A Combinatorial Auction with Multiple Winners for Universal Service

Frank Kelly and Richard Steinberg
University of Cambridge

June 27, 1998

Abstract

We describe a discrete-time auction procedure called PAUSE (Progressive Adaptive User Selection Environment) for use in assigning COLR (Carrier of Last Resort) responsibility for Universal Service. The auction incorporates synergies by permitting all combinatorial bids, allows for multiple winners, and minimizes the possibility of bidder collusion. The procedure is computationally manageable for the auctioneer and thus is very efficient to run. The inherent computational complexity of combinatorial bidding cannot be eliminated. However, in this auction the computational burden of evaluating synergies rests with the bidders claiming those synergies, while the auctioneer simply checks that a bid is valid.

(*Auctions; Combinatorial Bidding; Universal Service; Carrier of Last Resort; Synergies*)

1 Introduction

Many auctions involve the sale of more than one property. It is not difficult to think of examples of auctions in which the value of a property to a bidder is increased if another property or group of properties is won by that bidder; this superadditive or synergistic effect may be bidder-specific. Auction authorities clearly have an incentive to structure their auctions so as to allow bidders to realize their synergies on combinations of properties in such a way that will be both fair to the bidders and practical to implement. The most obvious approach is to permit bids on groups of properties, called *combinatorial bids*.[1]

Without the allowance of combinatorial bids, bidders will face 'exposure risk' (Rothkopf, Pekec and Harstad 1997). Suppose that an individual bidder has a synergy that is specific to him on a particular block of properties. The bidder may find that an unsuccessful attempt to acquire the block may lead him to commit to a price for a group of properties that is higher than what they are worth to him. Alternatively, the bidder may be unwilling to risk bidding above the sum of his individual valuations, and thus may not be able to obtain the block for which the synergy makes him the efficient recipient.

Given the considerable importance of combinatorial bids, it may be surprising that few auctions—in either theory or practice—have allowed combinatorial bidding. This is undoubtedly due at least in part to the fact that combinatorial bidding is computationally burdensome. For example, Rassenti, Smith and Bulfin (1982) developed a combinatorial auction procedure for airport time slots to competing airlines. (See also Grether, Isaac and Plott (1989) and McCabe, Rassenti and Smith (1991).) Their optimization procedure (called 'RSB') is based on a mathematical programming formulation. However, as the authors themselves point out: "A problem of the enormous dimensions dictated by even a four-city application (perhaps 15,000 constraints and 100,000 variables) will present a significant challenge for the finest configuration of hardware and software available." In general, for a combinatorial auction the determination of an optimal set of bids is an NP-complete problem.

1.1 Selecting FCC Licensees

In the mid 1980s, the Office of Plans and Policy (OPP) of the Federal Communications Commission (FCC) issued a working paper that was soon to have an enormous impact. The paper, by Evan Kwerel and Alex D. Felker, proposed that FCC licensees be selected not by lottery or through hearings—as had been the standard practice to that point—but rather via auction (Kwerel and Felker 1985). This proposal came to fruition with the passage of the Omnibus Budget Reconciliation Act (1993), whereby the US Congress authorized the use of auctions for assigning the electromagnetic spectrum for Personal Communications Services (PCS).[2] The Office of Plans and Pol-

[1]Rothkopf, Pekec and Harstad (1997) prefer the term 'combinational bids'. Our terminology is consistent with that generally used in the telecommunications literature.

[2]Personal Communications Services are a broad family of mobile communications services that allow people access to the Public Switched Telephone Network (PSTN) regardless of where they are located. PCS includes not only POTS ('Plain Old Telephone

icy was given the responsibility for managing the process, which included developing general spectrum auction rules.

1.2 Structure of the PCS Auction

The designers of the auction[3] were especially mindful to ensure there would be a robust and competitive market for PCS, and that the auction design would permit broad participation in the provision of PCS, including participation by existing cellular providers. As of 1 December 1997 the FCC has held 16 auctions.

Specifically, the FCC adopted a structure consisting of a simultaneous multiple-round auction in which collections of licenses are auctioned simultaneously during discrete rounds, where no sale takes place until the bidding is concluded on all licenses. The auction is structured in three stages, each with an unspecified number of bidding rounds. At the end of each round, the highest bid becomes the leading bid, and the results are made available to all bidders before the start of the next round. At the end of the last round of the third stage, the leading bidder on each property is designated the sole winner on that property. The detailed rules of the auction can be found in FCC documents, most notably FCC (1993). Key features of the auction include the following:

Activity Rules

Eligibility Requirements. Before the start of the auction, each bidder is required to specify the number of licenses he hopes to win and, based on this number, is required to post a bond. During the auction, a bidder is designated active on a particular property if either he has the leading bid from the previous round, or if he has submitted an acceptable improving bid in the current round. In the first stage of the auction, bidders are required to remain active on licenses covering a population which is at least A_1 percent of the total population for which they wish to remain eligible to bid; in the second stage, to be eligible they are required to remain active on licenses covering at least A_2 percent (with $A_2 > A_1$), and in the final stage A_3 percent ($A_3 > A_2$).[4]

The eligibility requirements are intended to thwart the 'deception effect', whereby a firm might bid cautiously, waiting to see how the others bid while

Service') but also data, facsimile, video communication, and other services.

[3]The PCS auction was developed through the efforts of many people. Rothkopf, Pekec and Harstad (1997) provide complete details.

