

SUPPORTING INFORMATION

Derivations related to the initial and secondary investigations appear in §1 and §2, respectively. The solution to the optimization problem is derived in §3, the parameters of the model are estimated in §4, and supporting numerical results are presented in §5.

1 The Initial Investigation

It is possible to refine the model by partitioning the nodes of each criminal network into levels that have their own unique statistical characteristics; e.g., a drug network could have levels for wholesale dealers, retail dealers and users (Caulkins, 1997). Including levels in the analysis is notationally cumbersome and is not necessary to formulate and evaluate the model, and therefore we do not incorporate levels of networks in this paper. However, the analysis done in this section can be extended to the situation where there are levels within networks. See Atkinson (2009) for a formulation and analysis of this model with levels.

In this section, we discuss asymptotic results for large networks (i.e., $N_k \rightarrow \infty$). In §1.1, we derive equation (1) in the main text, and in §1.2 we argue that a node is identified in at most one network during the initial investigation. If there is no degree correlation among the nodes, the analysis in §1.1 and §1.2 is straightforward and is based upon the convergence of a binomial random variable to a Poisson random variable. For details of this approach see Atkinson (2009). However, the terror network we describe in §4 has a positive degree correlation between neighbors (see §4.2.3), and therefore we do not assume that the degrees are uncorrelated (which is needed for the Poisson convergence proofs). The presence of correlation does not affect our results. Because in reality the size of the networks will be finite, we omit the technical details and provide only heuristic arguments in §1.1 and §1.2. Rigorous proofs for analysis related to §1.1 and §1.2 can be found in Atkinson (2009). It is

possible to show other properties about the networks (e.g., that the nodes identified during the initial investigation will not be neighbors with any other node identified in the initial investigation) with the machinery described in §1.1 and §1.2.

1.1 Derivation of Equation (1) in the Main Text

If the government detects any activity while investigating a node in network k , then the government identifies that node as belonging to network k . Therefore, the probability of detecting a node can be determined from the probability that the government detects zero activities involving that node during its investigation. Because activities on edges in network k occur according to a Poisson process at rate λ_k and we assume a node's activities occur independently along each of its edges, the total number of activities a node engages in is a Poisson process with parameter $n_k\lambda_k$. This parameter is random, and so the total activity process of a node in a network is a Cox (or doubly stochastic Poisson) process (§6.2 in Daley and Vere-Jones (2003)). The detection probability is therefore:

$$\mathbf{P}[\text{node detected in network } k] = \mathbf{E}\left[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}\right], \quad (1)$$

where the expectation is with respect to the random degree distribution, n_k .

The number of nodes identified during the initial investigation of network k is characterized by

$$Y_k^I(N_k) = \sum_{i=1}^{N_k} I(\text{node } i \text{ detected in network } k), \quad (2)$$

where we have written Y_k^I in (2) as a function of N_k to stress the dependence of Y_k^I on the size of the network. The value $I(x)$ is an indicator function that equals 1 if the statement x is true and 0 otherwise. The probability that one of the indicators in equation (2) equals 1 is given by equation (1). For our model, we only need to compute the expected number of

nodes identified during the initial investigation, which is

$$\mathbf{E}[Y_k^I(N_k)] = N_k \mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}]. \quad (3)$$

Because of the linearity of expectations, the derivation of equation (3) from equation (2) does not depend upon the degree correlations among the nodes. For large values of N_k , the detection probability in equations (1) and (3) can be approximated as

$$\mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}] \approx \mathbf{E}\left[\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k\right]. \quad (4)$$

Some care needs to be taken to ensure that this approximation is valid. If we assume that the degree distribution n_k has a finite variance then the approximation in (4) is reasonable (see Atkinson (2009) for more details). Substituting (4) into equation (3) yields

$$\mathbf{E}[Y_k^I(N_k)] \approx \theta_k^I (b_k + B_k^I) \mathbf{E}[n_k] \lambda_k. \quad (5)$$

Equation (1) in the main text is equivalent to (5), but we have replaced $\mathbf{E}[Y_k^I(N_k)]$ with $\mathbf{E}[Y_k^I]$. We use an equality rather than an approximation in equation (1) in the main text because the error in the approximation can be made arbitrarily small by increasing N_k .

1.2 No Multiple Detections of a Node in the Initial Investigation

In this subsection, we argue that asymptotically (i.e., as $N_k \rightarrow \infty$) a node can be identified in at most one network during the initial investigation. This guarantees there is no double counting of nodes and allows us to compute the number of terrorists the government detects during the secondary investigation by examining each criminal network separately and then aggregating the results. To show that a node is identified in at most one network, we consider two networks, network k and network q , and show that no nodes are identified in both of these networks during the initial investigation.

Our analysis is similar to that in §1.1. We will not provide the technical details (see Atkinson (2009) for a rigorous analysis) but will provide some mathematical intuition for why this result will hold. Let us assume there are N_k nodes in network k , N_q nodes in network q , and N_{kq} nodes in both network k and network q . Note that $N_{kq} \leq N_k$ and $N_{kq} \leq N_q$. To be identified in both network k and network q during the initial investigation, the government has to detect the node interacting in both networks. Because we assume that the degree of a node in network k is independent of its degree in network q and that detection of activities is independent, the probability a node is identified in network k and network q during the initial investigation is

$$\mathbf{P}[\text{node detected in networks } k \text{ and } q] = \mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}] \mathbf{E}[1 - e^{-\theta_q^I \frac{(b_q + B_q^I)}{N_q} n_q \lambda_q}]. \quad (6)$$

The number of nodes identified during the initial investigation in both network k and network q is

$$Y_{kq}^I(N_{kq}) = \sum_{i=1}^{N_{kq}} I(\text{node } i \text{ detected in networks } k \text{ and } q). \quad (7)$$

The probability that $Y_{kq}^I(N_{kq})$ is greater than 0 can be made arbitrarily small as we increase N_{kq} . This implies that the probability that an individual will be identified in at most one network during the initial investigation can be made arbitrarily close to 1. To show this we will argue that $\mathbf{E}[Y_{kq}^I(N_{kq})] \rightarrow 0$. For large values of N_{kq} the detection probability in equation (6) can be approximated as

$$\mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}] \mathbf{E}[1 - e^{-\theta_q^I \frac{(b_q + B_q^I)}{N_q} n_q \lambda_q}] \approx \mathbf{E} \left[\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k \right] \mathbf{E} \left[\theta_q^I \frac{(b_q + B_q^I)}{N_q} n_q \lambda_q \right]. \quad (8)$$

The expectation of $Y_{kq}^I(N_{kq})$ will therefore converge to 0:

$$\mathbf{E}[Y_{kq}^I(N_{kq})] = N_{kq} \mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}] \mathbf{E}[1 - e^{-\theta_q^I \frac{(b_q + B_q^I)}{N_q} n_q \lambda_q}], \quad (9)$$

$$\approx N_{kq} \mathbf{E} \left[\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k \right] \mathbf{E} \left[\theta_q^I \frac{(b_q + B_q^I)}{N_q} n_q \lambda_q \right], \quad (10)$$

$$= \frac{N_{kq}}{N_k N_q} \theta_k^I (b_k + B_k^I) \mathbf{E}[n_k] \lambda_k \theta_q^I (b_q + B_q^I) \mathbf{E}[n_q] \lambda_q \quad \text{because } N_{kq} \leq N_k, \quad (11)$$

$$\leq \frac{1}{N_q} \theta_k^I (b_k + B_k^I) \mathbf{E}[n_k] \lambda_k \theta_q^I (b_q + B_q^I) \mathbf{E}[n_q] \lambda_q, \quad (12)$$

$$\rightarrow 0. \quad (13)$$

2 The Secondary Investigation

The number of terrorists identified during the secondary investigation of network k satisfies

$$T_k^S = \sum_{i=1}^{Y_k} I_k(\text{node } i \text{ detected as a terrorist}). \quad (14)$$

The subscript k on the indicator (and in the analysis below) denotes that the random variable is conditioned on the fact that the node was detected in network k during the initial investigation. Whether a node is detected as a terrorist during the secondary investigation is independent of how many nodes were detected during the initial investigation. Therefore, the expected number of terrorists detected during the secondary investigation of network k is

$$\mathbf{E}[T_k^S] = \mathbf{E}[Y_k^I] \mathbf{P}_k[\text{node detected as terrorist}], \quad (15)$$

where $\mathbf{E}[Y_k^I]$ is given in equation (1) of the main text. Manipulating conditional probabilities, we find that the second term in (15) is

$$\begin{aligned} \mathbf{P}_k[\text{node detected as terrorist}] &= \mathbf{P}_k[\text{node detected as terrorist} \mid \text{node in terror network}] \\ &\quad \times \mathbf{P}_k[\text{node in terror network}], \end{aligned} \quad (16)$$

$$= \mathbf{P}_k[\text{node detected as terrorist} \mid m_1 = 1] \mathbf{P}_k[m_1 = 1], \quad (17)$$

$$= \mathbf{E}_k[1 - e^{-\theta_1^S \frac{B_k^S}{\mathbf{E}[Y_k^I]} n_1 \lambda_1}] \mathbf{P}_k[m_1 = 1]. \quad (18)$$

The remainder of this section is devoted to calculating the two terms in (18), starting with $\mathbf{P}_k[m_1 = 1]$. The fact that a node is detected in network k during the initial investigation tells us two additional pieces of information: $m_k = 1$ and this node was not identified in the terror network during the initial investigation. For ease of notation, we define the following two events

$$A = \text{the node was detected in network } k \text{ during the initial investigation}, \quad (19)$$

$$B = \text{the node was not detected in the terror network during the initial investigation}. \quad (20)$$

Manipulating conditional probabilities yields

$$\mathbf{P}_k[m_1 = 1] = \mathbf{P}[m_1 = 1 \mid A], \quad (21)$$

$$= \mathbf{P}[m_1 = 1 \mid A, B, m_k = 1], \quad (22)$$

$$= \frac{\mathbf{P}[m_1 = 1, A, B, m_k = 1]}{\mathbf{P}[A, B, m_k = 1]}, \quad (23)$$

$$= \frac{\mathbf{P}[A, B \mid m_1 = 1, m_k = 1] \mathbf{P}[m_1 = 1, m_k = 1]}{\mathbf{P}[A, B, m_1 = 0, m_k = 1] + \mathbf{P}[A, B, m_1 = 1, m_k = 1]}. \quad (24)$$

