Bidding
Robert Wilson

Auctions are studied because they are market institutions of practical importance. Their simple procedural rules to resolve multilateral bargaining over the terms of trade enjoy enduring popularity. They also present simply several basic issues of price determination: the role of private information, the consequences of strategic behavior, and the effect of many traders. These issues have influenced the subject since the initial work of Vickrey (1961), the early contribution of Griesmer, Levitan, and Shubik (1967), and the influential dissertation by Ortega-Reichert (1968). Useful introductory surveys are by Engelbrecht-Wiggans (1980), Engelbrecht-Wiggans, Shubik, and Stark (1983), Milgrom (1985, 1987, 1989), and MacAfee and McMillan (1987); bibliographies are in MacAfee and McMillan (1987) and Stark and Rothkopf (1979); and Cassady (1967) provides an historical perspective.

This note supplements the entry on Auctions by summarizing some additional theoretical contributions to these issues. This literature relies on the game-theoretic perspective that emphasizes the implications of complete optimizing behavior. Omitted here are the experimental studies that offer alternative predictions of bidder behavior. It remains to determine which better describes the behavior of experienced, savvy bidders in the major auction markets [Hendricks and Porter (1988), Thiel (1988)]. Although general equilibrium models of closed economies have been studied [Schmeidler (1980), Shapley and Shubik (1977), Wilson (1978)], we focus on partial equilibrium models with bids and offers denominated in money terms. Also omitted are studies of markets with intermediaries such as brokers and specialists; models without private information [Dubey (1982), Milgrom (1986)]; and auctions in which losers also pay, as in price wars and wars of attrition.

In the traditional view, price determination is a consequence of market clearing: prices equate supply and demand. This clearing process is especially transparent in the case of auction markets. Essentially, auctions are markets with explicit trading rules that specify precisely how market clearing determines prices. For example, in a sealed-
bid auction of one or more identical indivisible items, the (interval of) clearing prices is
determined by intersecting the seller’s supply schedule (reflecting the number of units
available and announced reservation prices) with the demand schedule formed by array-
ing the buyers’ bids in descending order. Nondiscriminatory pricing sets the price at the
highest rejected bid, discriminatory pricing charges each successful bidder the amount of
his bid, and various intermediate cases are possible. Double auctions operate similarly
except that the supply schedule is constructed by arraying the sellers’ offers in ascending
order. With divisible commodities, the aggregate schedules are obtained by constructing
the sums of the traders’ demand and supply schedules at each price. Oral auctions, such
as the English auction, find a clearing price by calling for bids in ascending order. An
oral double auction, or ‘bid-ask’ market, allows free outcry of bids and offers that can be
accepted immediately and therefore depends on participants’ judgments about the likely
clearing price.

The variety of possible procedural rules is large, so theoretical studies emphasize
the characterization of efficient trading rules, such as rules that are optimal for the buy-
ers or the sellers. The design of trading rules is subject to the incentive compatibility
constraints induced by the traders’ private information and the option of any trader
to forego participation or trade. Auctions are especially restrictive trading mechanisms
because their rules are specified independently of information about the distribution of
traders’ attributes, even if this information is common knowledge. On the other hand,
auctions have been important market institutions for millennia precisely because they
are efficient or nearly so in a wide variety of environments.

Much of the theory of efficient trading rules studies ‘direct revelation’ games in
which, in equilibrium, each trader’s action consists of a direct report of his private in-
formation. This approach loses no generality in static models but the resulting optimal
rules depend on the distribution of traders’ attributes: only in special cases can they be
implemented fully as auctions. (In the extreme case of highly correlated private infor-
mation, an optimal trading rule can be designed by the seller to extract most or all of the
potential revenue [Crémer and McLean (1985), Myerson (1981)].) The theory therefore
divides between the study of auctions, in which traders’ strategies take account of the
distribution of attributes, and the study of optimal direct revelation games, in which
the trading rule incorporates this data. We concentrate on auctions here, but mention intersections with the general theory.

