\beta}$ such that

$$\tilde{\beta}'(x) = \beta'(x) + (1-\delta)\, v'(x)$$

and $\tilde{\beta}(x) = \beta(y) + \int_y^x \tilde{\beta}'(s)\, ds = \beta(x) + \int_y^x (1-\delta)\, v'(s)\, ds$. This new payoff function is non-decreasing and satisfies

$$\frac{\tilde{\beta}'(x)}{v'(x)} = \begin{cases} \frac{\beta'(x)}{v'(x)} & x \leq y \\ \frac{\beta'(x)}{v'(x)} + (1-\delta) & x > y \end{cases}$$

Thus the ratio $\frac{\tilde{\beta}'(x)}{v'(x)}$ is non-decreasing. By definition

$$\psi\left(\tilde{\beta}\right) =$$

$$\int_r^y \left\{ s(x) \frac{\beta'(x)}{v'(x)} - \beta(x) \right\} d\left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}} + \int_y^1 \left\{ s(x) \frac{\tilde{\beta}'(x)}{v'(x)} - \tilde{\beta}(x) \right\} d\left(\frac{\tilde{\beta}'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}}$$

This converges to

$$\int_r^1 \left\{ s(x) \frac{\beta'(x)}{v'(x)} - \beta(x) \right\} d\left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}}$$

as y goes to 1. Hence for y close enough to 1, $\psi\left(\tilde{\beta}\right)$ strictly exceeds the true profit associated with β and the original mechanism cannot be optimal.

6.0.3. Finally: The implications of the terminal condition $\beta'(1)/v'(1) = 1$.

Next suppose that $\beta'(1) = v'(1)$ so that $\hat{\psi}(\beta) = \psi(\beta)$. It must be that $\beta(r) = \beta^*(r)$. To see this, note that $\beta(r) \geq \beta^*(r)$ for all feasible mechanisms, so suppose to the contrary that $\beta(r) > \beta^*(r)$. Then there is an interval $[r, y]$, say along which $\beta(x) > \beta^*(x)$ by the continuity of β and β^*. If $y = 1$, then β can be reduced for all x without changing the derivative, increasing profits according to (6.3). So suppose that $\beta(y) = \beta^*(y)$. Without loss of generality, we may assume that $\beta'(y) = \beta^{*\prime}(y)$ for if this is not true then the technique of the previous paragraph can be used to construct a $\hat{\beta}$ such that $\hat{\beta}'(y) = \beta^{*\prime}(y)$ for which $\psi\left(\hat{\beta}\right)$ is arbitrarily close to $\psi(\beta^*)$.

Now for small δ, let $x(\delta)$ denote the solution to $\beta(x) - \delta = \beta^*(x)$. Since β and β^* are both differentiable, straightforward calculation gives $x'(\delta) = \beta'(x) - \beta^{*\prime}(x)$.

Replace $\beta(x)$ by

$$\hat{\beta}(x,\delta) = \begin{cases} \beta(x) - \delta & x \in [r, x(\delta)] \\ \beta^*(x) & x \in (x(\delta), y] \\ \beta(x) & otherwise \end{cases}$$

By (6.3), the derivative of $\psi\left(\hat{\beta}(x,\delta)\right)$ with respect to δ is given by

$$\int_r^{x(\delta)} d\left(\frac{\beta'(x)}{v'(x)}\right) + x'(\delta)\left[S(x(\delta))\frac{\beta'(x(\delta))}{v'(x(\delta))} - \beta(x(\delta)) + \delta - S(x(\delta))\frac{\beta^{*\prime}(x(\delta))}{v'(x(\delta))} - \beta^*(x(\delta))\right]$$

$$= \int_r^{x(\delta)} d\left(\frac{\beta'(x)}{v'(x)}\right) > 0$$

Hence $\psi\left(\hat{\beta}(x,\delta)\right) > \psi(\beta)$ for some δ small enough. Then β cannot be a best reply.