The relevant values to substitute into equation (24) are

$$\mathbf{P}[A, B \mid m_1 = 1, m_k = 1] = \mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}] \mathbf{E}[e^{-\theta_1^I \frac{B_1^I}{N_1} n_1 \lambda_1}], \quad (25)$$

$$\mathbf{P}[A, B \mid m_1 = 0, m_k = 1] = \mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}]. \quad (26)$$

Substituting equations (25) and (26) into (24) yields

$$\mathbf{P}_k[m_1 = 1] = \frac{\mathbf{E}[e^{-\theta_1^I \frac{B_1^I}{N_1} n_1 \lambda_1}] \mathbf{P}[m_1 = 1, m_k = 1]}{\mathbf{E}[e^{-\theta_1^I \frac{B_1^I}{N_1} n_1 \lambda_1}] \mathbf{P}[m_1 = 1, m_k = 1] + \mathbf{P}[m_1 = 0, m_k = 1]}. \quad (27)$$

If we let $N_1 \rightarrow \infty$ (because we are assuming $n_1 < \infty$ almost surely, it is valid to interchange the limits and expectations, which are bounded by 1, in (27)) then our final expression for the conditional probability is

$$\mathbf{P}_k[m_1 = 1] = \frac{\mathbf{P}[m_1 = 1, m_k = 1]}{\mathbf{P}[m_k = 1]}, \quad (28)$$

$$= \mathbf{P}[m_1 = 1 \mid m_k = 1]. \quad (29)$$

That is, the conditional probability that a node initially detected in network k is a terrorist is simply the unconditional probability that a node network k is a terrorist.

Turning to the first factor in equation (18), we need to determine the conditional degree distribution $\mathbf{P}_k[n_1 = q]$ for all $q \geq 0$. This analysis is similar to the derivation above that yielded (29). Using the events defined in (19)-(20), we get

$$\mathbf{P}_k[n_1 = q] = \mathbf{P}[n_1 = q \mid A], \quad (30)$$

$$= \mathbf{P}[n_1 = q \mid A, B, m_k = 1], \quad (31)$$

$$= \frac{\mathbf{P}[n_1 = q, A, B, m_k = 1]}{\mathbf{P}[A, B, m_k = 1]}, \quad (32)$$

$$= \frac{\mathbf{P}[A, B \mid n_1 = q, m_k = 1] \mathbf{P}[n_1 = q, m_k = 1]}{\sum_{r=0}^{\infty} \mathbf{P}[A, B \mid n_1 = r, m_k = 1] \mathbf{P}[n_1 = r, m_k = 1]}. \quad (33)$$

The conditional probabilities in equation (33) are given by

$$\mathbf{P}[A, B \mid n_1 = r, m_k = 1] = \mathbf{E}[1 - e^{-\theta_k^I \frac{(b_k + B_k^I)}{N_k} n_k \lambda_k}] e^{-\theta_1^I \frac{B_1^I}{N_1} r \lambda_1}. \quad (34)$$

Substituting equation (34) into (33) and using the fact that the degree distribution in the terror network, n_1 , is independent of whether the node is in network k (i.e., whether $m_k = 1$),

yields

$$\mathbf{P}_k[n_1 = q] = \frac{e^{-\theta_1^I \frac{B_1^I}{N_1} q \lambda_1} \mathbf{P}[n_1 = q]}{\sum_{r=0}^{\infty} e^{-\theta_1^I \frac{B_1^I}{N_1} r \lambda_1} \mathbf{P}[n_1 = r]}. \quad (35)$$

The denominator of equation (35) is $\mathbf{E}[e^{-\theta_1^I \frac{B_1^I}{N_1} n_1 \lambda_1}]$. If we send $N_1 \rightarrow \infty$ (and again interchange the limits and expectation because $n_1 < \infty$ almost surely) then our final expression for the conditional distribution is

$$\mathbf{P}_k[n_1 = q] = \mathbf{P}[n_1 = q], \quad (36)$$

which is the unconditional degree distribution. Substituting (29) and (36) into (18), and substituting (18) into (15) gives equation (2) in the main text.

3 Solution to the Optimization Problem

In this section, we solve problem (3)-(6) in the main text. The problem is solved in §3.1, the uniqueness of this solution is derived in §3.2, and the solution to the uncoordinated case where $b_k = 0$ is given in §3.3.

3.1 The Solution in the General Case

After plugging in equations (1)-(2) in the main text into equation (3) in the main text, we can express the optimization problem in terms of the primitive parameters:

$$\begin{aligned} \max_{B_k^I, B_k^S} \quad & \sum_{k=2}^K \mathbf{P}[m_1 = 1 | m_k = 1] \mathbf{E} \left[1 - e^{-\frac{\theta_1^S B_k^S n_1 \lambda_1}{\min\{\theta_k^I (b_k + B_k^I) \mathbf{E}[n_k] \lambda_k, N_k\}}} \right] \min\{\theta_k^I (b_k + B_k^I) \mathbf{E}[n_k] \lambda_k, N_k\} \\ & + \min\{\theta_1^I B_1^I \mathbf{E}[n_1] \lambda_1, N_1\}, \end{aligned} \quad (37)$$

$$\text{s.t.} \quad B_1^I + \sum_{k=2}^K (B_k^I + B_k^S) = B, \quad (38)$$

$$B_k^I \geq 0 \text{ for } k = 1, \dots, K, \quad (39)$$

$$B_k^S \geq 0 \text{ for } k = 2, \dots, K. \quad (40)$$

We next transform the decisions variables from (B_k^I, B_k^S) to $B_k = B_k^I + B_k^S$ and $\gamma_k = \frac{B_k^I}{B_k}$ and define the second term in (37) to be

$$P_k^S(\gamma_k, B_k) = 1 - \mathbf{E} \left[\exp \left(- \frac{\theta_1^S n_1 \lambda_1 (1 - \gamma_k) B_k}{\min\{\theta_k^I \mathbf{E}[n_k] \lambda_k (b_k + \gamma_k B_k), N_k\}} \right) \right], \quad (41)$$

which is the probability that the government will determine that a criminal is also a terrorist during the secondary investigation, given that the criminal is also a terrorist. We first solve for the optimal γ_k^* in terms of B_k via

$$\gamma_k^*(B_k) = \arg \max_{0 \leq \gamma_k \leq 1} P_k^S(\gamma_k, B_k) \min\{\theta_k^I \mathbf{E}[n_k] \lambda_k (b_k + \gamma_k B_k), N_k\}. \quad (42)$$

If the min operator in (41) and (42) returns the second term (i.e., N_k), then inspection of equations (41) and (42) yields that $\gamma_k^*(B_k) = 0$. If the min operator in (41) and (42) returns the first term, then by equation (41), the solution to (42) depends only upon the ratio of the law enforcement resources to the counterterrorism resources $\left(\frac{b_k}{B_k}\right)$, the relative investigative efficiencies between the initial and secondary investigations $\left(\frac{\theta_1^I}{\theta_k^S}\right)$, and the relative total activity rates between the terror network and network k $\left(\frac{n_1 \lambda_1}{\mathbf{E}[n_k] \lambda_k}\right)$. Although there is no general closed-form solution to (42), in §3.2 we show that there is a unique solution $\gamma_k^*(B_k) \in [0, 1)$ to (42).

We define the government's cost-effectiveness during the initial investigation to be

$$e_k^I = \theta_k^I \mathbf{E}[n_k] \lambda_k, \quad (43)$$

which is the average number of nodes that the government identifies per dollar spent in the initial investigation of network k . After substituting $\gamma_k^*(B_k)$ into (37), the remaining optimization problem is

$$\max_{B_k} \min\{e_1^I B_1, N_1\} + \sum_{k=2}^K \mathbf{P}[m_1 = 1 | m_k = l] P_k^S(\gamma_k^*(B_k), B_k) \min\{e_k^I (b_k + B_k \gamma_k^*(B_k)), N_k\}, \quad (44)$$

which can be computed numerically.

3.2 Solution to Equation (42)

In this subsection, we derive some properties about $\gamma_k^*(B_k)$, which is the solution to equation (42). For ease of presentation, we define the moment generating function $\psi(t) = \mathbf{E}[e^{tn_1}]$. We begin by reviewing the properties of the moment generating function. Because $n_1 \geq 0$ we have that $\lim_{t \rightarrow -\infty} \psi(t) = 0$, $\lim_{t \rightarrow \infty} \psi(t) \rightarrow \infty$, and $\psi(0) = 1$. Furthermore, if we assume n_1 is square integrable, then both $\psi'(t)$ and $\psi''(t)$ exist for $t \leq 0$ and $\psi'(t) \geq 0$ and $\psi''(t) \geq 0$. The technical details to show these relationships are very similar to the analysis in §1. Defining the objective function of equation (42) to be $f_k(\gamma_k, B_k)$, we assume $B_k > 0$ because $f_k(\gamma_k, 0) = 0$ for all $\gamma_k \in [0, 1]$. The min operator in equations (41) and (42) leads us to split the analysis into three cases:

$$\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k b_k \geq N_k, \quad (45)$$

$$\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k (b_k + B_k) \leq N_k, \quad (46)$$

$$\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k b_k < N_k < \theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k (b_k + B_k). \quad (47)$$

For the case in (45), the min operator returns the second term for all $\gamma_k \in [0, 1]$, for the case in (46) the min operator returns the first term for all $\gamma_k \in [0, 1]$, and for the case in (47) there exists a $\hat{\gamma} \in (0, 1)$ such that the min operator returns the first term for $\gamma_k \in [0, \hat{\gamma})$ and returns the second term for $\gamma_k \in [\hat{\gamma}, 1]$. For the case in (45), γ_k appears in $f_k(\gamma_k, B_k)$ only in the numerator of the moment generating term in $P_k^S(\gamma_k, B_k)$ (see equations (41) and (42)). In this case, $P_k^S(\gamma_k, B_k)$ is a decreasing function for $\gamma_k \in [0, 1]$, and thus $\gamma_k = 0$ maximizes $P_k^S(\gamma_k, B_k)$ and $f_k(\gamma_k, B_k)$. Therefore, $\gamma_k^*(B_k) = 0$ for the case in (45). That is, if the government has already identified all criminals in a network during the initial investigation, it would not allocate more resources to the initial investigation of that network.