A trading rule specifies each trader’s feasible actions and the prices and trades resulting from their joint actions. Models also specify each trader’s information and preferences. Typically each trader $i$ knows privately an observation $s_i$ affecting his preferences, and the restrictive assumption is adopted that the joint probability distribution of these observations and any salient unobserved random variable $v$ is common knowledge among the participants. (The observation $s_i$ is often taken to be real valued for simplicity; it could be the bidder’s posterior certainty-equivalent valuation of the item based on his private information.) A strategy therefore specifies a trader’s actions depending on his observation and any further observations (such as others’ bids) made in process. Trader $i$’s expected utility $u_i$ depends on the received quantity $q_i$, the price(s) $p_i$ at which these units are traded, his observation $s_i$, the array $S_i = \{s_j \mid j \neq i\}$ of others’ observations, and possibly on other variables $v$.

Interesting special cases of the probabilistic structure are: independent and identically distributed (iid) observations; conditionally iid observations given $v$; and more generally, affiliated observations (e.g., nonnegative correlation on any rectangle). In each case assume that the (conditional) distribution of an observation satisfies the monotone hazard rate or likelihood ratio property. Most of the familiar probability distributions satisfy these assumptions; e.g., lognormal distributions are often used in applications to oil-lease bidding.

Interesting special cases of the preference structure for a single item include: private values, $u_i = s_i - p_i$; a common value, $u_i = v - p_i$; mixed values, $u_i = u(s_i, v) - p_i$, where $u$ is increasing; and private-value cases with common risk aversion, $u_i = U(s_i - p_i)$, where $U$ is increasing and concave. Relevant features are summarized in the expected utility $\bar{u}(s_i, S_i) = \mathcal{E}\{u(s_i, v) \mid s_i, S_i\}$.

Other features are also addressed in some formulations: the seller’s optimal reservation price, a trader’s option to obtain costly further observations to improve his information, bids submitted jointly by syndicates of traders, and entry fees and auxiliary contingent payments such as royalties. (Bidding on the royalty rather than the price has been used in auctions of oil leases.) Uncertainty about the number of bidders is easily
included if this number is independent of the bidders’ observations; however, somewhat
different comparative statics results ensue. If there are bid preparation costs, exposure
constraints (total amount of bids submitted) or portfolio motives, then participation in
an auction is itself a strategic action and may involve randomization if there are too
many potential bidders for all to expect to recoup their costs. If information is costly
and subject to choice then even with many bidders there is typically an upper bound
on the bidders’ total expenditures and each bidder may choose to collect relatively lit-
tle information [Matthews (1984)]. Bids in an auction can be affected by motivations
to signal information relevant in subsequent markets [Bikhchandani and Huang (1980)].
Incentives for collusion and the operation of ‘rings’ of colluding bidders have also been
studied [Graham and Marshall (1987)]. Repeated auctions introduce novel features, such
as reputation effects, that severely alter the results; e.g., one bidder with privileged in-
formation can win systematically [Bikhchandani (1985)].

Most theoretical studies assume that the traders’ strategies form a Nash equilibrium,
or in dynamic formulations, a sequential equilibrium: each strategy in each contingency
is optimal for the remainder of the game. For many auction models the equilibrium
strategies can be characterized elegantly in terms of the joint distribution of observations
and bids [Milgrom and Weber (1985)]. If the bidders (on the same side of the market)
are positioned symmetrically \textit{ex ante} then one focuses on the symmetric equilibrium in
which all bidders use the same strategy, which is an increasing function of one’s observa-
tion. A large class of symmetric discriminatory auctions have \textit{only} symmetric equilibria
[Maskin and Riley (1986)]; they are usually characterized by differential equations, as
illustrated for various cases in Milgrom and Weber (1982), Reece (1978), Wilson (1977,
1985). In nondiscriminatory auctions a single equation specifies the optimal bid as the
most one would be willing to pay conditional on one’s observation being the most op-
timistic. Results about symmetric equilibria are fairly robust: examples indicate that
under- or over-bidding by one participant engenders a similar but muted response by
others, and the difference from the symmetric equilibrium varies smoothly. In sealed-bid
discriminatory auctions with iid private values, one’s bid is essentially the conditional
expectation of the highest rejected valuation given that one’s valuation is acceptable. An
analogous property applies to mixed-value preferences. The important asymmetric cases
occur when some bidders’ information is superior to others’ (e.g., direct information about $v$); in these cases any bidder with strictly inferior information obtains expected profits at most zero, and bidders may use randomized strategies. If all bidders have private information (with a positive density satisfying technical restrictions) then typically equilibrium strategies are not randomized and positive expected profits result.