[4]For example, bounds that have been used are $(A_1, A_2, A_3) = (33, 67, 100)$.

not revealing its own interests until late in the auction.

Bid Increments. In order to be acceptable, a bid must improve the previous leading bid by at least the specified minimum amount (e.g., 5% of the leading bid) set by the auction authority. This helps maintain the speed of the auction.

Transition Between Stages. The auction moves from the first stage to the second stage when there are no bids on more than T_1 percent of the population base for three consecutive rounds, and from the second stage to the third stage when there are bids on no more than T_2 percent of the population base.[5] However, in order to speed the progress of the auction, the auction authority has, on occasion, moved an auction to a subsequent stage before reaching one of the benchmark transition points. The auction closes when bidding stops on every license.

Bid Waivers

The activity rules are balanced by an allocation of a small number of bid waivers to each player to be used at will to maintain eligibility for a round without meeting the eligibility criteria. This allowance for bid waivers may be viewed as an effort to generally increase bidder flexibility.

Bid Withdrawals

A leading bidder is permitted to withdraw his bid during the course of the auction, but is penalized by being required to pay the difference between his bid and the price for which the license ultimately sold; a winning bidder withdrawing after the close of the auction suffers an extra penalty.

This allowance for bid withdrawals may be viewed as an effort to reduce the exposure risk to bidders attempting to realize their synergies; the FCC has not yet permitted combinatorial bids in the Spectrum auctions. Rothkopf, Pekec and Harstad (1997) persuasively argue the disallowal of combinatorial bids is a consequence of the economists' briefs—in particular that of McAfee (1993)—that argued that the only choice was between completely disallowing, or permitting all possible, combinatorial bids, and the latter option in the worst-case scenario would be computationally intractable, as pointed out above.

[5] The values $(T_1, T_2) = (5, 10)$ have been used.

1.3 Universal Service and Carrier of Last Resort

Combinatorial auction design is a live issue with regard the Telecommunications Act of 1996, which includes a requirement that a joint board of Federal and state regulators consider ways to reform the current methods of providing Universal Service subsidies for high-cost areas.[6] Several parties have advocated that competitive bidding be used to determine these subsidies. A policy paper issued by the public interest group, Citizens for a Sound Economy Foundation (Leighton 1996), states the case clearly:

> Various methods have been discussed, but the best way may be to allow the marketplace—through competitive bidding—to determine the size and scope of any subsidies. Appropriately designed auctions may serve as an effective means to minimize the costs of service in these areas, while providing equal or greater quality of service.

Leighton also points out an additional feature that such an auction should ideally have, viz., provision for multiple winners:

> More than one bidder could win a subsidy in the auction, though rules would have to be set forth to clarify which bidders would be eligible for support. While no provider would be denied the right of entry, the subsidy might be limited to those who participated in the auction and offered a sufficiently low bid. Support would be distributed according to the number of subscribers served by the provider, with the subsidy guaranteed at this level for a specified period of time, say three to five years.

Leighton sums up the argument for competitive bidding as follows:

> An obvious advantage of this subsidy arrangement is that it decreases the likelihood that providers will inflate their costs and make it more likely that efficient providers will be rewarded. This tendency toward efficient operations stems from the best of enforcement mechanisms: competition. As competing providers bid for a subsidy, the value of the combined return (retail price plus

[6] Universal Service subsidies are government subsidies given to telecommunications service providers. These subsidies are intended to promote the availability of quality services at just, reasonable and affordable rates to all consumers, including those in high-cost areas (as well as those in low-income, rural and insular areas) at rates that are reasonably comparable to those charged in urban areas. Universal Service directives are provided in the Telecommunications Act (1996).

> subsidy) should approximate the cost incurred by the most efficient provider. The end result would be service provided to high-cost areas at a lower cost to the taxpayer.

A U.S. telephone carrier that undertakes various obligations as a condition for receipt of Universal Service support is called a *Carrier of Last Resort* (COLR). In a COLR auction, firms would bid for COLR subsidies on geographic areas, where the winning firms are those that have submitted the lowest subsidy bids.

Synergies—in particular, cost synergies—may be a significant consideration in the design of a COLR auction; i.e., it is conceivable that a firm might find it less costly to provide COLR service for a particular geographic area if it serves it together with some collection of other areas. Hence the relevance of combinatorial bidding.

1.4 Computationally Manageable Auctions

Attempts to make the combinatorial auction design problem tractable through specific restrictions on the bidding mechanism have taken the approach of considering specialized structures that are amenable to analysis. This work includes, for example, that of Krishna and Rosenthal (1995) and Rosenthal and Wang (1996), in which sets of properties are considered by a number of 'local' bidders interested in only a single property each, as well as by a number of 'global' bidders each interested in multiple properties.

The following question might reasonably be asked: What are the least restrictive structures that would result in an auction being computationally tractable for the auctioneer to determine the revenue-maximizing outcome?[7] This is the question considered by Rothkopf, Pekec and Harstad (1997). They thoroughly analyze this problem, consider several different structures for combinatorial bids, and constructively show that the structures described are indeed computationally tractable (they use the term 'computationally manageable'). While this manner of dealing with the computational intractability violates what might be termed the 'McAfee principle' of allowing *all if any* combinatorial bids, Rothkopf, Pekec and Harstad (1997) show in a certain sense how far one can go in this direction: in each instance, they demonstrate their approach to be best possible by proving that the next level of generality will result in an NP-complete problem.