Next we show that there exists a unique $\gamma_k^*(B_k) \in [0, 1)$ that maximizes $f_k(\gamma_k, B_k)$ for

the case in (46). We have that $f_k(1, B_k) = 0$, $\lim_{\gamma_k \rightarrow -\frac{b_k}{B_k}} f_k(\gamma_k, B_k) = 0$, and $f_k(\gamma_k, B_k) > 0$ for all $\gamma_k \in (-\frac{b_k}{B_k}, 1)$. Therefore the solution $\gamma_k^*(B_k) \in [0, 1)$. If $b_k = 0$ then the solution $\gamma_k^*(B_k)$ must be strictly in the interior of $(0, 1)$ because to detect terrorists in criminal networks requires both an initial investigation and a secondary investigation. The function $f_k(\gamma_k, B_k)$ is differentiable (with respect to γ_k) on $\gamma_k \in (-\frac{b_k}{B_k}, 1)$, and therefore there must be a global maximum on $\gamma_k \in (-\frac{b_k}{B_k}, 1)$ that is also a stationary point. Consequently, $\gamma_k^*(B_k)$ is either a zero of the derivative of $f_k(\gamma_k, B_k)$ in $(0, 1)$, or if there are no such zeros in $(0, 1)$ then $\gamma_k^*(B_k)$ is 0. The first derivative is

$$\begin{aligned} \frac{\partial}{\partial \gamma_k} f_k(\gamma_k, B_k) &= 1 - \psi \left(-\frac{\theta_1^S \lambda_1}{\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k} \frac{(1 - \gamma_k) B_k}{(b_k + \gamma_k B_k)} \right) \\ &\quad - \psi' \left(-\frac{\theta_1^S \lambda_1}{\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k} \frac{(1 - \gamma_k) B_k}{(b_k + \gamma_k B_k)} \right) \\ &\quad \times \frac{\theta_1^S \lambda_1}{\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k} \frac{b_k + B_k}{(b_k + \gamma_k B_k)}, \end{aligned} \quad (48)$$

and the second derivative is

$$\begin{aligned} \frac{\partial^2}{\partial \gamma_k^2} f_k(\gamma_k, B_k) &= -\psi'' \left(-\frac{\theta_1^S \lambda_1}{\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k} \frac{(1 - \gamma_k) B_k}{(b_k + \gamma_k B_k)} \right) \\ &\quad \times \left(\frac{\theta_1^S \lambda_1}{\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k} \frac{b_k + B_k}{(b_k + \gamma_k B_k)} \right)^2 \frac{B_k}{(b_k + \gamma_k B_k)}, \end{aligned} \quad (49)$$

which is negative on $\gamma_k \in (-\frac{b_k}{B_k}, 1)$. Therefore, $f_k(\gamma_k, B_k)$ is concave on $\gamma_k \in (-\frac{b_k}{B_k}, 1)$, and there is a unique $\gamma_k \in (-\frac{b_k}{B_k}, 1)$ that is a root of (48). This maximizer corresponds to $\gamma_k^*(B_k)$ in equation (42) if the maximizer is positive, otherwise by concavity $\gamma_k^*(B_k)$ is 0.

Showing that there is a unique $\gamma_k^*(B_k)$ for the final case in (47) is similar to the analysis for the cases in (45) and (46). We define $\hat{\gamma}$ such that $\theta_k^I \sum_{p=1}^{L_k} \mathbf{E}[n_k] \lambda_k (b_k + \hat{\gamma} B_k) = N_k$. From the analysis of the case in (45), $\gamma_k^*(B_k) \leq \hat{\gamma}$ because $f_k(\gamma_k, B_k)$ is decreasing for $\gamma_k \in [\hat{\gamma}, 1]$. If there is a local maximum of $f_k(\gamma_k, B_k)$ on $(-\frac{b_k}{B_k}, \hat{\gamma}]$ then $\gamma_k^*(B_k)$ equals the maximizer if

this maximizer is positive, and is 0 otherwise (this analysis is the same as for the case in (46)). If there is not a local maximum of $f_k(\gamma_k, B_k)$ on $(-\frac{b_k}{B_k}, \hat{\gamma}]$ then $\gamma_k^*(B_k) = \hat{\gamma}$.

3.3 Solution to the Uncoordinated Case

We conclude this section with a discussion of the uncoordinated case ($b_k = 0$). If the min operator in (44) returns the first term, then $P_k^S(\gamma_k)$ in (41) is no longer a function of the total investigative resources allocated to that network (B_k) and there is a unique solution $\gamma_k^* \in (0, 1)$ that is independent of B_k . After substituting γ_k^* into (37), the objective function simplifies to a linear combination of the decision variables B_k , and the optimal solution is to allocate the entire budget B to the network with the largest e_k , where

$$e_1 = e_1^I, \tag{50}$$

$$e_k = e_k^I \mathbf{P}[m_1 = 1 \mid m_k = 1] \gamma_k^* P_k^S(\gamma_k^*). \tag{51}$$

The quantity e_k represents the average number of terrorists identified per dollar spent investigating network k .

4 Parameter Estimation

In this section, we estimate the parameters of the objective function in equation (37).

Our main data source is a report entitled *Pre-Incident Indicators of Terrorist Incidents* (Smith, Damphousse, and Roberts, 2006) and is described in §4.1. We estimate the terror parameters in §4.2 and the criminal parameters in §4.3.

4.1 *Pre-incident Indicators of Terrorist Incidents*

The *Pre-incident Indicators of Terrorist Incidents* report is partially derived from the American Terrorism Study (ATS) (Smith and Damphousse, 2002), which is the primary database for existing empirical studies on the connections between crime and terror (Smith et al., 2006; Smith, Cothren, Roberts, and Damphousse, 2008; Hamm, 2005). The ATS collects information on individuals indicted as a result of “domestic security/terrorism investigations” (Smith and Damphousse, 2002). Indictments and other court documents also provide the detailed information necessary for our empirical study: which terrorists interact with each other, how frequently they interact, and what criminal activities each terrorist is involved with.

We use the population of terrorists in the 60 cases analyzed in Smith et al. (2006), which focuses on the activities (both legal and illegal) terrorists are involved with prior to an attack. Court documents associated with the 60 terror cases in Smith et al. (2006) were available through the Terrorism Knowledge Base (TKB) (Memorial Institute for the Prevention of Terrorism, 2008), but unfortunately as of March 2008 the TKB is no longer available. The TKB was created and sponsored by the Memorial Institute for the Prevention of Terrorism and has transferred to the University of Maryland’s Study of Terrorism and Responses to Terrorism (START). However, it is uncertain if and when the court documents from the TKB will be available through START (University of Maryland, 2008; Straw, 2008). We did not gain access to court documents for every case in Smith et al. (2006) because either the TKB did not have court documents associated with a given case (17 cases) or we were unable to download the court documents before the TKB became unavailable (four cases). For those cases for which we did not have court documents, we had to appeal to other sources (including the descriptions in Smith et al. (2006)).

Each case focuses primarily on one individual or group and a particular incident or string of incidents associated with that group. We aggregated the 60 cases into 54 cases by merging several cases from Smith et al. (2006) when the same terrorists were associated with multiple cases. These 54 cases occur in the U.S. between 1971–2003 and – according to the classification scheme used by Smith et al. (2006) and the FBI (Federal Bureau of Investigation, 2002) – contain 26 right-wing cases, six left-wing cases, 14 single-issue cases, and eight international cases.

Table 1 (for the 26 right-wing cases) and table 2 (for the 28 cases from the remaining three categories) provide a list of the terror cases from Smith et al. (2006) we include in our analysis, information about each case – including the number of terrorist interactions that occur prior to an attack and the number of edges (connecting terrorist nodes) upon which these interactions occur – and sources for this information. We identified 452 terrorists in these 54 cases, and present data on the number of terrorists in each of the four categories in table 3.

The results presented in table 2 of the main text are derived from all 54 terror cases. However, in §5.1 we analyze each of the four terror categories separately. The right-wing cases involve individuals associated with Aryan groups, the Ku Klux Klan, or anti-government groups (Smith et al., 2006). The Oklahoma City Bombing is the most well known and devastating example. The left-wing cases include leftist student groups, all-black groups, and Puerto Rican independence movement groups (Smith et al., 2006). There has been limited activity from these groups since the mid 1980s. The single-issue cases focus either on anti-abortion acts or environmental groups associated with the Earth Liberation Front or the Animal Liberation front. The international cases involve foreign individuals plotting an attack in the U.S. or collecting money or other goods to send to terrorist groups abroad (such as Hezbollah or the IRA). The first World Trade Center attack in 1993 is one of the

eight international cases, but the September 11, 2001 attacks are not included.

4.2 Terror Network

We construct a terror network of the 452 individuals involved in the cases in tables 1 and 2. In §4.2.1, we determine what fraction of these 452 individuals are involved in various crimes to estimate $\mathbf{P}[m_k = 1 \mid m_1 = 1]$, which also guides our choice for the criminal types to include in the analysis. In §4.2.2 and §4.2.3, we use the constructed terror network to estimate the interaction rate λ_1 and the distribution of the degree n_1 , respectively. We use other data sources (i.e., not based on the terror population in tables 1 and 2) to estimate the size of the terror network N_1 (which is proportional to $\mathbf{P}[m_1 = 1]$) in §4.2.4 and the efficiency parameters θ_1^I and θ_1^S in §4.2.5.

4.2.1 Overlap Distribution

Table 4 presents the relevant part of the overlap distribution, which is $\mathbf{P}[m_k = 1 \mid m_1 = 1]$. The values in table 4 are the fraction of terrorists in our data set that are involved with each criminal activity; 98% of the criminal connections are included in this table, which represent all but the rarest of overlap criminal activities. We determine which criminal networks each terrorist is a member of by going through the sources listed in tables 1 and 2. Four of the criminal networks in table 4 are divided into different levels that correspond to being a consumer or a distributor of some illicit good. In three of the networks we distinguish between “retail distributors” and “wholesale distributors,” where retail distributors deal in smaller quantities, primarily to users, and wholesale distributors deal in larger quantities, primarily to other distributors. While it is possible to refine the model to account for these levels explicitly (see Atkinson (2009)), for the purposes of this paper the separate levels of a criminal network are equivalent to different criminal networks.