We summarize results mainly for the special probabilistic and preference structures mentioned above, and for symmetric equilibria. Also, multiple-item auctions introduce few novelties when there is a single seller offering a fixed supply of identical items and each bidder wants at most one, so we focus on the single item case. An exception is a ‘share auction’, in which bidders offer demand schedules for shares of a divisible item in fixed supply: in this case there can be a continuum of symmetric equilibria, and the seller’s expected revenue can be unaffected by more bidders [Wilson (1979), Bikhchandani and Huang (1988)].

A main effect of risk aversion is to increase bids in symmetric discriminatory auctions with iid private values. The seller can enhance this effect by imposing an entry fee (preferably decreasing in the amount of the bid and ultimately negative for the highest bids) [Matthews (1983), Maskin and Riley (1984)]. Risk aversion induces bidders to bid higher under discriminatory pricing, and in fact this rule makes the winning bid a less risky random variable. The seller therefore prefers discriminatory pricing, and more so if he too is risk averse. However, if bidders have decreasing absolute risk aversion (ARA), they have the reverse preference [Matthews (1987)]. With constant (or zero) ARA, a bidder’s higher price with discriminatory pricing is exactly balanced by the riskier price associated with nondiscriminatory pricing. With affiliated observations, the bidders prefer discriminatory pricing if they have constant ARA, and will be indifferent again at some degree of decreasing ARA. Affiliation biases the seller’s preferences in the opposite direction, towards nondiscriminatory pricing [Riley (1989)].

Hereafter we assume no risk aversion. Then, in the iid private-values model of bidders’ preferences, the seller’s expected revenue is the same for discriminatory and nondiscriminatory pricing [Harris and Raviv (1981ab), Myerson (1981), Riley and Samuelson (1981)]. Moreover, subject to a technical restriction, either of these is optimal among all possible trading rules provided the seller adopts an optimal reservation price [Harris and
Raviv (1981ab), Myerson (1981). With more general preferences, whenever \( \theta \) is increasing affiliation produces a distinct preference of the seller for (and the bidders against) nondiscriminatory pricing vs. discriminatory; indeed, the seller further prefers an oral auction [Milgrom and Weber (1982)]. This illustrates the `linkage principle`: the seller wants to reduce the bidders’ profits from their private information, and auction rules that reveal affiliated information publicly (inferences from bids in the case of oral auctions) or otherwise positively link one bidder’s price to another’s bid (nondiscriminatory pricing) are advantageous when observations are affiliated and therefore positively correlated. Similarly, the seller prefers to reveal publically any relevant affiliated information he has so as to reduce the bidders’ informational advantages vis-à-vis each other. (However, revealing non-affiliated information may be disadvantageous, and in particular this applies to the number of bidders, even when it is independent of other data [Matthews (1987)].) The seller can gain further by conditioning payments \textit{ex post} on realized values, as in the case of a royalty [Riley (1986)].