[7]Here 'computationally tractable' is the standard concept that an upper bound on computation time for the given class of computational problems can be expressed as a polynomial function of the size of the input.

For example, they show that allowing bids on arbitrary doubletons reduces to finding a maximum weight matching in a graph. By the algorithm due to Edmonds (1965), such a matching can be found in $O(n^3)$ time. However, they also show that allowing bids on arbitrary tripletons reduces to the 3-set packing problem, which is NP-complete (Karp 1972).

Rothkopf, Pekec and Harstad point out that several of the structures they identify as being computationally manageable have potential relevance to real-world auctions. For example, they consider the case where there exists a total ordering of the properties, and show that allowing bids on intervals, i.e., consecutive properties, is computational manageable. They suggest that such a structure might be applicable to bidding on oil leases, and report that some years ago the State of California sold oil leases in Southern California that formed a single swath of offshore waters. They also show, however, that for the two-dimensional analogue of intervals, i.e., rectangles of properties, the problem is NP-complete.

Rothkopf, Pekec and Harstad do make a convincing case that there are many natural ways to restrict the nature of the bids so as to make combinatorial auctions computationally tractable. However, they also warn that this then raises the problem of deciding *which* combinations of bids to allow.

1.5 Adaptive User Selection Mechanism (AUSM)

Thus far, the only auction approach that has allowed for all possible combinatorial bids is the AUSM (Adaptive User Selection Mechanism), which has been tested experimentally at the California Institute of Technology (Banks, Ledyard and Porter 1989). In AUSM, bids can be submitted at any time, with bidding stopping according to some pre-specified closing rule. (The original stopping rule specified that the auction stops when no new bid is made soon enough after the last bid.) However, as pointed out by Bykowsky, Cull and Ledyard (1995), this procedure is susceptible to the 'threshold problem'. Specifically, a group of players who jointly desire a subset of properties may have difficulty coordinating their bids to displace a single bid on the subset of properties.

To address this problem, Banks, Ledyard and Porter (1989) designed a modification of AUSM that makes use of a 'stand-by queue' in which bidders announce to all other bidders via a bulletin board their willingness to pay a certain price for a specific combination of licenses. As described in Bykowsky and Cull (1993): "In essence, the stand-by queue serves as a voluntary contribution mechanism in which perspective contributors attempt to move, in a repeated game context, to a mutually desirable equilibrium."

While a stand-by queue can help overcome the threshold problem, it is less effective at dealing with the related 'free-rider' problem. A group of players who jointly desire a subset of properties may, in principle, be able to coordinate their bidding via a bulletin board, but each player from the group will have an incentive to wait for the others in the group to improve their bids, thus retaining more of the benefit from the joint bid for itself.

In response to criticism that a combinatorial auction is difficult to implement, Bykowsky, Cull and Ledyard (1995) point out that it has in fact been used in the type of environment that exists in the PCS auction and that, in this real-world case, AUSM performed very successfully. Specifically:

> An AUSM-like mechanism was recently used successfully to acquire logistics services across 850 connected routes. The version employed allowed bidders to submit bids (for as many packages as they wished) on spreadsheets. In between each bidding round, bids were processed using an R/S 6000. The maximum computation time in each round was less than 30 minutes, although bidders had more time that that to rebid. Bidders received information about the current winning allocation in the form of a spreadsheet. The mechanism saved the firm an estimated $10–15 million on a total cost of $150 million.

They conclude that "the growing body of experimental, theoretical, and actual auction data concerning bidding in such environments will further demonstrate the theoretical desirability and the practical usefulness of AUSM-like mechanisms".

1.6 Jump Bidding

A potential problem that can occur with simultaneous, multiple-round auctions has been reported by McAfee and McMillan (1996). In the portion of the PCS auction known as the MTA auction,[8] aggressive bidding in early rounds took the form of 'jump bidding': entering bids far above that required by the minimum bid increment. The intention of this tactic, which we will call more specifically *price-jump bidding*, is to warn weaker rivals against competing on specific properties. In a COLR auction, which would be a low-bid auction, jump bidding would mean entering bids far below that required by the minimum bid increment.

Although McAfee and McMillan describe jump bidding as being not a problem but rather only a part of bidder strategies, clearly jump bidding is

[8] The MTA auction ran from December 1994 to March 1995 and sold broadband licenses covering the 51 'Major Trading Areas', or MTAs, into which the United States is divided.

of concern to the FCC. In the Commission's Third Report and Order (FCC 1997)—a document self-described as making "substantive amendments and modifications to our general competitive bidding rules for all auctionable services"–jump bidding is addressed in paragraph 143, which begins:

> Several commentators suggest that jump bidding is not a problem of serious concern. Some theoretical literature, however, suggests that bidders could use jump bidding to manipulate the auction process and potentially reduce efficiency of the auction.

In a combinatorial auction, there is also the possibility of another type of jump bidding, *block-jump bidding*, in which a bid by a powerful player for a block of several properties could be effective at preventing small players from piecing together a comparable composite bid, i.e., the threshold problem.

1.7 Bidder Collusion

Experience with the PCS auction has revealed the possibility for a type of bidder collusion, which we designate as *property-preference signaling*. The idea is that bidders signal their strong interest to win specific properties in subsequent rounds by encoding this information in the bid prices they submit. This information is collectively used by bidders to bid noncompetitively on each others' designated properties. This tactic can carry the additional benefit—like price-jump bidding—of warning away weaker rivals.