Because of the effort involved in estimating the parameters for each criminal network, coupled with the fact that the overlap probabilities are small (e.g., $< 10\%$) for most of the criminal networks in table 4, we restrict ourselves to six criminal networks: explosives, illegal firearms distributor, illegal firearms user, bank robbery, false documents distributor, and false documents user.

4.2.2 Interaction Rate

For the terror cases in tables 1 and 2, we tabulate how many terror interactions or activities the nodes associated with each case participate in. These interactions might be meetings to plan the logistics of an attack or discuss a group’s radical ideology, or some other preparatory event. If there is a meeting of several individuals then we assume that each individual takes part in only one interaction (the meeting); however, that one interaction is assumed to occur between that individual and every other participant at the meeting, and therefore there can be fractional interactions between neighbors. The primary sources we use to tabulate these interactions are affidavits, transcripts, indictments, and other court documents.

To compute the interaction rate λ_1 we need the total number of interactions within the terror network, the number of edges in the network (upon which these interactions occur), and the time period when the interactions take place. With these three pieces of information, the interaction rate is $\lambda_1 = \frac{\text{interactions}}{\text{edges} \times \text{time}}$. If we define interactions_j to be the number of interactions associated with terror case j and edges_j to be the number of edges associated with terror case j , and we set $t = 32.5$ years to be the length of the time period these terror cases pertain to (1971–2003), then the interaction rate is

$$\lambda_1 = \frac{\sum_{j=1}^{54} \text{interactions}_j}{t \times \sum_{j=1}^{54} \text{edges}_j}. \quad (52)$$

The values of interactions_j and edges_j for each of the 54 cases appear in tables 1 and 2, where we count each interaction between neighbors only once when tabulating interactions_j . The

interaction rate is $\lambda_1 = 5.5 \times 10^{-2}/\text{year}$.

This estimate assumes that the terror network is static over the period between 1971–2003, while in reality these terrorists are only active for a short period of time. Therefore, the interaction rate for active terrorists will be greater than the estimate given by equation (52). However, λ_1 appears in equation (37) only in the product $\lambda_1\theta_1^I$ or $\lambda_1\theta_1^S$. In §4.2.5 we estimate θ_1^I as a function of λ_1 (see equation (53)), and thus the product $\lambda_1\theta_1^I$ is determined by equation (53). We could absorb λ_1 into θ_1^I and still compute θ_1^S as we do in §4.2.5, and therefore the estimate of λ_1 is not crucial for our analysis.

4.2.3 Degree Distribution

We define two nodes as neighbors in the terror network if we determine that they interact in a terror activity (i.e., we count their interactions when tabulating interactions_j in §4.2.2). For meetings involving several individuals, we assume that every participant of the meeting is a neighbor with every other participant. The empirical degree distribution is illustrated in fig. 1 and the mean degree is $\mathbf{E}[n_1] = 5.1$. We use this empirical distribution to compute the moment generating term, $\mathbf{E}[e^{tn_1}]$, appearing in equation (37).

We do not expect our network to have the same properties (e.g., small-world or scale-free) as the criminal and terror networks in Xu and Chen (2008) because our network consists of 54 disconnected and relatively small cells. Using the methods described in Clauset, Shalizi, and Newman (2009), we find that a power law distribution is a poor fit to the degree distribution of our network, as are Poisson and discretized log-normal distributions. The geometric distribution does not fit the data well, but we cannot reject it at the 0.05 level. Our terror network has a high degree correlation among neighboring nodes which is consistent with the Global Salafi Jihad terror network analyzed in Xu and Chen (2008) and many other social networks (Newman, 2002). The assortativity coefficient (defined in

equation (4) of Newman (2002) and equivalent to the Pearson correlation coefficient) for our network is 0.473.

4.2.4 Size of Terror Network

Several sources report that there have been on the order of 100 terror incidents in the U.S. over the last decade (see Lawson Terrorism Information Center (2008); Federal Bureau of Investigation (2008c); Jarboe (2002); Blejwas, Griggs, and Potok (2005)). These incidents primarily involve environmental or right-wing terrorists. As an order-of-magnitude estimate, we assume there are 1000 domestic terrorists. We further assume that there is an equal number of international terrorists in the U.S., which is not inconsistent with a report that states a “very small fraction” of the over 200,000 individuals on an international terrorist list are in the U.S. (Washington Post, 2006). Therefore we assume there are $N_1 = 2000$ nodes in the terror network.

4.2.5 Efficiency Parameters

The efficiency parameters, θ_1^I and θ_1^S , are aggregate parameters that account for several different factors, and therefore are difficult to directly estimate. Our approach is to indirectly estimate θ_1^I using equation (1) in the main text and then estimate the ratio $\frac{\theta_1^I}{\theta_1^S}$. We have an estimate of $\mathbf{E}[n_1]$ from §4.2.3 and an estimate of λ_1 from §4.2.2. If we also estimate the resources spent on an initial investigation of the terror network, \hat{B}_1^I , and the number of terrorists identified during that initial investigation, \hat{Y}_1^I , then we can estimate θ_1^I via equation (1) in the main text,

$$\theta_1^I = \frac{\hat{Y}_1^I}{\hat{B}_1^I \mathbf{E}[n_1] \lambda_1}, \quad (53)$$

where we assume that the estimate \hat{Y}_1^I is a reasonable approximation to $\mathbf{E}[Y_1^I]$ in equation (1) in the main text. We now estimate \hat{B}_1^I and \hat{Y}_1^I . We set $\hat{B}_1^I = \$1.4\text{B}$, which is the

FBI domestic counterterrorism budget for 2005 (Harlow, 2006). The FBI investigated 248 suspects in 2005 for offenses related to terrorist activity (Bureau of Justice Statistics, 2005) (using the category “Suspects in investigations initiated” from Bureau of Justice Statistics (2005), which we also use in §4.3). In addition, out of 1067 individuals referred to federal prosecutors and classified as “international terrorists” during 2001–2006, only 372 had a lead charge directly related to terrorism (Transactional Records Access Clearinghouse, 2007). We therefore multiply 248 by $\frac{1067}{372}$ to estimate the number of terrorists identified by the FBI in 2005. Rounding down this product, we set $\hat{Y}_1^I = 700$. Substituting these estimates into the right side of (53) yields $\theta_1^I = 1.79 \times 10^{-6}$.

Because much of the initial investigation budget involves investigating false leads, we estimate $\frac{\theta_1^I}{\theta_1^S}$ by the fraction of preliminary investigations that lead to legitimate terror investigations. One report states that of nearly 10k terrorism investigations in 2000, only about 500 individuals were charged, which leads to an estimate of 0.05 (Transactional Records Access Clearinghouse, 2003). A Department of Justice report states that between 2004 and 2007 there were 108k terrorism related threats, but only 600 terrorism related investigations, which leads to an estimate of 0.006 (Office of the Inspector General, 2008). This same report later states that in 2006 there were 219k terrorism tips by the public to the FBI, and this resulted in 2800 terror threats entered into its system, yielding an estimate of 0.01 (Office of the Inspector General, 2008). We use the median of these three estimates and set $\theta_1^S = 100\theta_1^I = 1.79 \times 10^{-4}$, although we vary θ_1^S in §5.2 because there is uncertainty about its relationship to θ_1^I .

4.3 Criminal Networks

The four subsections in this subsection are devoted to estimating the parameters for the four types of criminal networks with the greatest overlap with the terror network: explo-

sives, illegal firearms (distributor and user), bank robbery, and false documents (distributor and user) (although we have six criminal networks, it is easier to present the parameter estimates for the two illegal firearms networks together and for the two false documents networks together). By equation (37), we need to estimate the size of network k , N_k , which is proportional to $\mathbf{P}[m_k = 1]$, and the aggregate quantity $\theta_k^I \mathbf{E}[n_k] \lambda_k$. Using the same method we used to estimate θ_1^I in (53), if we estimate the resources spent on an initial investigation of network k , \hat{B}_k^I , and the number of nodes identified during that initial investigation, \hat{Y}_k^I , then we can estimate the aggregate quantity via equation (1) in the main text:

$$\theta_k^I \mathbf{E}[n_k] \lambda_k = \frac{\hat{Y}_k^I}{\hat{B}_k^I}. \quad (54)$$

For each criminal network we focus on one agency that investigates that crime and estimate the resources the agency spent to investigate that criminal network, \hat{B}_k^I , and the number of nodes the agency identified during its investigations, \hat{Y}_k^I ; for consistency, we use the year 2005 for these calculations whenever possible. In addition, for the criminal networks our estimates for \hat{B}_k^I are also used for b_k in the coordinated version of the optimization problem.

4.3.1 Explosives Network

Out of 452 terrorists in our study, 192 use explosives, 10 are retail distributors, and 2 are wholesale distributors (table 4). However, because of a lack of data for N_k , \hat{B}_k^I and \hat{Y}_k^I for these 3 levels, we do not distinguish between different levels of the explosives network and only consider the aggregate explosives network.

Network Size. There were 3693 explosives investigations by the Bureau of Alcohol, Tobacco, Firearms and Explosives (ATF) in 2005 (Bureau of Alcohol, Tobacco, Firearms and Explosives, 2005), which is a similar value to the number of reported explosives incidents per year between 2004 and 2006

(Bureau of Alcohol, Tobacco, Firearms and Explosives, 2006). These values are approximately 50% more than the reported number of bombing incidents per year in the 1990s (Pastore, A.L. and Maguire, K., 2003; Bureau of Alcohol, Tobacco, and Firearms, 1996).

There were 369 defendants in 218 explosives-related cases in 1997 (Bureau of Alcohol, Tobacco, Firearms, 1997). These values are 315 and 196, respectively, for 1996 (Bureau of Alcohol, Tobacco, Firearms, 1997) and 409 and 244, respectively, for 1995 (Bureau of Alcohol, Tobacco, Firearms, 1996), implying that there were ≈ 1.66 defendants per case during these years. If we assume there were 1.66 people per bombing incident in 2005, then the total population in the explosives network would be approximately 6000 individuals. Although many criminals are repeat offenders that commit several crimes per year (Blumstein, Cohen, Roth, and Visser, 1986), we could not find any information to estimate how many explosives incidents each criminal is involved with, and therefore we assume there are $N_k = 6000$ nodes in the the explosives network.