The main results about bidders’ strategies in single-item sealed-bid discriminatory auctions can be summarized for bidders with symmetric conditionally iid mixed-value preferences. \textit{Ex ante} each bidder has an equal chance of winning and the bidder with the most optimistic observation is predicted to win, namely \( i \) wins in the event \( W(s_i) \equiv \{ s_i > \max S_i \mid s_i \} \). (Failure to recognize that winning is an informative event, signaling that others’ observations were less optimistic, is called the winner’s curse [Capen, Clapp, and Campbell (1971)]; it is distressingly common in practice as well as in experiments. The implications of the fact that the maximum of several unbiased estimates is biased upward are apparently difficult to appreciate.) The most that \( i \) can profitably bid is therefore \( \hat{\theta}(s_i) = \mathbb{E}\{ \theta(s_i, S_i) \mid W(s_i) \} \), whereas the optimal bid is less than this, by a percentage that is of the order of \( 1/n \) when there are \( n \) bidders, reflecting the bidder’s monopoly rent both in terms of the limited number of bidders and the advantage of his private information, which are the two sources of bidders’ expected profits. [In a nondiscriminatory auction, \( i \) bids \( \hat{\theta}(s_i) \) computed from \( \hat{W}(s_i) \equiv \{ s_i = \max S_i \mid s_i \} \) but in equilibrium pays \( \hat{\theta}(\max S_i) \) if he wins.] With many bidders these rents are dissipated and, remarkably, the winning bid conveys essentially all the information about \( v \) contained in \( \max s_i \). In the common value model, the winning bid is a consistent estimator.
of the value whenever any consistent estimator exists that is a function of \( \max_i s_i \); the winning bid is asymptotically as good an estimator as is possible from extrema of the bidders’ observations. In particular, if the relative likelihood of a large observation is small for smaller values of \( v \), then the maximum bid converges in probability to \( v \) as the number of bidders increases [Milgrom (1979ab), Palfrey (1985), Wilson (1977)].

These features are reflected in the detailed calculations reported for models of oillease bidding [Reece (1978)]. Other examples are shown in Table 1, which exhibits the equilibrium strategies for a model that roughly approximates firms’ bidding for oil leases. Each bidder \( i \) observes \( s_i = (s_{i1}, s_{i2}) \) and \( u(s_i, v) = s_{i1}v \), where \( s_{i1} \) represents a private factor (e.g., price or discount factor), and \( s_{i2} \) represents an estimate of the common factor \( v \). Assume that, conditional on a location parameter \( \bar{s}_i \), the private factors are conditionally independent and \( \ln s_{i1} \) has mean \( \ln \bar{s}_i \) and variance \( \sigma_{i1}^2 \); and marginally \( \ln s_i \) has variance \( \sigma_i^2 \). Similarly, conditional on \( v \) the estimates are conditionally independent and \( \ln s_{i2} \) has mean \( \ln v \) and variance \( \sigma_{i2}^2 \); and marginally \( \ln v \) has variance \( \sigma_v^2 \). Consider the case adapted to the empirical fact that for Gulf of Mexico oil leases the logarithm of the bids typically has conditional variance about 1.0 whereas the estimating precision implies a variance of about 0.36 given that the prior variances (\( \sigma_{i1}^2, \sigma_{i2}^2 \)) are comparatively so large that they can be considered infinite: assume that the conditional variance of the private factors accounts for the difference. In this case, the symmetric equilibrium bidding strategy specifies that each firm submits a bid that is a specified fraction (the bid factor) of the product of its private factor and its posterior expectation of the common factor given its estimate. The tabulation shows the percentage bid factor for four numbers \( n \) of bidders, assuming the seller’s reservation price is zero, and it shows the winning bidder’s expected percentage profit. The seemingly low bid factors are necessary to avoid the winner’s curse; whereas the surprisingly large profit percentages reflect the role of the private valuation factors.

Analogous models in which a bidder can increase the precision of his information at increasing cost differ in that, even though bidders’ total expenditures converge to a positive level as the number of bidders increases, each bidder’s expenditure converges to zero. In this case the winning bid is not a consistent estimator of the common value \( v \) and the seller’s expected revenue is reduced by the amount of the bidders’ total expenditure.
Table 1  
Examples of Equilibrium Bidding Strategies  
Lognormal Distributions  

\[ u_i = s_{11} v, \sigma_1^2 + \sigma_2^2 = 1.0, \sigma_2^2 = 0.36. \]

<table>
<thead>
<tr>
<th>Number of Bidders (n)</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid Factor (%)</td>
<td>30.9</td>
<td>39.0</td>
<td>40.5</td>
<td>39.6</td>
</tr>
<tr>
<td>Expected Profit (%)</td>
<td>53.45</td>
<td>33.85</td>
<td>23.78</td>
<td>17.82</td>
</tr>
</tbody>
</table>

on information, since in equilibrium this is necessarily recouped in expectation by the participating bidders [Matthews (1984)]. An important policy conclusion is that bidders’ expenditures on information are inefficiently large.