As reported in *The Economist* (1997), a small firm in Texas called High Plains Wireless has claimed that Mercury PCS, a rival for licenses in Texas, made use of property-preference signalling during a January 1997 auction. According to High Plains Wireless, Mercury signaled its intention to go after specific licenses in subsequent rounds by submitting bids in earlier rounds that ended with the area codes of the cities it strongly desired. Presumably, what made this possible was that bids in the auction could be submitted to a high order of precision. Although this is generally a good idea in auction design, as high precision tends to prevent against tie bids, high precision bidding can also permit opportunities for encoding information. According to *The Economist*, the FCC is considering in future auctions to continue to accept bids to a high order of precision, but to reveal them to a lower order of precision, thus rendering difficult this form of signaling.

A second form of collusive signaling, which we call *price-level signaling*, is conceivable in the situation where multiple winners for properties are permitted. In this case, two or more players become aware that they are actively interested in the same property on which they have the potential to share as multiple winners. Consequently each firm bids non-competitively so as to maximize its subsidy as a joint winner on that property.

1.8 COLR Auction Desiderata

Based on the preceding, we can present a list of properties that we might want to see in a a combinatorial auction with multiple winners for COLR.[9] The deception effect can be addressed in activity rules along the lines of those used in the PCS auction. However, in addition, a COLR auction should:

(a) Present the auctioneer with a tractable bid evaluation problem;

(b) Have a bounded completion time;

(c) Prevent against jump bidding and the threshold problem; and

(d) Minimize the opportunity for bidder collusion.

2 The PAUSE Auction

We describe a discrete-time auction procedure called PAUSE (Progressive Adaptive User Selection Environment) for use in assigning COLR (Carrier of Last Resort) responsibility. (Note that this auction structure can, with minor modifications, be adapted for use as a combinatorial PCS auction.) More specifically, PAUSE is a two-stage procedure, where:

> Stage 1 is a simultaneous, multiple-round auction, conducted in three substages, with progressive eligibility requirements and an improvement margin requirement, with bidders submitting bids on individual properties; and
>
> Stage 2 is a simultaneous, multiple-round auction, conducted in two substages, with progressive eligibility requirements and an improvement margin requirement, with combinatorial bids submitted via an AUSM (Adaptive User Selection Mechanism), to facilitate realization of player synergies.

[9]Following Milgrom (1996), we make the simplifying assumption that the fixed costs of service are the same across bidders.

Bid waivers can be included in both stages, but no bid withdrawals are permitted.

The PAUSE auction is designed to be fully general in that every possible combinatorial bid is available to the bidders. If, however, the auctioneer wishes to restrict the bids in any manner that he finds convenient to verify, the auction structure will accommodate this, and the auctioneer can announce to the bidders a list of attributes a bid must have. (An example of such an attribute might be: "Bids that are combinatorial are to be composed of geographically contiguous subsets of the properties".) This is formalized in the next section.

2.1 Definitions

Label *properties* $j \in J$, and *blocks* $k \in K$, where $K = K(J, A)$ is a subset of J defined by a set of *attributes* A that are computationally tractable for the auctioneer to verify for each member of K. Let

$$K_n = \{k \in K(J, A) : 1 \leq |k| \leq n\},$$

where $|k|$ is the number of properties in block k. (Thus, K_1 is the set of blocks allowed by the attribute set and consisting of a single property, K_2 is the set of allowed blocks consisting of at most two properties, K_3 is the set of allowed blocks consisting of at most three properties, and so forth.)

A *partition* $P = (p_1, p_2, \ldots, p_r)$ is a collection $p_1, p_2, \ldots, p_r \in K$ such that $\bigcup_{i=1}^{r} p_i = J$, and $p_i \cap p_j = \emptyset, i \neq j$. (In words, a partition is a grouping of all the properties in the auction into sets that do not overlap.)

A *composite bid* comprises a partition $P = (p_1, p_2, \ldots, p_r)$, together with an *evaluation*:

$$\big(C(P); c(p_1), c(p_2), \ldots, c(p_r)\big)$$

where

$$C(P) = \sum_{i=1}^{r} c(p_i), \tag{1}$$

and $c(p_i)$ is the *bid* for block p_i.

To be more precise, $c(p_i)$ is the *value of the bid for block* p_i. A composite bid consists of $3r + 1$ pieces of information, capable of registration in a database. The first piece of information is the total value of the composite bid, $C(P)$. The $3r$ pieces of information are, for each i ($i = 1, 2, \ldots, r$):
(1) the specification of the block p_i, (2) the value of the bid on the block

$c(p_i)$, and (3) the identity of the bidder for block p_i. All $3r + 1$ items of information are available from the database to all bidders.

Note that $c(p_i)$ is the *total subsidy for block* p_i. It corresponds to a *subsidy per subscriber in block* p_i of $c(p_i)/\|p_i\|$, where $\|p_i\|$ is the total number of subscribers[10] in all the properties in p_i.

2.2 Two Stages of the PAUSE Auction

Stage 1: Bidding on Individual Properties

The Bidders. Each bidder submits a collection of bids on individual properties. In each round there is an *improvement margin requirement*:

> The new bid must improve on the previous best bid on that property by *at least* ε and *strictly less than* 2ε.

The Auctioneer. In each round, for each property the auctioneer checks that a bid on that property is *valid* by checking:

> *Increment validity*: The bid satisfies the bounds of the improvement margin requirement.