Budget. We set $\hat{B}_k^I = b_k = \$119\text{M}$, which is the amount allocated by the ATF to explosives enforcement in 2005 (Bureau of Alcohol, Tobacco, Firearms and Explosives, 2005).

Number of Nodes Identified During the Investigation. We set $\hat{Y}_k^I = 228$ for the explosives network, which is the number of suspects investigated by the ATF in 2005 for offenses related to explosives (Bureau of Justice Statistics, 2005).

4.3.2 Illegal Firearms Network

Out of 452 terrorists, 158 use illegal firearms, 21 are retail distributors, and 8 are wholesale distributors (table 4). In our analysis, we consider users and distributors of illegal firearms as being in separate networks.

Network Size. There are $\approx 2\text{M}$ criminal firearm acquisitions per year (Pierce, Braga, Koper, McDevitt, Carlson, Roth, Saiz, Hyatt, and Griffith, 2004), and offend-

ers purchase about one handgun per year (Koper and Reuter, 1996). Therefore, we assume there are $N_k = 2\text{M}$ users in the illegal firearms user network. Approximately 30% of criminals obtain their firearm through a drug dealer, off the street, a fence, or the black market (Harlow, 2001). We assume these suppliers are the population of illegal firearm distributors, and therefore 600K criminals obtain their firearm from an illegal firearms distributor each year. The average gun distributor sells on the order of 100 firearms per year (Koper and Reuter, 1996), and thus we assume there are $N_k = 6000$ distributors in the illegal firearms distributor network.

Budget. The 2005 ATF Report states \$591M went to firearms enforcement (Bureau of Alcohol, Tobacco, Firearms and Explosives, 2005). In the ATF’s budget for 2008, \$337.5M were allocated to firearms trafficking out of \$730.1M allocated to firearms enforcement (46.2%) (Bureau of Alcohol, Tobacco, Firearms and Explosives, 2007). Assuming the same percentage allocation in 2005, we set $\hat{B}_k^I = b_k = \$318\text{M}$ as the investigative budget for users of illegal firearms, and $\hat{B}_k^I = b_k = \$273\text{M}$ as the investigative budget for distributors of illegal firearms.

Number of Nodes Identified During the Investigation. The ATF investigated 11,068 suspects in 2005 for offenses related to firearms (Bureau of Justice Statistics, 2005). Out of 8353 convictions for firearm offenses in 2005, 1448 were for trafficking offenses (Bureau of Alcohol, Tobacco, Firearms and Explosives, 2005). We assume this fraction $\left(\frac{1448}{8353}\right)$ from (Bureau of Alcohol, Tobacco, Firearms and Explosives, 2005) also holds for the values from (Bureau of Justice Statistics, 2005), and set the number of individuals identified during the investigation of the illegal firearms network in 2005 to be $\hat{Y}_k^I = 9149$ for users of illegal firearms and $\hat{Y}_k^I = 1919$ for distributors of illegal firearms.

4.3.3 Bank Robbery Network

Network Size. There are on the order of 10k bank robberies per year

(Federal Bureau of Investigation (2006); Weisel (2007); Federal Bureau of Investigation (2005); Federal Bureau of Investigation (2002)). A Department of Justice report on bank robberies states that in London each apprehended banker robber is associated with an average of 2.8 bank robberies (Weisel, 2007). The Bank Crime Statistics report published by the FBI in 2005 states that there were an average of 1.2 known people associated with each bank robbery (Federal Bureau of Investigation, 2005), and the 2002 Uniform Crime Report states that 80% of bank robberies are carried out by one offender and 15% involve two offenders (Federal Bureau of Investigation, 2002) (and therefore the average number of offenders per robbery is at least 1.25 according to Federal Bureau of Investigation (2002)). If we assume there are 1.3 offenders per bank robbery, 10k bank robberies per year, and 2.8 bank robberies per individual, then the population of bank robbers would be 4643. Rounding up, we assume there are $N_k = 5000$ nodes in the the bank robbery network.

Budget. The FBI allocated \$2.1B in 2008 for federal criminal law enforcement, and \$1.1B of this amount was allocated to reduce violent crime (Federal Bureau of Investigation, 2008a) (robbery is considered a violent crime (Federal Bureau of Investigation, 2006)). The FBI spent \$2.0B on federal criminal law enforcement in 2005 (Federal Bureau of Investigation, 2006). If we assume the FBI allocated the same fraction of the criminal enforcement budget to reduce violent crime in 2005 as it did in 2008 $\left(\frac{1.1}{2.1}\right)$, then the FBI spent \$1.0B to reduce violent crime in 2005. Unfortunately, we could not find more specific information on the amount of resources the FBI spends investigating bank robberies. The FBI investigates several types of activities related to violent crime including bank robberies, murder for hire, and crimes against children (Federal Bureau of Investigation, 2008a,b). We make the rough estimate that the FBI spends 10% of its budget to reduce violent crime on bank robbery

investigations. We therefore set $\hat{B}_k^I = b_k = \$100\text{M}$ as the investigative budget for the bank robbery network.

Number of Nodes Identified During the Investigation. We set $\hat{Y}_k^I = 1877$ for the bank robbery network, which is the number of suspects the FBI investigated in 2005 for offenses related to bank robberies (Bureau of Justice Statistics, 2005).

4.3.4 False Documents Network

Out of 452 terrorists, 70 use false documents and 10 are distributors (table 4). We consider both a user network and a distributor network for false documents.

Our focus is on the use of false documents to obscure or alter one's identity. However there are many related criminal activities involving identity theft, fraud, forgery, counterfeiting, and immigrations violations that we are not including in this analysis. Unfortunately, the definitions in reports and databases do not always clearly distinguish these various criminal categories (Federal Bureau of Investigation, 2006; Koops and Leenes, 2006). There are also many populations who use false documents (Associated Press, 2006), there are several types of false documents (Dinerstein, 2002), and there are various agencies who investigate false documents (General Accounting Office, 1998). Therefore, it can be difficult to obtain information about false documents that is relevant for our purposes (Gordon and Willox, 2003). We use data from Immigration and Customs Enforcement (ICE) and Customs and Border Protection (CBP) because those agencies provide data that are most pertinent to our analysis.

Network Size. The largest group of false document users is illegal immigrants (Associated Press, 2006). We assume there are $N_k = 10\text{M}$ users in the false documents user network, which is roughly the illegal immigrant population in the U.S. in 2005 (Passel, 2006).

Approximately 500k new illegal immigrants enter the U.S. every year (Passel, 2006).

If we assume all of these individuals need false documents and another 500k people already in the U.S. also need false documents, then there are on the order of 1M consumers of false documents per year. A sophisticated false documents operation called the Castorena Family Organization sells 50–100 document sets per day (Fitzgerald, 2007) and their cells consist of 10–20 individuals (Immigrations and Customs Enforcement, 2005). Thus, a distributor would sell to ≈ 5 consumers a day or ≈ 1000 consumers per year, implying that there are ≈ 1000 false document distributors. However, this value assumes the characteristics of one of the largest and most complex operations is the norm. Because many distributors are involved with much smaller operations and only serve a few clients, we assume there are $N_k = 5000$ distributors in the false documents distributor network.

Budget. The budget for the CBP in 2005 was 6.2 billion dollars (Department of Homeland Security, 2005a). The investigative category that is most relevant to our analysis is “Border Security Inspections and Trade Facilitation at Points of Entry,” which has a budget of \$2.72B (Department of Homeland Security, 2005a). Unfortunately, there is no more information regarding how much of these resources are spent investigating users of false documents. However, a reasonable amount of the effort at ports of entry is analyzing documents to ensure that the people who enter the U.S. are who they claim to be and are entering for legitimate purposes. Therefore, we roughly estimate that $\hat{B}_k^I = b_k =$ \$500M for users of false documents.

The 2007 ICE budget was \$4.7B, and \$1.3B of that went to domestic investigations (Immigrations and Customs Enforcement, 2006). These investigations include visa security, illegal arms exports, financial and smuggling violations, immigration and customs fraud, human trafficking, identity and benefit fraud, child pornography, and sex tourism (Immigrations and Customs Enforcement, 2006). The category “Identity and Benefit Fraud” is most relevant to our analysis. There are eight categories that ICE investigates, and we

assume that roughly 10% of the budget is allocated to each category. We further assume that ICE targets primarily distributors of false documents with these resources, and therefore we set $\hat{B}_k^I = b_k = \$100\text{M}$ for distributors of false documents.

Number of Nodes Identified During the Investigation. CBP confiscated 75k false documents in 2005 and apprehended 84k individuals trying to enter the U.S. with false documents (Department of Homeland Security, 2005b). Therefore, we set $\hat{Y}_k^I = 80\text{k}$ for users of false documents. In 2007, ICE initiated 1309 fraud investigations that targeted document and immigration benefit fraud that supported illegal immigrants (Immigrations and Customs Enforcement, 2007), and thus we assume $\hat{Y}_k^I = 1309$ for distributors of false documents.

5 Supporting Numerical Results

In this section, we discuss how the results change when we modify the parameter values estimated in §4. We analyze each of the four terror categories separately in §5.1, we vary the efficiency parameter θ_1^S in §5.2, and we compute confidence intervals for the results in §5.3.

5.1 Analysis of Different Terror Categories

The terror cases in tables 1 and 2 are defined to be in one of four categories: right-wing, left-wing, single-issue, and international. In this subsection, we solve the uncoordinated version of the optimization problem for each separate category. We present the parameter estimates for each category in tables 4, 5 and 6 for $\mathbf{P}[m_k = l \mid m_1 = 1]$, λ_1 , and $\mathbf{E}[n_1]$, respectively. The degree distribution appears in fig. 2 for each category. We set N_k equal to $(\frac{190}{452}, \frac{108}{452}, \frac{60}{452}, \frac{94}{452}) \times 2000$ for the four categories (so that the proportion of terrorists in each category is the same as in table 3), thereby assuming that the probability for a terrorist

to be included in the cases in tables 1 and 2 does not vary by terror category. We compute the efficiency parameters for each category in the manner described in §4.2.5 and use the parameter values for \hat{B}_1^I and \hat{Y}_1^I from §4.