Single-item auctions with dynamic rules add a few new aspects. In a Dutch auction an exogenously specified price is lowered until a bidder accepts. This rule induces a game that for the bidders is strategically equivalent to a sealed-bid auction; it is also payoff equivalent unless they are impatient to trade, in which case a bidder is concerned about the sum of the interest rate and the hazard rate that trade will be usurped by a competitor. For the seller it differs if he can not commit to forego trading by stopping the price at a reservation price exceeding his valuation. In the iid private-values model of preferences, a generalized multi-item Dutch auction is an optimal selling strategy for a monopolist seller whenever potential demand exceeds supply [Harris and Raviv (1981ab)]. Auctions with exogenously ascending prices, in which the items are awarded to the remaining bidders at the price at which the last of the others drops out, are a form of nondiscriminatory auction; but with affiliated observations, a bidder’s strategy accounts for the learning enabled by seeing the prices at which others drop out — an instance of the linkage principle.

Double auctions have been studied only for the case of iid private values and nondiscriminatory pricing. The price chosen is the midpoint of the interval of clearing prices derived from intersecting the schedules of bids (arrayed in descending order) and offers (arrayed in ascending order). Such an auction is actually an ex ante efficient trading rule for the case of equal numbers of buyers and sellers with values distributed uniformly on the same interval [Chatterjee and Samuelson (1983), Myerson and Satterthwaite (1983)].
and by implication from the previous results for auctions, for one buyer or one seller. With several buyers and sellers and fairly general distributions, the \textit{ex ante} efficient trading rule bears a strong resemblance to a double auction and has the remarkable property that the expected efficiency losses (compared to \textit{ex post} efficiency) from strategic behavior decline nearly quadratically to zero as the numbers of traders increase [Gresik and Satterthwaite (1984)]. The weaker criterion of \textit{interim} efficiency requires that no other trading rule is sure to improve every trader’s expected gains from trade: a double auction satisfies this criterion if there are sufficiently many buyers and sellers [Wilson (1985)].

Oral multi-item discriminatory double auctions, allowing free outcry of bids and offers, are the most important practically (e.g., commodity markets) and the most challenging theoretically. Since trades are consummated in process at differing prices, ‘market clearing’ is dynamic and, for example, traders with extra-marginal valuations in the static sense can obtain gains from trade early on. Since traders are continually motivated to estimate the distribution of subsequent bids and offers, the learning process is a key feature. Theoretical studies have been attempted for both complete equilibrium models [Wilson (1986)] and others invoking some plausible behavioral assumptions [Easley and Ledyard (1982), Friedman (1984)]. These studies aim to explain the dramatic efficiency attained in experiments and the tendency for transaction prices to approximate or converge to the static Walrasian clearing prices [Smith (1982), Plott and Sunder (1982)], especially with replication, even when the subjects lack a base of common knowledge about distributional features. The efficiency realized in experimental settings is a major puzzle deserving better theoretical explanations.

In summary, auctions are important market institutions that ensure market clearing via explicit trading rules that are independent of the distribution of preferences and information among the participants. Over wide ranges of models of preferences and information, these trading rules are \textit{ex ante} or \textit{interim} efficient or nearly so, and both practically and experimentally they are evidently robust. The theory elaborates these properties and demonstrates the role of private information and strategic behavior. The explicit construction of equilibrium strategies establishes the magnitudes of these effects and enables comparisons of trading rules, preference structures, informational conditions, and the number of participants; and additionally it explains phenomena such as
the winner’s curse that stem from adverse selection effects when there is dispersed information. Some models predict that the choice of pricing rule is inconsequential because bidders alter their strategies to compensate: the market clearing condition is the main determinant of welfare consequences.

In relation to the general economic theory of markets, the theory of auctions addresses the special case of markets with explicit market-clearing trading rules and elaborates in fine detail the determination of prices and the efficiency and distributional consequences of particular assumptions about the attributes of participants. This endeavor is a useful step in the construction of a general theory of the micro-structure of markets that encompasses the full range from bilateral bargaining to ‘perfectly competitive’ markets.

References


Milton Harris and Artur Raviv (1981b), “Allocation Mechanisms and the Design of Auc-


Thomas R. Palfrey (1985), “Uncertainty Resolution, Private Information Aggregation and