In each round, the lowest valid bid on each property is accepted. The round ends when bidding ends on all properties. Stage 1 is divided into three substages. At the conclusion of the third substage, the leading (i.e., lowest) bids on the properties are registered to their respective owners, and the auctioneer announces the number of multiple winners that will be accepted and necessary for property j, as determined by the rule described below.

Activity Rules. A bidder is designated *active* on a property if he has the leading bid from the previous round or submits an acceptable bid in the current round. Each of the three substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the three stages, 60 percent, 70 percent and 80 percent of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to 2 occurs when there are bids on no more than 10 percent of the subscribers for three consecutive rounds, from substage 2 to substage 3 when there are bids on no more than 5 percent of the subscribers for three consecutive rounds.

[10] In this document, by subscribers we mean subscribers counted under the Universal Service provisions for support for high cost areas.

Multiple Winners. At the conclusion of the third substage, the auctioneer announces the number of winners on each property as determined by the *outcome rule*:[11] (1) if at least one competing bid is within $M_1 = 15$ percent of the lowest bid, then all who bid within M_1 percent of the lowest bid are designated as winners; (2) if no competing bid is within M_1 percent of the lowest bid but one is within $M_2 = 25$ percent, then the two lowest bidders are winners, and (3) if no bid is within M_2 percent of the lowest bid, then there is a single winner, viz., the lowest bidder. The number of multiple winners on each property j at the end of Stage 1 is denoted by $m(j)$.

Before the start of Stage 2, property j is replaced by $m(j)$ properties $j_1, j_2, j_3, \dots, j_{m(j)}$, each allocated a nominal number of subscribers equal to $\text{sub}(j)/m(j)$, where $\text{sub}(j)$ denotes the numbers of subscribers in property j.

Stage 2: Combinatorial Bidding

The Bidders. Each bidder submits a single composite bid on a collection of properties. In each round there is an *improvement margin requirement*:

> Let b be the number of *new* bids in the composite bid. The new evaluation must improve on the previous best evaluation by *at least* $b\varepsilon$ and *strictly less than* $2b\varepsilon$ (i.e., an average improvement per block of at least $b\varepsilon$ but less than $2b\varepsilon$).

Each bidder's partition $P = (p_1, p_2, \dots, p_r)$ is restricted to $p_i \in K_n$, where $c(p_i)$ is either a new bid for block i, or a registered bid. Initially, $n = 2$. For a composite bid to be valid,[12] for each property j the bid must not allocate j_s and j_t $(s \neq t)$ to the same player. In this stage of the auction, the bidder identities are made public.[13] Thus the validity of a composite bid—and in particular the requirements that the bid does not allocate j_s and j_t $(s \neq t)$ to

[11] This is the outcome rule proposed by Paul Milgrom (1996), who points out that other formulas for multiple winners are possible. The concept of the outcome rule itself is due to Milgrom.

[12] Note that synergies are accounted for via composite bids. Thus, to allow for multiple winners, players need to check the validity of their composite bids, which is not possible with a sealed-bid auction.

[13] R.P. McAfee and J. McMillan (1996) report that in the MTA broadband PCS auction the FCC revealed bidders' identities, judging that the risk of collusion was outweighed by the benefits of information. As McAfee and McMillan point out: "Bidders' identities are useful to the bidders for evaluating the meaning of others' bids, reducing the winner's curse and generally assisting sensible bidding." They add that "it takes only one maverick bidder to upset an attempt at collusion", and provide an illustrative example from the MTA auction. With synergies, one might expect the overlapping nature of composite bids would tend to make collusion all the more difficult.

the same player—can be checked by the player constructing the composite bid.

The Auctioneer. In each round, the auctioneer checks that a composite bid is *valid* by checking:

(i) *Bid validity*: Each bid which is asserted to be registered in the database is indeed so registered; that new bids satisfy $p_i \in K_n$, that is, that new bids are on allowed blocks of not more than n properties; and, for each property j, the composite bid does not allocate j_s and j_t $(s \neq t)$ to the same player.

(ii) *Evaluation validity*: Equation (1) holds, i.e., the value $C(P)$ of the composite bid is indeed the sum of the bids on each of its blocks, and

(iii) *Increment validity*: The bid evaluation $C(P)$ satisfies the bounds of the improvement margin requirement.

In each round of Stage 2, the new collection of bids on the blocks $\{c(p_i)\}$ are registered to their respective owners, and the lowest valid composite bid is accepted. The round ends when bidding ends. Stage 2 is divided into two substages.

The size of the bid increment, ε, and the rate of increase of the block size limit, n, are used by the auctioneer to control the speed of the auction, in conjunction with the activity rules.

Activity Rules. A bidder is *active* on a property if his bid on a block containing that property forms part of the accepted composite bid of the previous round, or if he submits a valid bid in the current round on a block containing that property. Each of the two substages contains an unspecified number of bidding rounds. The bidders must remain active on properties covering, respectively in the two stages, 90 percent and 98 percent of the number of subscribers for which they wish to remain eligible to bid. The transition from substage 1 to substage 2 occurs when there are bids on no more than 10 percent of the subscribers for three consecutive rounds.