The value γ_k^* from equation (42) changes only slightly for each of these scenarios (table 7), suggesting that the specific form of the degree distribution has a minimal impact on the results. To further test this hypothesis, we fit the base-case empirical distribution in fig. 1 to several discrete distributions and then computed the values corresponding to table 2 of the main text; the results from this exercise were very similar to the base-case results (data not shown).

Because γ_k^* is similar to the base-case value for each terror category, the values of the overall efficiencies e_k are going to vary from the base-case because N_k , which is proportional to $\mathbf{P}[m_k = l]$, and $\mathbf{P}[m_k = l \mid m_1 = 1]$ (table 4) will be different for each category. While there are certainly variations in the criminal activities the different categories of terrorists are involved with (table 4), these differences do not change the rankings of e_k much from the base-case results in table 2 of the main text. The most noticeable difference between table 2 of the main text and table 7 is that the bank robbery network has the smallest value of e_k in the single issue and international categories because those terrorists in our population are not involved in that activity. The terror network has the largest value of e_k for every category. Because N_1 is smaller for each category, the value $\frac{\mathbf{P}[m_1=1]}{\mathbf{P}[m_k=l]} = \frac{N_1}{N_k}$ in $\mathbf{P}[m_1 = 1 \mid m_k = l]$ (see equation (7) in the main text) will be smaller, which reduces the probability that criminals will be involved in any one specific category of terrorism. Therefore, the government is less effective at identifying terrorists (for a given category) in criminal networks than in the base-case (table 2 of the main text).

5.2 Secondary Investigative Efficiency Parameter

In the base case, the efficiency parameter for the secondary investigation satisfies $\theta_1^S = 100\theta_1^I = 1.79 \times 10^{-4}$. In this subsection, we consider four other values of θ_1^S : $\theta_1^S = \theta_1^I$, $\theta_1^S = 10\theta_1^I$, $\theta_1^S = 1000\theta_1^I$, $\theta_1^S = 10000\theta_1^I$. The results from the uncoordinated version of the optimization problem for these scenarios (table 8) are nearly identical to the results in table 2 of the main text. There is some variation in the rankings of the three network with the largest value of e_k (terror, explosives, and bank robbery). The fraction of the resources allocated to the initial investigation, γ_k^* , increases as θ_1^S increases. The relationship between γ_k^* and θ_1^S occurs because as the secondary investigation becomes more efficient, the government shifts its resources away from the secondary investigation to the initial investigation because the government is now able to maintain the same effectiveness in the secondary investigation with fewer resources. Furthermore, e_k increases as θ_1^S increases because when the government is more effective at identifying terror interactions during the secondary investigation, it will detect more terrorists throughout the investigation.

5.3 Uncertainty Analysis

There is uncertainty with the parameter estimates in §4. Unfortunately, for most of the parameters we do not have the underlying distributions for their true values and therefore cannot adequately determine how the uncertainty in the inputs affects the results. We can compute standard errors for the estimates of the conditional probabilities $\mathbf{P}[m_k = 1 \mid m_1 = 1]$ in table 4, but most of the other parameter estimates in §4 are derived from single values in government reports. While we cannot do an exact uncertainty analysis for the model, we can illustrate how uncertainty in the input values may propagate through the system. First we assume that all inputs are normally distributed. While the parameters are positive and some are probabilities, the normality assumption is reasonable for this purpose.

We assume the conditional probabilities $\mathbf{P}[m_k = 1 \mid m_1 = 1]$ in table 4 have a mean of \hat{p} and a standard deviation of $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where \hat{p} is the point estimate in table 4 and n is the 452 nodes in the terror network. For the remaining input parameters (λ_1 , N_k , and \hat{Y}_k^I and \hat{B}_k^I from equation (54)), we assume the values are normally distributed with mean \hat{x} and standard deviation $0.3\hat{x}$, where \hat{x} is the point estimate from §4. Finally, we do not change the degree distribution of the terror network n_1 and we assume that $\theta_1^S = a\theta_1^I$ where a is normally distributed with mean 100 and standard deviation 30.

We present the results from the uncoordinated version of the optimization problem in table 9. To construct table 9 we first generated the parameter values according to the distributions described in the previous paragraph and then computed the values of e_k corresponding to these inputs. We repeated this 1 million times and calculated a 95% confidence interval for e_k based upon these runs (column 3 of table 9). Furthermore, for each run we can rank the networks according to the value of e_k . In the last column of table 9 we list which place each network is ranked most frequently and what fraction of the 1 million runs the network is ranked in that place. For example, the explosives network has the third largest value of e_k in 0.745 of the runs.

The ratio of the upper limit of the confidence intervals to the lower limit is about a factor of 10. The main results are similar to those in table 2 of the main text. The two largest networks are still the least effective networks. They have the two smallest values of e_k in 0.989 of the runs. The terror network, explosives network, and bank robbery network have the three largest values of e_k in 0.913 of the runs, and either the terror network or bank robbery network has the largest value of e_k in 0.949 of the runs.

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Figure Legends

Figure 1. Empirical degree distribution for the terror population in tables 1 and 2.

Figure 2. Empirical degree distribution for the terror population in tables 1 and 2 by category. **(a)** Right-wing; **(b)** Left-wing; **(c)** Single-issue; **(d)** International.

Name of Terror Case	Category	Terrorists	Interactions	Edges	Case Number in Smith et al. (2006)	Sources
Arizona Patriots	Right-wing	18	35.50	41	1.1	Smith et al. (2006); United States of America vs. Oliphant, Schlect, Arthur, and Ross (1986) United States of America vs. Hagan (1986); United States of America vs. Christensen (1986) United States of America vs. Gumaer and Christensen (1986)
Aryan Nations I	Right-wing	5	18	7	1.2	Smith et al. (2006); United States of America vs. Winslow, Nelson, and Baker (1990)
Aryan Nations II	Right-wing	3	0	0	1.2	Smith et al. (2006); Keenan vs. Aryan Nations, Saphine Inc, Butler, Teague, Warfield, Yeager, and Wright (1999)
Buford Furrows Incident	Right-wing	1	0	0	1.3	Smith et al. (2006)
Aryan People's Republic	Right-wing	6	31.5	14	1.4	Smith et al. (2006); United States of America vs. Thomas, Langan, Stedeford, McCarthy, and Brescia (1997)
The Kehoe Cell	Right-wing	5	1	0	1.5	Smith et al. (2006); United States of America vs. Kehoe (2002) Southern Poverty Law Center (1998)
Bixby SC	Right-wing	3	2.5	3	1.6	Smith et al. (2006)
The Order	Right-wing	49	189.5	231	1.7, 1.15, 1.17	Smith et al. (2006); United States of America vs. Ellison and Thomas (1985) United States of America vs. Ellison and Noble (1985); United States of America vs. Ellison and Yates (1985) United States of America vs. Ellison, Stone, Russell, Noble, Loewen, and Giles (1985) United States of America vs. Miles, Beam, Butler, Scutari, Pierce, Barnhill, McBrearty, Lane, Miller, et al. (1988) United States of America vs. Smalley and Brugle (1985); United States of America vs. Scott (1985) United States of America vs. Lane and Pierce (1985)
Jewish Defense League	Right-wing	3	13	3	1.8	Smith et al. (2006); United States of America vs. Lane and Pierce (2002)
Ku Klux Klan Barefoot	Right-wing	6	14	10	1.9	Smith et al. (2006); Holthouse and Sanchez (2007)
Ku Klux Klan Fusilier	Right-wing	6	0	0	1.10	Smith et al. (2006)
Ku Klux Klan Hull	Right-wing	5	10	10	1.11	Smith et al. (2006)
Krar IDC	Right-wing	3	7	1	1.12	Smith et al. (2006); United States of America vs. Krar, Bruey, and Feltus (2003)
Minnesota Patriots Council	Right-wing	5	14	6	1.13	Smith et al. (2006); United States of America vs. Henderson and Oelrich (1995) United States of America vs. Wheeler and Baker (1995)
Oklahoma City Bombing	Right-wing	3	9	2	1.14	Smith et al. (2006); United States of America vs. McVeigh and Nichols (1995) United States of America vs. McVeigh and Nichols (1996)
Phineas Priests	Right-wing	4	10	3	1.18	Smith et al. (2006); United States of America vs. Merrell, Barbee, and Berry (1996)
Sence Conspiracy	Right-wing	3	5	2	1.19	Smith et al. (2006)
Third Continental Congress	Right-wing	10	39.5	45	1.20	Smith et al. (2006); United States of America vs. Dorsett and Glover (1997) United States of America vs. Hobeck and Hobeck (1997); United States of America vs. Newman and Newman (1997) United States of America vs. Glover (1997); United States of America vs. Lingenfelter (1997)
Up the IRS, Inc	Right-wing	1	6	0	1.21	Smith et al. (2006); United States of America vs. Hicks (1991a) United States of America vs. Hicks (1991b)
Washington State Militia	Right-wing	18	90.5	97	1.22	Smith et al. (2006) United States of America vs. Pitner, Mack, Kuehnoel, Fisher, Kirk, Kirk, Burton, Smith, Carter, and Hansen (1996)
Woodring Homrich Standoff	Right-wing	1	0	0	1.23	Smith et al. (2006)
West Virginia Mountaineer Milita	Right-wing	9	33.5	14	1.24	Smith et al. (2006); United States of America vs. Looker, Johnson, and Lewis (1996) United States of America vs. Looker, Phillips, and Moore (1996); United States of America vs. Rogers (1996) United States of America vs. Coon and Phillips, (1996); United States of America vs. Looker, and Coon (1996)
Vance Assassination	Right-wing	2	3	1	1.25	Smith et al. (2006); United States of America vs. Moody (1992)
White Patriot Party	Right-wing	11	36	26	1.26	Smith et al. (2006); United States of America vs. Miller, Jackson, Wydra, Lane, and Davis (1987) United States of America vs. Jackson, Sheets, and Miller (1987); United States of America vs. Miller (1987)
Oklahoma Constitutional Party	Right-wing	5	16	7	1.27	Smith et al. (2006); United States of America vs. Lampley, Lampley, Crow, and Baird (1995)
Felton-Chase	Right-wing	5	30.5	8	1.28	Smith et al. (2006); United States of America vs. Felton and Chase (2001)

Table 1: List of right-wing terror cases from Smith et al. (2006), along with the number of terrorists, the number of terror interactions, and the number of edges in the terror network.