Finally, since $\beta(r) = \beta^*(r)$ for any best reply, if r is different from r^* then the seller can raise profits by raising or lowering r slightly by the definition of r^*. ■

7. Appendix - Proof of Theorem 3.2

Proof. To begin, strategy rules are specified for all the players in the market. It is then shown that these strategy rules are all best replies in the sense required by the definition of the competitive equilibrium in mechanisms. First suppose that every seller holds an english auction in which the price starts at the reserve price specified in the theorem and rises continuously. Buyers can drop out at any time, and the winner is the last buyer remaining in the auction.

Equilibrium bidding strategies for this kind of auction are well know, buyers remain in the auction as long as the price is below their valuation for the commodity conditional on all available information. Rather than writing out the rules by which buyers update their beliefs when other buyers drop out, simply observe that (continuation) equilibrium behavior will ensure that the buyer participating in the auction who has the highest valuation will win the auction.

Next suppose that buyers select each seller with the same probability $1/J$ independent of their type. Under these conditions, the probability with which any buyer wins a particular seller's auction is equal to $z(x)^{kJ-1} = \left[1 - \int_x^1 \pi(s) f(s) ds\right]^{kJ-1} = \left\{1 - \frac{1-F(x)}{J}\right\}^{kJ-1}$. By (2.2), this gives the buyer a payoff equal to $\beta(x) = \beta(0) + \int_0^x \left\{1 - \frac{1-F(s)}{J}\right\}^{kJ-1} v'(s) ds$ given that all the other buyers are behaving this way. The payoff that a buyer of type 0 gets is equal to the value of trade to a buyer of type 0 less the seller's reserve price. The value of trade is equal to the expected value of trade conditional on the fact that no other buyer has chosen the seller's mechanism. This is given by

$$\frac{1}{z^{kJ-1}(0)} \int \cdots \int \gamma(b_2, \ldots b_{kJ}) (1 - \pi(b_2)) f(b_2) \ldots (1 - \pi(b_{kJ}) f(b_{kJ})) db_2 \ldots db_{kJ}$$

$$= \left(\frac{J}{J-1}\right)^{kJ-1} \int \ldots \int \gamma(b_2, \ldots b_{kJ}) \left(\frac{J-1}{J}\right) f(b_1) \ldots \left(\frac{J-1}{J}\right) f(b_{kJ}) db_1 \ldots db_{kJ}$$

$$= \int \cdots \int \gamma(b_2, \ldots b_{kJ}) f(b_2) \ldots f(b_{kJ}) db_2 \ldots db_{kJ}$$

which gives $\beta(0) = 0$.

Since every seller offers the same mechanism, and consequently the same payoff function, the selection rule "choose each seller with probability $1/J$" will be a best reply to what the other buyers are doing. Thus condition (3) in the definition of the competitive equilibrium in mechanisms is satisfied.

It remains to show that the english auctions that sellers are using are in fact best replies to the market payoff function β. The excess surplus function is

$$S_j(x)\frac{\beta'(x)}{v'(x)} - \beta(x) = [v(x) - \psi(x)]\frac{\beta'(x)}{v'(x)} - \beta(x)$$

$$= [v(x) - \psi(x)]\left\{1 - \frac{1 - F(x)}{J}\right\}^{kJ-1} - \int_0^x \left\{1 - \frac{1 - F(s)}{J}\right\}^{kJ-1} v'(s)\,ds$$

The condition specified in the theorem states that the derivative of this function is positive. Furthermore, since $v(0) = \psi(0) = 0$ this function is non-negative for all x.

Since β satisfies all the conditions of Theorem 3.1, the distribution z that maximizes the seller's profits is uniquely determined by the requirement that for all $x \geq 0$

$$z(x) = \left(\frac{\beta'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}} = \left(\frac{\left\{1 - \frac{1-F(x)}{J}\right\}^{kJ-1} v'(x)}{v'(x)}\right)^{\frac{1}{kJ-1}} = \left\{1 - \frac{1 - F(x)}{J}\right\}^{kJ-1}$$

The first equality is from Theorem 3.1, while the others follow by substitution for β. This guarantees that the sellers' mechanisms and the beliefs defined by the buyers' continuation equilibrium satisfy condition (2) in the definition of the competitive equilibrium in mechanisms. ■

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