Multiple Winners. At the conclusion of Stage 2, the $m(j)$ winners on property j are each designated a $1/m(j)$ share of the responsibility on property j. Specifically, the contractual obligation carried by each player is as follows:

(i) *The player will receive his bid subsidy per subscriber on* up to $1/m(j)$ *of the total number of subscribers in that property;*

(ii) *The regulatory authority may require any winner in that property who is not serving the full amount of his contractual share to serve any unserved subscriber in that property.*

The particular subscribers that constitute a contractual share are not specified; the player will compete for these subscribers with the other winners on that property. There is thus an incentive for players to actively seek to serve their share of subscribers, lest they be required to serve subscribers not of their choice.

A player's winning bid on property j will, in general, be part of a composite bid. Thus, the limitation on the fraction of customers for which a player will be subsidized prevents the player from cross-subsidizing property j from the other properties that comprise his bid. Of course, each player is free to compete for any or all the customers in property j, although the player will not receive subsidy for any customers beyond the fraction he has won in the auction.

Finally, note that it is essential that, before the start of Stage 2, the auctioneer specifies the rules that need to be satisfied by a valid composite bid in a manner that can be checked by players, as well as by the auctioneer. In particular, the auctioneer should not attempt to decide the number of multiple winners after the conclusion of the auction, since to do so would involve the auctioneer in a task of some considerable computational complexity.

2.3 Other Auction Rules

Bid Waivers

The auction can include bid waivers, especially if the time between rounds is short. In Stage 1, the number of waivers would be concurrent with the number of waivers issued in the PCS auctions, and in Stage 2 a number 1.5 times this amount.

Bid Withdrawals

No bid withdrawals are allowed in either stage. It may be asked why bid withdrawals were permitted the PCS auction, since they complicate the auction. The motivation is stated explicitly by P. Milgrom in his statement attached to GTE's comments (Milgrom 1996): "In effect, a bid withdrawal substitutes partially and quite imperfectly for combinatorial bidding."

David Porter (1996) reports results of experiments on auctions with bid withdrawals with penalties. He found that efficiency and revenue increase, but individual losses are larger. He also found that the increased efficiency does not outweigh the higher prices paid; thus, the bidder surplus falls.

More General Auctions

For simplicity of exposition, we have assumed that the properties $j \in J$ are disjoint, but other possibilities may be of interest. For example, in the auction of frequency bands for PCS, there may be two or more incompatible band plans, with a simple definition of disjoint properties within each band plan. In this case, Stage 1 could be used to set an initial price for each property in each band plan, with a composite bid in Stage 2 relating to a player's preferred band plan.

3 The Propositions

The PAUSE auction described in the previous section clearly allows for all combinatorial bids and allows for multiple winners. In addition, it satisfies items (a), (b), (c) and (d) listed under the desiderata of Section 1.

The auctioneer's procedure described in Section 2 for evaluating the combinatorial bids clearly is computationally tractable, assuming that the associated information storage problem is manageable. This is easily settled by Proposition 1, which addresses desiderata item (a). However, Proposition 1 first addresses desiderata item (b) by showing that the completion time of the auction is bounded.

Proposition 1. Bounds on Number of Rounds

Since in each round of Stage 2 the value of the accepted composite bid must decrease by at least ε over the previously accepted composite bid, the number of rounds in total is bounded above by $C_0(P_0)/\varepsilon$, where $C_0(P_0)$ is the value of the opening composite bid.

Let B be the number of bidders. Since each bidder is allowed to make at most one composite bid per round, the maximum number of bids that needs to be registered by the auctioneer is bounded above by $\frac{C_0(P_0)}{\varepsilon}B|J|$. □

Remarks. Although for the procedure we present here, the auctioneer's problem of determining the winning bid is computationally manageable, for a bidder, it may be an NP-complete problem to determine whether he can make a composite bid that beats the currently accepted composite bid. However, there is very little computational burden for small players interested in only a small number of properties. If no synergies are claimed, then the auction reduces to an auction of the type utilized for the PCS licenses. As discussed in the Introduction, the results of Rothkopf, Pekec and Harstad (1997) show that, if the form of composite bids is restricted in one or other of several possible ways, then the problem can be made computationally manageable. However, in cases where the bidders are unlikely to agree upon the form of the appropriate restriction on composite bids, we view the elicitation of the form and size of potential synergies as a major purpose of the auction proposed here.

Work on computationally difficult problems shows that in several situations where finding the exact optimum is hard, finding a good approximation to the optimum with high probability may be relatively easy (Jerrum and Sinclair 1996). It is our belief that the traditional problems of elicitation and gaming are more serious difficulties than the possible computational burden on those bidders claiming complex synergies.

Proposition 2 responds to desiderata item (c).

Proposition 2. Prevention against Jump Bidding

The rule that the improvement margin is bounded above by 2ε reduces the possibility of price-jump bidding. The progression of allowable block size n reduces the possibility of block-jump bidding and mitigates the threshold problem. □

Remarks. The upper bound on the improvement margin also tends to reduce the bidders' computation requirements, by limiting the range of possibilities that need to be considered. The existence of the block size also aids the auctioneer in controlling the speed at which the auction progresses.

Proposition 3 addresses desiderata item (d).

Proposition 3. Minimization of Bidder Collusion

The opportunity for both types of bidder signaling are minimized. Property-preference signaling is minimized via the improvement margin requirement. Price-level signaling is minimized by having the number of multiple winners determined prior to Stage 2; thus, most of the bidder surplus for the individual properties has already been extracted. Any additional surplus is most likely to be due to synergies, for which price-level signaling is very difficult.