Name of Terror Case	Category	Terrorists	Interactions	Edges	Case Number in Smith et al. (2006)	Sources
El Runks	Left-wing	13	131	38	2.1	Smith et al. (2006) United States of America vs. Fort, Mayes, Knox, Davis, Cranshaw, McAnderson, and Hawkins (1986) United States of America vs. McAnderson, Hawkins, Fort, Knox, and Cranshaw (1990)
FALN	Left-wing	6	34.5	12	2.2	Smith et al. (2006); United States of America vs. Jordan (2000)
EPB Macheteros	Left-wing	24	108.5	143	2.3	Smith et al. (2006) United States of America vs. Gerena, Rios, Palmer, Claudio, Diamante, Garcia, Claudio, et al. (1985) United States of America vs. Carrion, Diamante, Osorio, Rios, Negron, Claudio, Ramos, Palmer (1987)
May 19TH Communist Organization	Left-wing	25	6	30	2.4, 2.5, 2.6	Smith et al. (2006); United States of America vs. Shakur and Buck (1989) United States of America vs. Whitehorn (1987); Gado (2008) United States of America vs. Chumurenga (1985)
United Freedom Front	Left-wing	22	508	81	2.7	Smith et al. (2006); United States of America vs. Gros (1985) United States of America vs. Levasseur, Gros, Manning, Manning, Laaman, Curzi, Williams (1986)
Yahweh	Left-wing	18	5	12	2.8	Smith et al. (2006); Scheeres (2008)
Griffin Florida Assassination	Single-issue	3	21	2	3.1A, 3.2A	Smith et al. (2006); Booth (1993)
Kopp Amherst Assassination	Single-issue	3	9	3	3.4A	Smith et al. (2006); Arena (2001)
Ellerman Utah Bombing	Single-issue	6	3	6	3.5B	Smith et al. (2006)
EcoRaiders	Single-issue	6	18	11	3.6B	Smith et al. (2006)
Sherman Oregon Firebombing	Single-issue	4	16	6	3.7B	Smith et al. (2006); United States of America vs. Cesario (2002) United States of America vs. Sherman, Cesario, Rosenbloom, and Scarpetti (2002) United States of America vs. Sherman and Scarpetti (2002)
ELF Long Island Arsons	Single-issue	4	0	0	3.8B	Smith et al. (2006)
Fairfield Snow Bowl Vandalism (EMETIC)	Single-issue	13	87.5	32	3.9B	Smith et al. (2006); United States of America vs. Davis, Millett, Foreman, Baker, and Asplund (1989)
Free Critter Eugene Arson	Single-issue	2	4	1	3.10B	Smith et al. (2006)
Nebraska Golf Vandals	Single-issue	3	0	0	3.11B	Smith et al. (2006)
Coronado MSU Arson	Single-issue	8	45.5	11	3.12B	Smith et al. (2006); United States of America vs. Coronado (1993)
Santa Cruz 2	Single-issue	2	4	0	3.13B	Smith et al. (2006)
Unabomber	Single-issue	1	12	0	3.14B	Smith et al. (2006)
Wisconsin Mink Release	Single-issue	2	1	1	3.15B	Smith et al. (2006)
Dr. Robert Goldstein	Single-issue	3	9	3	3.16C	Smith et al. (2006); McGraw (2008a)
Hezbollah: Cigarette Smuggling	International	9	39	19	4.1	Smith et al. (2006); Crowley (2004)
Japanese Red Army	International	2	11	1	4.2	Smith et al. (2006); United States of America vs. Kikumura (1988)
Millenium Conspiracy	International	20	88	48	4.3	Smith et al. (2006); United States of America vs. Haouri and Meskini (2000)
New York City Subway Bombing	International	4	19	5	4.4	Smith et al. (2006); United States of America vs. Mezer and Khalil (1997) United States of America vs. Mezer and Khalil (2000); Office of the Inspector General (1998)
Omega - 7	International	12	59	25	4.5	Smith et al. (2006); United States of America vs. Remon, Garcia, and Fernandez (1985) United States of America vs. Arocena and Badia (1983); United States of America vs. Arocena (1983)
Provisional IRA: Valhalla Incident	International	7	16	21	4.6	Smith et al. (2006); United States of America vs. Murray and Andersen (1987) United States of America vs. Murray, Andersen, Nee, Crawley, McIntyre, Nigro, and Winn (1986)
Provisional IRA: Tuscon Incident	International	14	39.5	22	4.6	Smith et al. (2006)
NYC Conspiracy	International	26	135.5	86	4.7, 4.8	Smith et al. (2006); McGraw (2008b) United States of America vs. Rahman, Nossair, El-Gabrowni, Ali, Hampton-El Abdelgani, et al. (1993) United States of America vs. Salameh, Ayyad, Abouhalima, Ajaj, Yousef, Alkaisi, Yasin (1997) United States of America vs. Rahman, El-Gabrowni, Nossair, Elhassan, Hampton-El, et al. (1999)

Table 2: List of left-wing, single-issue, and international terror cases from Smith et al. (2006), along with the number of terrorists, the number of terror interactions, and the number of edges in the terror network.

	Number of Cases	Number of Terrorists	Mean Number of Terrorists per Case	Median Number of Terrorists per Case	Standard Deviation of Terrorists per Case
Total	54	452	8.37	5	8.76
Right-wing	26	190	7.31	5	9.59
Left-wing	6	108	18	20	7.35
Single-issue	14	60	4.29	3	3.15
International	8	94	11.75	10.5	8.12

Table 3: Number of terrorists in each case, in total and by terror category.

	Total	Right-wing	Left-wing	Single-issue	International
Explosives, total	0.451	0.558	0.398	0.150	0.489
Explosives, users	0.425	0.542	0.398	0.150	0.394
Explosives, retail distributors	0.022	0.011	0.000	0.000	0.085
Explosives, wholesale distributors	0.004	0.005	0.000	0.000	0.011
Firearms, total	0.414	0.563	0.398	0.083	0.340
Firearms, users	0.350	0.489	0.389	0.083	0.191
Firearms, retail distributors	0.046	0.074	0.009	0.000	0.064
Firearms, wholesale distributors	0.018	0.000	0.000	0.000	0.085
Bank Robbery	0.217	0.242	0.482	0.000	0.000
False Documents, total	0.177	0.121	0.259	0.100	0.245
False Documents, users	0.155	0.095	0.250	0.100	0.202
False Documents, distributors	0.022	0.026	0.009	0.000	0.043
Violent Acts	0.091	0.089	0.176	0.017	0.043
Drugs, total	0.053	0.011	0.074	0.050	0.117
Drugs, users	0.013	0.011	0.019	0.017	0.011
Drugs, retail distributors	0.029	0.000	0.056	0.033	0.053
Drugs, wholesale distributors	0.011	0.000	0.000	0.000	0.053
Fraud	0.051	0.042	0.009	0.017	0.138
Counterfeit Money	0.035	0.084	0.000	0.000	0.000
Immigration Violations	0.033	0.000	0.000	0.000	0.160
Illegal Smuggling	0.027	0.000	0.000	0.000	0.128
Money Laundering	0.027	0.000	0.028	0.000	0.096
Arson	0.020	0.016	0.056	0.000	0.000
Extortion	0.011	0.000	0.000	0.000	0.053
Kidnapping	0.007	0.016	0.000	0.000	0.000
Tax Evasion	0.004	0.000	0.000	0.017	0.011

Table 4: Fraction of terrorists involved with other criminal activities, $\mathbf{P}[m_k = 1 \mid m_1 = 1]$.

	Interaction Rate
Total	5.47×10^{-2}
Right-wing	3.56×10^{-2}
Left-wing	7.73×10^{-2}
Single-issue	9.32×10^{-2}
International	5.52×10^{-2}

Table 5: Average annual interactions per edge in the terror network, λ_1 .

	Mean Degree	Median Degree	Standard Deviation of Degree
Total	5.09	4	4.91
Right-wing	5.59	4	5.36
Left-wing	5.85	4	5.62
Single-issue	2.53	2	2.28
International	4.83	5	3.66

Table 6: Characteristics of the degree distribution of the terror network, n_1 .

	γ_k^*				e_k				Rank of e_k			
	Right-wing	Left-wing	Single-issue	International	Right-wing	Left-wing	Single-issue	International	Right-wing	Left-wing	Single-issue	International
Terror	–	–	–	–	5.00×10^{-7}	5.00×10^{-7}	5.00×10^{-7}	5.00×10^{-7}	1	1	1	1
Explosives	0.83	0.85	0.86	0.84	1.20×10^{-7}	4.52×10^{-8}	9.98×10^{-9}	5.87×10^{-8}	3	3	2	2
Firearms User	0.58	0.60	0.59	0.57	1.87×10^{-9}	8.24×10^{-10}	1.02×10^{-10}	4.04×10^{-10}	6	6	3	5
Firearms Distributor	0.74	0.76	0.76	0.74	3.70×10^{-8}	2.43×10^{-9}	0	4.16×10^{-8}	4	5	5	3
Bank Robbery	0.64	0.65	0.64	0.62	2.86×10^{-7}	3.13×10^{-7}	0	0	2	2	5	7
False Docs User	0.37	0.37	0.36	0.34	1.64×10^{-10}	2.43×10^{-10}	5.54×10^{-11}	1.85×10^{-10}	7	7	4	6
False Docs Distributor	0.68	0.69	0.69	0.67	2.42×10^{-8}	4.58×10^{-9}	0	2.24×10^{-8}	5	4	5	4

Table 7: Results for the uncoordinated version of the optimization problem by terror category, including the optimal allocation γ_k^* from equation (42), the efficiency e_k from equations (50)-(51), and the rank, in descending order, of e_k .