Further price-level signaling is very difficult to take advantage of, even if successful, due to the very nature of combinatorial bids, viz., that they involve more than one property, and they overlap in ways that usually cannot be easily disaggregated. □

Remarks. Since a fixed number of multiple winners can be accepted on a given individual property, and the contracts for each will carry the same obligations, then rational behavior by the bidders will generally lead to them achieving the same price (within ε) on successful bids on blocks comprising just that property. This is simply the law of one price, i.e., a bidder is unlikely to pay more for something identical available at a lower price.

Note that a price for a property cannot be determined from a composite bid if, within that composite bid, the property is part of a larger block. Similarly, if the contracts carry different obligations, then rational behavior by the bidders will lead to a variety of achieved prices reflecting the bidders' views about the value to the bidders of the various obligations.

4 Implementation

Before the start of a PCS auction, each bidder is required to specify the number of licenses he hopes to win. Based on this number, he is required to post a bond. A similar bonding procedure may be appropriate for a COLR auction.

There are two additional issues associated with this auction: (1) determining the opening bids; and (2) whether there is a need for bidders to conduct cost studies, as well as the administrative costs of the auction.

4.1 Opening Bids

Opening bids for each property could be set as the lower of the historical cost and the forward-looking cost for that property. By 'forward-looking cost' we mean, for example, the Total Service Long Run Incremental Cost (Hatfield 1994) or the cost obtained from the Benchmark Cost Model (MCI et al. 1995). Leighton (1996) provides further discussion.

If the lower of these two costs is the historical cost, then it would be announced that historically service has been provided on this property at a certain subsidy level and it is expected that service will be provided at no higher than that level in the future.

If, on the other hand, the lower of these two costs is the forward-looking cost, then forward-looking cost would serve as a starting point for the analysis to determine the minimum subsidy to provide service in a given market.

4.2 Bidder Costs Studies and Auction Authority's Administrative Costs

Note that no new bidder cost studies are required for participation in this auction. To participate, each firm needs to know only the value of its synergies, something that such a firm most likely already calculates and is part of the firm's information.

The costs of auction administration will be minimal, as shown by Proposition 1, which provides bounds on the number of rounds and on the number of bids that need to be registered.

5 Testing and Validation

Like the PCS and AUSM auction structures, PAUSE is probably too complex to admit much theoretical analysis. However, to prove its value as a combinatorial bidding design, the PAUSE auction could be tested and validated in an experimental setting. We suggest a two-phase testing procedure. In the first phase, the PAUSE auction can be refined by adjusting the parameters through simulation; general guidelines are provided. In the second phase, the PAUSE auction can be tested against other auction approaches.

Testing—Phase 1: Refinement of the PAUSE auction

We suggest that simulation be used to adjust, specifically, the sets of parameters described below.

Eligibility Requirements: The parameters A_i that determine the activity levels for the stages.

In the early PCS auctions these bounds were $(A_1, A_2, A_3) = (33, 67, 100)$. However, the first two bounds were found to be insufficiently stringent, and were increased in later auctions. Rothkopf, Pekec and Harstad (1997) report: "The particular percentages attached to eligibility rules have been tweaked from auction to auction, most notably backing away from 100% in stage 3 to 95–98%."

Bid Increments and Block Size: The bid increment ε and the progression of the block size n.

As mentioned in Section 2, the bid increment, ε, and the rate of increase of the block size limit, n, are used to control the speed of the auction in conjunction with the activity rules (as well as preventing against both types of jump bidding). For example, the auctioneer might move n from the starting value of 2, to 3, 4, 5, ...; however, the auctioneer might instead move n to 4, 8, 16,.... In either case the value of the bid increment would decrease, and the activity rule percentage increase, as n increases.

Transition between Substages: The parameters T_i that determine the substage transition points.

The FCC has found that, on occasion, it has been desirable to make *ad hoc* modifications to the T_i values.

Outcome Rule: The parameters M_1 and M_2 that specify the outcome rule.

Milgrom (1996) points out that the cut-off values he presents of 15 and 25 percent respectively are illustrative and not based on any detailed analysis. He explains that a more restrictive standard should be set for including competitors beyond the second, because these additional competitors are expected to contribute less to consumer welfare.

Testing—Phase 2: Comparison of the PAUSE auction with other auction processes

Among the possible validation procedures, that PAUSE auction could be tested by adapting either or both of the procedures employed by the teams of Rassenti, Smith and Bulfin (1982) and Ledyard, Porter and Rangel (1997), respectively.

Rassenti, Smith and Bulfin (1982) made use of a $2 \times 2 \times 2$ experimental design, with the three factors being: (1) an alternative method (control) versus RSB (treatment), (2) experience level of subject (experienced versus inexperienced), and (3) combinatorial complexity of resource utilization ('easy' versus 'difficult'). In an appendix to their paper, Rassenti, Smith and Bulfin include the instructions and forms used in their experiments.

Ledyard, Porter and Rangel (1997) conducted over 130 auctions under controlled conditions to examine the robustness of several auction mechanisms to allocate multiple objects. They compared three auction processes: (1) the simultaneous discrete process used in the PCS auction, (2) a sequential auction process, and (3) a continuous-time AUSM mechanism. Their paper provides complete details.

In this paper, we have focused on the computational challenges involved in combinatorial auctions, an essential step before testing. With the results of tests, it will be possible to explore the allocative efficiency of auction designs.

6 Conclusion

In March 1997, an outline of a preliminary version of this auction was submitted *ex parte* to the FCC by the Citizens for a Sound Economy Foundation (Kelly and Steinberg 1997). In March 1998, the FCC released a 'Request for Proposal' (RFP) for a combinatorial bidding system (FCC 1998).

In the near future, it seems likely that there will be substantial testing and validation of combinatorial auction mechanisms.

Acknowledgements

We are grateful to Richard Gibbens, the editors and reviewers for valuable comments on earlier drafts.

References

'Learning to play the game', *The Economist*, 17 May 1997, p. 120.

Omnibus Budget Reconciliation Act, Pub. L. No. 103–66, 107 Stat. 312 (1993).

Telecommunications Act of 1996, Pub. LA. No.104–104, 110 Stat. 56 (1996).

Federal Communications Commission, 'In the matter of implementation of Section 309(j) of Communication Act, Competitive Bidding: Notice of Proposed Rule Making', PP Docket No. FCC 93–253 (1993).

Federal Communications Commission, 'Benchmark Cost Model' (a joint submission by MCI Communications Corporation, NYNEX Corporation, Sprint Corporation, and US West, Inc.), CC Docket No. 80–296 (1995).

Federal Communications Commission, 'Third Report and Order and Second Further Notice of Proposed Rulemaking' (1997).

Federal Communications Commission, 'Combinatorial Bidding System', Solicitation S–9854013 (1998).

Hatfield Associates, 'The Cost of Basic Universal Service' (prepared for MCI Communications Corporation), Boulder, Colorado (1994).

Banks, J.S., Ledyard, J.O. and Porter, D., 'Allocating uncertain and unresponsive resources: an experimental approach', *Rand Journal of Economics* **20** (1989) 1–25.

Bykowsky, M.M. and Cull, R.J., ‘Issues in implementing a personal communications services auction’, attached to Comments of the National Telecommunications and Information Administration in PP Docket No. 93–253 (1993).

Bykowsky, M.M., Cull, R.J. and Ledyard, J.O., ‘Mutually Destructive Bidding: The FCC Auction Design Problem’, Social Science Working Paper 916, California Institute of Technology, January 1995.

Edmonds, J., ‘Maximum matching and a polyhedron with 0,1 vertices’, *Journal of Research of the National Bureau of Standards* **69B** (1965) 125–130.

Grether, D.M. Isaac, R.M. and Plott, C.R., *The Allocation of Scarce Resources, Experimental Economics and the Problem of Allocating Airport Slots*, Westview Press (1989).

Jerrum, M. and Sinclair, A., ‘The Markov Chain Monte Carlo method: an approach to approximate counting and integration’, in *Approximation Algorithms for NP-Hard Problems* (ed. Dorit S. Hochbaum), PWS Publishing Company, Boston, Massachusetts (1996).

Karp, R.M., ‘Reducibility among combinatorial problems’, in *Complexity of Computer Computations* (ed. R.E. Miller and J.W. Thatcher), Plenum Press, New York (1972) 85–103.

Kelly, F. and Steinberg, R., ‘A Combinatorial Auction with Multiple Winners for COLR’, University of Cambridge, 17 March 1997. [Submitted *ex parte* 18 March 1997 by Citizens for a Sound Economy Foundation to US Federal Communications Commission re CC Docket No. 96–45, Federal-State Board on Universal Service.]

Krishna, V. and Rosenthal, R.W., ‘Simultaneous auctions with synergies’, *Games and Economic Behavior* **17** (1996) 1–31.

Kwerel, E. and Felker, A.D., ‘Using Auctions to Select FCC Licensees’, Office of Plans and Policy Working Paper No. 16, Federal Communications Commission, May 1985.

Ledyard, J.O., Porter, D. and Rangel, A., ‘Experiments testing multiobject allocation mechanisms’, *Journal of Economics and Management Strategy* **6** (1997) 639–675.

Leighton, W., ‘What to do about Universal Service Subsidies: Support for High-Cost Areas’, Issue Analysis 39 (September 30, 1996), Citizens for a Sound Economy Foundation, Washington, D.C.

McAfee, R.P., ‘Auction Design for Personal Communications Services: Reply Comments’, PacTel Exhibit in PP Docket No. 93–253 (1993).

McAfee, R.P. and McMillan, J., ‘Analyzing the airwaves auction’, *Journal of Economic Perspectives* **10** (1996) 159–175.

McCabe, K.A., Rassenti, S.J. and Smith, V.L., ‘Smart computer assisted markets’, *Science* **254** (1991) 534–538.

Milgrom, P.R., ‘Statement attached to GTE’s Comments in response to questions’, Federal Communications Commission, Notice of Proposed Rulemaking, Federal-State Joint Board on Universal Service, CC Docket No. 96–45 (1996).

Porter, D., ‘The Effect of Bid Withdrawal in a Multi-Object Auction’, Working Paper, California Institute of Technology, May 1996.

Rassenti, S.J., Smith, V.L. and Bulfin, R.L., ‘A combinatorial auction mechanism for airport time slot allocation’, *Bell Journal of Economics* **13** (1982) 402–417.

Rosenthal, R.W. and Wang, R., ‘Simultaneous auctions with synergies and common values’, *Games and Economic Behavior* **17** (1996) 32–55.

Rothkopf, M.H., Pekec, A. and Harstad, R.M., ‘Computationally manageable combinational auctions’, Working Paper, May 20, 1997. To appear in *Management Science*.