	γ_k^*				e_k				Rank of e_k			
	$\theta_1^S = \theta_1^I$	$\theta_1^S = 10\theta_1^I$	$\theta_1^S = 10^3\theta_1^I$	$\theta_1^S = 10^4\theta_1^I$	$\theta_1^S = \theta_1^I$	$\theta_1^S = 10\theta_1^I$	$\theta_1^S = 10^3\theta_1^I$	$\theta_1^S = 10^4\theta_1^I$	$\theta_1^S = \theta_1^I$	$\theta_1^S = 10\theta_1^I$	$\theta_1^S = 10^3\theta_1^I$	$\theta_1^S = 10^4\theta_1^I$
Terror	–	–	–	–	5.00×10^{-7}	5.00×10^{-7}	5.00×10^{-7}	5.00×10^{-7}	1	1	2	2
Explosives	0.32	0.60	0.95	0.99	3.56×10^{-8}	1.24×10^{-7}	2.88×10^{-7}	3.05×10^{-7}	2	3	3	3
Firearms User	0.12	0.30	0.82	0.95	1.37×10^{-10}	8.72×10^{-10}	6.21×10^{-9}	7.91×10^{-9}	6	6	6	6
Firearms Distributor	0.21	0.47	0.91	0.98	6.70×10^{-9}	3.11×10^{-8}	1.11×10^{-7}	1.24×10^{-7}	4	4	4	4
Bank Robbery	0.14	0.35	0.86	0.96	3.20×10^{-8}	1.88×10^{-7}	1.07×10^{-6}	1.31×10^{-6}	3	2	1	1
False Docs User	0.05	0.15	0.65	0.87	1.40×10^{-11}	1.12×10^{-10}	1.97×10^{-9}	3.34×10^{-9}	7	7	7	7
False Docs Distributor	0.16	0.39	0.88	0.97	3.09×10^{-9}	1.67×10^{-8}	7.97×10^{-8}	9.36×10^{-7}	5	5	5	5

Table 8: Results for the uncoordinated version of the optimization problem as θ_1^S varies, including the optimal allocation γ_k^* from equation (42), the efficiency e_k from equations (50)-(51), and the rank, in descending order, of e_k .

Network	Point estimate of e_k	95% Confidence Interval	Mode Rank (fraction of time the mode occurs)
Terror	5.00×10^{-7}	$(1.85 \times 10^{-7}, 13.4 \times 10^{-7})$	2 (0.496)
Explosives	2.31×10^{-7}	$(0.570 \times 10^{-7}, 8.08 \times 10^{-7})$	3 (0.745)
Illegal Firearms User	3.19×10^{-9}	$(0.790 \times 10^{-9}, 10.3 \times 10^{-9})$	6 (0.979)
Illegal Firearms Distributor	7.62×10^{-8}	$(1.77 \times 10^{-8}, 26.3 \times 10^{-8})$	4 (0.673)
Bank Robbery	6.11×10^{-7}	$(1.51 \times 10^{-7}, 20.0 \times 10^{-7})$	1 (0.564)
False Documents User	6.36×10^{-10}	$(1.42 \times 10^{-10}, 21.3 \times 10^{-10})$	7 (0.982)
False Documents Distributor	4.89×10^{-8}	$(0.870 \times 10^{-8}, 17.5 \times 10^{-8})$	5 (0.723)

Table 9: Uncertainty analysis for the uncoordinated version of the optimization problem. The point estimate of the overall cost-effectiveness e_k and its 95% confidence interval. The last column lists what place each network is ranked most frequently and how frequently this ranking occurs.

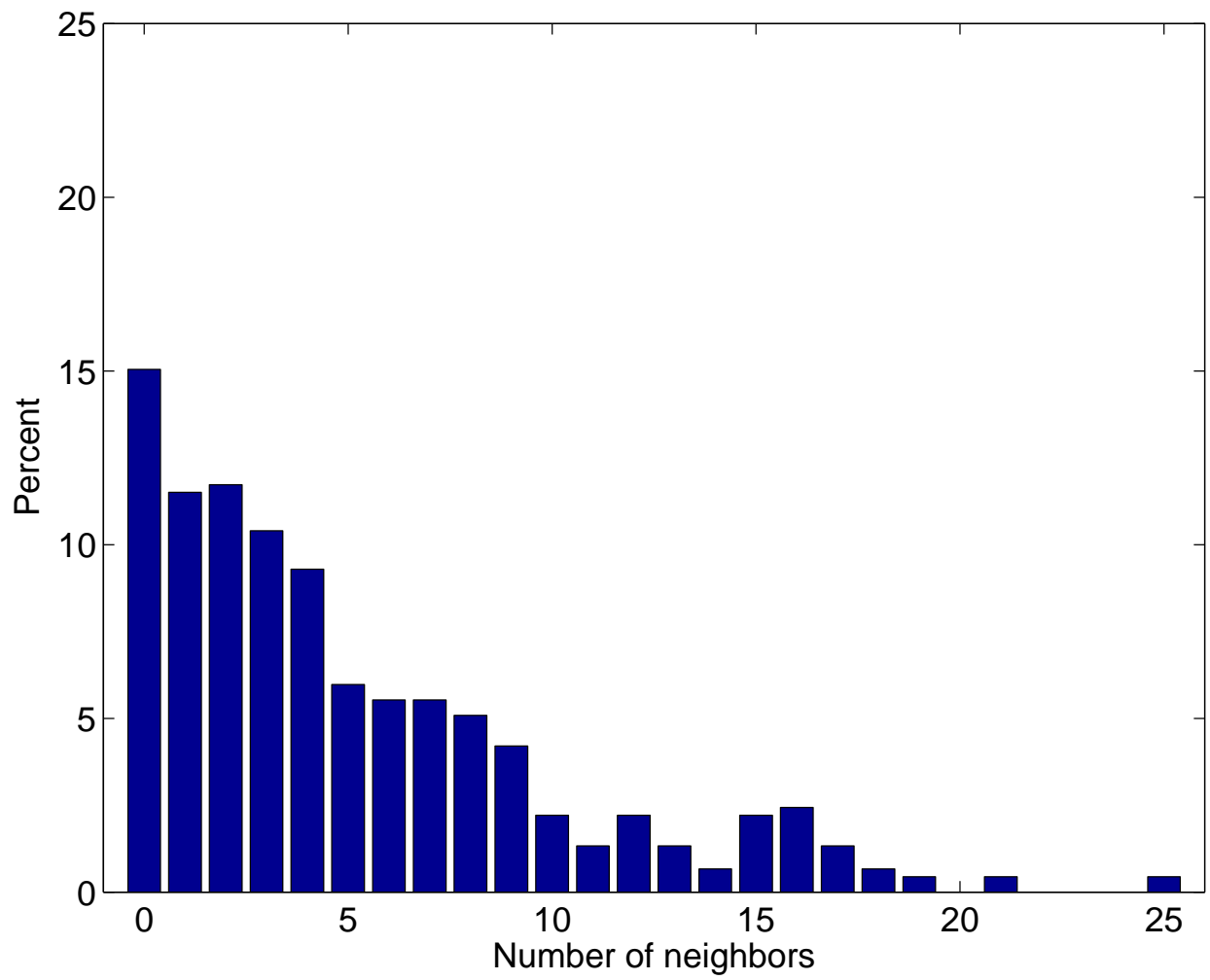
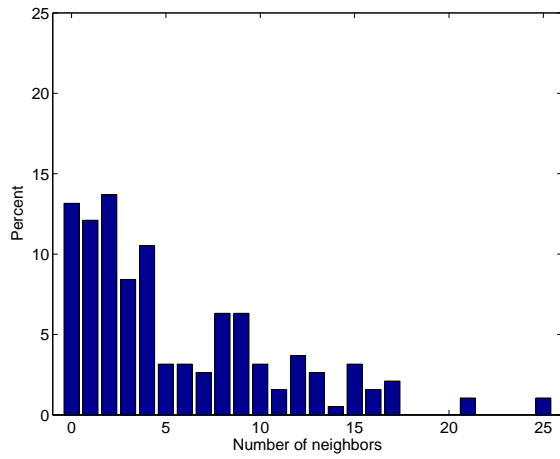
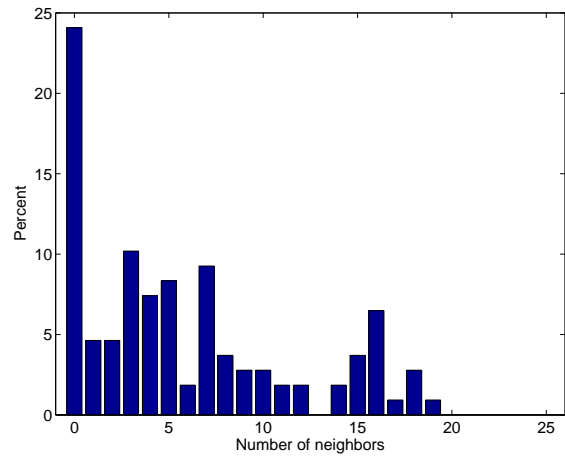


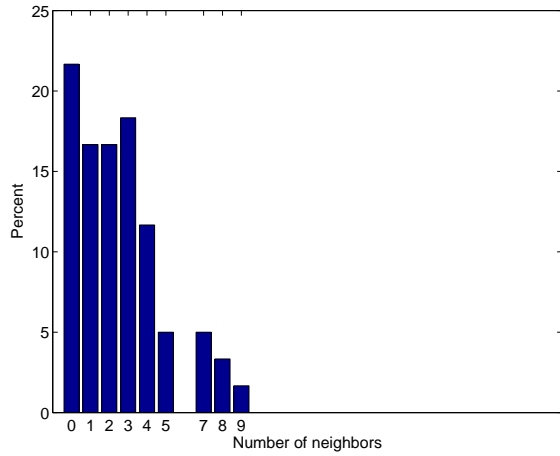
Figure 1:



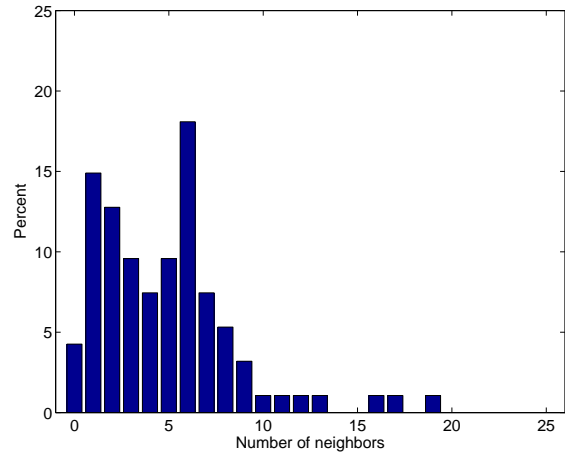
(a) Right-wing



(b) Left-wing



(c) Single-issue



(d) International

Figure 2: