APPENDIX

This appendix formulates and calibrates the mathematical model described in the main text. The deployment model is described in §1, the PTSD model parameter estimates are given in §2 and the detailed results are presented in §3.

1 The Deployment Model

In §1.1, we construct deployment schedules for major combat troops and their attached support troops in the active Army, National Guard and Marines from published data at the brigade level (for active Army and National Guard) or the more detailed battalion level (for Marines). In §1.2, we estimate deployment schedules for unattached support troops (e.g., Army Reserve) by using a mathematical model that incorporates cycles of deployment and rest. We present the resulting deployment schedules (which are through September 2008) in §1.3 and construct future deployment schedules (from October 2008 until complete withdrawal) under three possible withdrawal strategies in §1.4.

1.1 Brigade/Battalion Construction

The basic hierarchy of the Army is that several battalions make up a brigade, several brigades make up a division, several divisions make up a corps, and one or more corps form a theater. There are two types of units, nondeployable and deployable, and we model only the deployable troops. Of these deployable troops, there are combat units and support units. Combat units also include support troops, and we distinguish between support troops associated with a combat unit (attached support troops) and those that are not associated with a combat unit (unattached support troops). The unattached support troops are members of support units (e.g., engineering, medical, or logistics) that are assigned to
a corps or theater and may assist many other combat units when necessary [1]. For more
details on the structure of the Army and the different types of troops refer to [1]. In this
section we estimate the deployment schedules of Army combat brigades and Marine combat
battalions (with attached support troops) directly, and in §1.2 we compute the deployment
schedules of unattached support troops indirectly.

It is possible to approximately track the deployment of major combat units in the
Army and Marine corps deploying to and returning from OIF by examining multiple publicly
available sources, including government reports [1, 2, 3, 4, 5], media reports [6], and various
independent compilations and webpages [7, 8]. We built a dataset that approximates the
deployment start and end dates for each OIF deployment of each brigade/battalion to the
nearest month. Our dataset includes 42 active Army brigades, 21 National Guard brigades,
27 active Marine battalions, and 9 reserve Marine battalions. To construct the dataset,
we first consulted compiled documents [7, 8] and military press releases to get a list of
major combat units that may have served in OIF. We then performed a Lexis-Nexis [6]
search of media reports to independently verify the deployment times of each unit. Because
there is no official DOD database that contains the deployment schedules of each unit,
local media accounts that provide information about deployment dates are valuable sources.
These accounts report when units from local bases will either return from or deploy to
OIF. Therefore, many of our references are media accounts, such as newspaper articles.
The datasets along with primary references can be found in Table 1 for the active Army
brigades, Table 2 for the National Guard brigades, and Table 3 for the Marine battalions.
These tables includes estimates of deployment dates prior to September 2008 as well as
predictions of deployment dates after September 2008, which are used in §1.4.

Each of these combat units is assumed to have a constant number of troops at full
strength depending on unit type. There are between 2.5k to 5k troops for Army combat
brigades (see Table 4 for a list of Army brigade sizes by type; these numbers do not include all of the attached support troops). The Army is currently undergoing a transformation to a modular system (i.e., a reorganization aimed at improving agility [1]), and there are usually 3 major combat brigades for pre-modular divisions and 4 major combat brigades for modular divisions. There are approximately 15k troops per division [1] (this includes combat and attached support troops), and thus we assume there are 5k combat and attached support troops for a pre-modular brigade and 4k combat and attached support troops for a modular brigade. Because there are usually 3 major combat battalions per brigade [1], we assume there are 1.5k combat and attached support troops in each Marine battalion.

We assume that all troops starting a deployment stay until the end of the deployment. The fraction of troops from the previous deployment serving on the current deployment depends on the annual continuation rate of their service branch, which is derived in §1.2.2 from data in [218], and the length of time between the start of the two deployments. Discontinuing servicemembers are replaced by new servicemembers to maintain constant troop strength in each unit.

We denote the total number of major combat plus attached support troops of type $j$ at time $t$ by $\hat{N}_j(t)$, where $j = 1, 2, 3$ refer to active Army, reserve Army (which includes Army Reserve and National Guard, although all Army Reserve soldiers are unattached in our model), and (active plus reserve) Marines, respectively, and where $t$ is numbered consecutively with $t = 1$ corresponding to March 2003. This is calculated by a MATLAB program that cycles through each unit and computes the above steps. We define the vector $\hat{n}^{(j)} = (\hat{N}_j(1), \ldots, \hat{N}_j(M))^T$, where $M$ is the number of months.
1.2 Rotation Model for Unattached Troops

Because the unattached support units fall under the umbrella of larger units, it is difficult to find detailed information about when specific units deploy. Therefore, we assume unattached support units are deployed in repetitive cycles. Each unit deploys for a constant number of months, rests for a constant number of months, and continues in this manner throughout OIF. We need to estimate the deployment lengths, rest periods, and continuation rates from available data. Finally, the most difficult part of this estimation process is computing when the unattached units first deploy. We formulate a quadratic program to compute these quantities with constraints given by deployment characteristics at various time points. The mathematical model describing the cyclic deployments is formulated in §1.2.1, the parameter values are estimated in §1.2.2, and the construction of the quadratic program and its constraints is described in §1.2.3.

1.2.1 The Rotation Model

Let $A_j(t)$ be the number of type $j$ unattached support soldiers that first deploy during month $t$. A deployment cycle for type $j$ soldiers consists of deploying for $d_j$ months and returning home for $\delta_j$ months. After each deployment cycle, a fixed fraction $r_j$ of soldiers redeploy for another cycle. Hence, of the soldiers that first deploy in month $t$, $r_j^l A_j(t)$ remain after $l$ deployments. Discontinuing servicemembers are not replaced by new servicemembers, and thus the troop strength of each unit decreases with each deployment. While $t = 1$ corresponds to March 2003 we assume the unattached troops can deploy as early as October 2002 ($t = -4$) to allow for an initial buildup of troops.

Let $\tilde{N}_j(t)$ be the total number of type $j$ unattached support troops deployed during month $t$. If $I_{\{x\}}$ is the indicator function for the event $x$ (i.e., equals 1 if $x$ is true and equals 0 otherwise), $x \mod y$ is the modulo operator (i.e., the remainder after dividing $x$ by $y$), and
\[ \lfloor x \rfloor \text{ is the largest integer less than or equal to } x, \text{ then} \]
\[
\sum_{s=-4}^{t} I_{\lfloor (t-s) \text{mod}(d_j + \delta_j) \rfloor < d_j} r_j^j A_j(s) = \tilde{N}_j(t). \quad (1)
\]

It is convenient to write equation (1) in matrix notation, where \( M \) is the total number of months under consideration. If we define the vectors \( \tilde{n}^{(j)} = (\tilde{N}_j(-4), \tilde{N}_j(-3), \ldots, \tilde{N}_j(M))^T \)
and \( a^{(j)} = (A_j(-4), A_j(-3), \ldots, A_j(M))^T \), and the \((M + 5) \times (M + 5)\) deployment matrix \( B \) as
\[
B_{is}^{(j)} = \begin{cases} 
    r_j^j \lfloor \frac{t-s}{d_j+\delta_j} \rfloor & \text{if } s \leq t \text{ and } (t-s) \text{mod}(d_j + \delta_j) < d_j; \\
    0 & \text{otherwise},
\end{cases} \quad (2)
\]
then equation (1) can be expressed as
\[
B^{(j)} a^{(j)} = \tilde{n}^{(j)} \quad \text{for } j = 1, 2, 3. \quad (3)
\]

1.2.2 Parameter Estimation for Unattached Troops

For each type \( j \), we need to estimate the deployment length \( d_j \), the interdeployment length \( \delta_j \), the redeployment probability \( r_j \), and the arrival process \( A_j(t) \) of unattached troops. In this section we estimate \( d_j, \delta_j, \) and \( r_j \), and in 1.2.3 we estimate \( A_j(t) \).

We assume \( d_2 = 12 \) and \( d_3 = 7 \) \[219\]. We assume \( d_1 = 12 \) for \( t < 39 \) and \( t \geq 66 \), and \( d_1 = 15 \) for \( 39 \leq t < 66 \). The change in policy at month 39, which corresponds to May 2006, was announced on April 11, 2007, and applied to all currently deployed active Army soldiers \[220\]. The change in policy at month 66, which corresponds to August 2008, was announced on April 10, 2008, and applied to all troops deploying after August 1, 2008 \[221\].

Since 2003, the active Army has had 1.2 units out of deployment for every unit in Iraq, and the National Guard has had 4.3 units out of deployment for every unit in Iraq \[222\]. Hence, \( \delta_1 = 1.2(12) \), which we round up to 15 months, and \( \delta_2 = 4.3(12) \), which we round up to 52 months. The Marines have \( \delta_3 = 9 \) months \[219 \ 223\].
To estimate the redeployment probabilities, we use the annual continuation rates in [218], which measure the fraction of troops that started the fiscal year in the military who are still in the military at the end of the fiscal year. Figure 1-1 of [218] has continuation rates for 2003-2005 for active Army, National Guard and Army Reserve, and Table 1-1 of [218] gives the fraction of soldiers in each of these 3 parts of the military. Averaging over the 3 years yields an annual continuation rate of 0.838 for active soldiers. Taking a weighted (the weights can be derived from Table 1-1 of [218]) average of the annual continuation rates for the National Guard and Army Reserve gives a 3-year average of 0.823. Using Table 2-1 and Figure 2-1 in [218], we get a weighted (over active and reserve) average annual continuation rate for Marines of 0.822. To convert annual continuation rates into redeployment probabilities $r_j$, we raise the annual probabilities to the power of $d_j + \delta_j$ (measured here in years). This gives $r_1 = (0.838)^{2.25} = 0.672$, $r_2 = (0.823)^{5.33} = 0.354$, and $r_3 = (0.822)^{1.33} = 0.770$.

1.2.3 Estimation of Unattached Troops First Deployment Date

The most difficult manpower parameters to estimate are the $A_j(t)$’s. We have data on deployment characteristics throughout the first several years of OIF, and our approach is to convert the existing data into linear constraints for $A_j(t)$, and then find the $A_j(t)$’s that violate these constraints as little as possible (in the least squares sense). We derive five types of constraints, as described below.

**Total Deployment.** Table 5 gives the number of (active plus reserve) Army soldiers and the number of Marines deployed at 3-month time intervals, from March 2003 ($t = 1$) to June 2008 ($t = 64$) [224]. From these data, we subtract the estimated attached troop numbers ($\hat{n}^{(j)}$ from §1.1) to get the number of unattached support troops, then insert these numbers into the right side of 3 (for $\hat{n}^{(1)} + \hat{n}^{(2)}$ and $\hat{n}^{(3)}$) to create 42 constraints (there is no data
for March 2007). In addition, Table 6 [225] gives monthly totals over all Army soldiers and Marines (i.e., \( \hat{N}_1(t) + \hat{N}_2(t) + \hat{N}_3(t) + \bar{N}_1(t) + \bar{N}_2(t) + \bar{N}_3(t) \)) from July 2008 through September 2008 [225], which creates 3 more constraints from (3).

**Fraction of Army Soldiers that are Active.** Let \( g(t) \) be the fraction of Army soldiers that are active during month \( t \). Table 7 gives the values of \( g(t) \) for 5 months ([224] and Tables 2 through 6 of [226]). These 5 constraints can be expressed as \( \hat{N}_1(t) + \hat{N}_1(t) = g(t)[\hat{N}_1(t) + \hat{N}_2(t) + \hat{N}_3(t) + \bar{N}_1(t) + \bar{N}_2(t)] \), or equivalently \( [1-g(t)]\hat{N}_1(t) - g(t)\hat{N}_2(t) - [1-g(t)]\hat{N}_1(t) \), which can be rewritten in matrix notation as the linear constraints

\[
[1-g(t)] \sum_{s=-4}^{t} B_{t,s}^{(1)} A_1(s) - g(t) \sum_{s=-4}^{t} B_{t,s}^{(2)} A_2(s) = g(t)\hat{N}_2(t) - [1-g(t)]\hat{N}_1(t). \quad (4)
\]

**Fraction on First Deployment.** Table 8 states the fraction of Army soldiers on their first deployment (denoted by \( m_{12}(t) \)) during October 2005 (active only [227]) and September 2006 (active plus reserve [226]), and the fraction of Marines on their first deployment during September 2006 \( (m_3(t)) \) [226]. Let \( \hat{N}_{jk}(t) \) be the total number of combat plus attached servicemembers of type \( j \) who are on their \( k \)th deployment. The Army constraint for October 2005 is given by

\[
\hat{N}_{11}(t) + \sum_{s=0}^{d_1-1} A_1(t-s) = m_{12}(t) \left( \hat{N}_1(t) + \bar{N}_1(t) \right), \quad (5)
\]

the Army constraint for September 2006 is

\[
\sum_{j=1}^{2} \hat{N}_{j1}(t) + \sum_{j=1}^{2} \sum_{s=0}^{d_j-1} A_j(t-s) = m_{12}(t) \sum_{j=1}^{2} \left( \hat{N}_j(t) + \bar{N}_j(t) \right), \quad (6)
\]

and the Marines constraint is

\[
\hat{N}_{31}(t) + \sum_{s=0}^{d_3-1} A_3(t-s) = m_3(t) \left( \hat{N}_3(t) + \bar{N}_3(t) \right). \quad (7)
\]

**Median Number of Months Deployed on Current Deployment.** Table 9 gives the median number of months deployed on their current deployment for (active plus reserve) Army soldiers and for Marines during September 2006 \( (t = 43) \), which we denote by \( \bar{d}_{12}(t) \)
and \( \bar{d}_3(t) \), respectively \([226]\). Let \( \tilde{N}_{j,\text{dep.} \leq x}(t) \) (\( \tilde{N}_{j,\text{dep.} > x}(t) \), respectively) be the number of unattached troops of type \( j \) that have deployed for less than or equal to (greater than, respectively) \( x \) months on their current deployment at time \( t \), and define \( \hat{N}_{j,\text{dep.} \leq x}(t) \) and \( \hat{N}_{j,\text{dep.} > x}(t) \) accordingly for combat plus attached troops. Because the median can be represented by two linear inequality constraints, these data lead to the four constraints:

\[
\sum_{j=1}^{2} \left( \hat{N}_{j,\text{dep.} \leq \bar{d}_1}(t) + \tilde{N}_{j,\text{dep.} \leq \bar{d}_1}(t) \right) \geq \sum_{j=1}^{2} \left( \hat{N}_{j,\text{dep.} > \bar{d}_1}(t) + \tilde{N}_{j,\text{dep.} > \bar{d}_1}(t) \right), \quad (8)
\]

\[
\sum_{j=1}^{2} \left( \hat{N}_{j,\text{dep.} \geq \bar{d}_1}(t) + \tilde{N}_{j,\text{dep.} \geq \bar{d}_1}(t) \right) \geq \sum_{j=1}^{2} \left( \hat{N}_{j,\text{dep.} < \bar{d}_1}(t) + \tilde{N}_{j,\text{dep.} < \bar{d}_1}(t) \right), \quad (9)
\]

\[
\hat{N}_{3,\text{dep.} \leq \bar{d}_3}(t) + \tilde{N}_{3,\text{dep.} \leq \bar{d}_3}(t) \geq \hat{N}_{3,\text{dep.} > \bar{d}_3}(t) + \tilde{N}_{3,\text{dep.} > \bar{d}_3}(t), \quad (10)
\]

\[
\hat{N}_{3,\text{dep.} \geq \bar{d}_3}(t) + \tilde{N}_{3,\text{dep.} \geq \bar{d}_3}(t) \geq \hat{N}_{3,\text{dep.} < \bar{d}_3}(t) + \tilde{N}_{3,\text{dep.} < \bar{d}_3}(t). \quad (11)
\]

These constraints can be rewritten as (at \( t = 43 \), for unattached support Army soldiers it is possible to have troops on their second deployment, and for unattached support Marines it is possible to be on the third deployment)
\[
\sum_{j=1}^{2} \sum_{k=1}^{2} \left( \sum_{s=0}^{d_{j-1}} B^{(j)}_{t,s-(k-1)(d_j+\delta_j)} A_j(t - s - (k-1)(d_j + \delta_j)) - \sum_{s=d_{j-1}(t)}^{d_{j-1}} B^{(j)}_{t,s-(k-1)(d_j+\delta_j)} A_j(t - s - (k-1)(d_j + \delta_j)) \right) \\
\geq 2 \left( \hat{N}_{j,dep. > \bar{d}_{12}(t)}(t) - \hat{N}_{j,dep. \leq \bar{d}_{12}(t)}(t) \right),
\]

(12)

\[
\sum_{j=1}^{2} \sum_{k=1}^{2} \left( \sum_{s=\bar{d}_{12}(t)-2}^{\bar{d}_{12}(t)-1} B^{(j)}_{t,s-(k-1)(d_j+\delta_j)} A_j(t - s - (k-1)(d_j + \delta_j)) \right) \\
\geq 2 \left( \hat{N}_{j,dep. < \bar{d}_{12}(t)}(t) - \hat{N}_{j,dep. \geq \bar{d}_{12}(t)}(t) \right),
\]

(13)

\[
\sum_{k=1}^{3} \left( \sum_{s=0}^{d_{3-1}} B^{(3)}_{t,s-(k-1)(d_3+\delta_3)} A_3(t - s - (k-1)(d_3 + \delta_3)) - \sum_{s=d_3(t)}^{d_{3-1}} B^{(3)}_{t,s-(k-1)(d_3+\delta_3)} A_3(t - s - (k-1)(d_3 + \delta_3)) \right) \\
\geq \hat{N}_{3,dep. > \bar{d}_3(t)}(t) - \hat{N}_{3,dep. \leq \bar{d}_3(t)}(t),
\]

(14)

\[
\sum_{k=1}^{3} \left( \sum_{s=d_3(t)-1}^{d_{3-1}} B^{(3)}_{t,s-(k-1)(d_3+\delta_3)} A_3(t - s - (k-1)(d_3 + \delta_3)) - \sum_{s=0}^{\bar{d}_3(t)-2} B^{(3)}_{t,s-(k-1)(d_3+\delta_3)} A_3(t - s - (k-1)(d_3 + \delta_3)) \right) \\
\geq \hat{N}_{3,dep. < \bar{d}_3(t)}(t) - \hat{N}_{3,dep. \geq \bar{d}_3(t)}(t).
\]

(15)

**Mean Total Number of Months Deployed.** Finally, Table [110] gives the mean number of months deployed in total (on the current plus past deployments) for currently deployed
Army soldiers in October 2005 \((t = 32)\), both for those on their first deployment (denote by \(\mu_{1d}(t)\)) and for those who have been on multiple deployments \((\mu_{2d}(t))\) [227]. For the active Army troops that we analyze, at time \(t = 32\) all units have been on either one or two deployments, so we will denote those on multiple deployments as being on their second deployment. For \(k = 1, 2\), let \(\tilde{N}_{1,\text{deps}=k}(t)\) be the number of unattached active Army troops that are on their \(k\)th deployment at time \(t\), and define \(\hat{N}_{1,\text{deps}=k}(t)\) similarly for combat plus attached active Army troops. Finally, let \(\tilde{N}_{1,\text{dep.}=k,\text{deps}=l}(t)\) be the number of unattached active Army troops who are on their \(l\)th deployment and in their \(k\)th total (i.e., over all \(l\) deployments) month of deployment at time \(t\), and define \(\hat{N}_{j,\text{dep.}=k,\text{deps}=l}(t)\) analogously for combat plus attached troops.

For the unattached active Army troops to be in their second deployment at time \(t = 32\), they must have started their first deployment between time \(-4\) and time \(5\). For second-time deployers, the constraint is given by

\[
\sum_{i=1}^{t} i \tilde{N}_{1,\text{dep.}=i,\text{deps}=2}(t) + \sum_{s=-4}^{t-(d_1+\delta_1)} (d_1 + t - (s + d_1 + \delta_1) + 1)B_{ts}^{(1)}A_1(s) = \mu_{2d}(t) \left( \tilde{N}_{1,\text{deps}=2}(t) + \tilde{N}_{1,\text{deps}=2}(t) \right).
\]

(16)

Moving variables to the left and constant terms to the right gives a linear constraint:

\[
\sum_{s=-4}^{t-(d_1+\delta_1)} (d_1 + t - (s + d_1 + \delta_1) + 1)B_{ts}^{(1)}A_1(s) = \mu_{2d}(t) \tilde{N}_{1,\text{deps}=2}(t) - \sum_{i=1}^{t} i \tilde{N}_{1,\text{dep.}=i,\text{deps}=2}(t).
\]

(17)

Similarly for first deployers, we have

\[
\sum_{s=0}^{d_1-1} (s + 1)A_1(t - s) - \mu_{1d}(t) \tilde{N}_{1,\text{deps}=1}(t) = \mu_{1d}(t) \tilde{N}_{1,\text{deps}=1}(t) - \sum_{i=1}^{t} i \tilde{N}_{1,\text{dep.}=i,\text{deps}=1}(t).
\]

(18)

Taken together, we can write the constraints (3)-(18) as \(Ea = b\) and \(Ca \geq c\) for matrices \(E\) and \(C\) and vectors \(b\) and \(c\). Because there is no nonnegative solution that
satisfies this set of constraints, we resort to minimizing the weighted squared deviation of the equality constraints, subject to the inequality constraints and nonnegativity constraints. As explained in Table 11, we weight these constraints to account for the scaling inherent in each constraint as well as the relative importance of the constraints. We impose these weights by scaling the rows of $E$ and $b$ to create the weighted matrix $\tilde{E}$ and vector $\tilde{b}$. That is, we solve the quadratic program

$$\min_a a^T \tilde{E}^T \tilde{E} a - 2 \tilde{b}^T \tilde{E} a + \tilde{b}^T \tilde{b},$$

subject to

$$Ca \geq c,$$

$$a \geq 0.$$

Substituting the solution $A_j(t)$ to (19)-(21) into equation (11) yields $\tilde{N}_j(t)$, which is the number of unattached type $j$ troops during month $t$.

1.3 Results

The solution $A_j(t)$ to (19)-(21) appears in Table 12. Fig. 1 displays the monthly troop levels for Army and Marines, broken down by combat plus attached troops vs. unattached troops, and Fig. 2 gives the monthly troop levels for active Army ($j = 1$), reserve Army ($j = 2$) and Marines ($j = 3$). Fig. 3 shows that our computed troop levels track the official DOD and project post-surge troop numbers reasonably well. The average monthly relative deviation (i.e., $\frac{|\text{reported} - \text{computed}|}{\text{reported}} \times 100\%$) in Figs. 3a, 3b and 3c are 5.9%, 9.8% and 0.0%, respectively, with larger deviations occurring during the first 7 months of OIF. Tables 13-16 compare the computed vs. reported values of the fraction of Army soldiers that are active, the median number of months deployed during the current deployment, the mean number of months deployed over all OIF deployments, and the fraction of troops on first deployment. The average relative deviation over the 12 comparisons in Tables 13-16 is 6.1%, and the largest
relative deviation is 15.2%, which is for the mean time deployed for first deployers during October 2005.

1.4 Future Deployments

To determine the incidence of symptomatic PTSD over the coming years, we need to estimate troop deployments from October 2008 through the end of OIF.

For each of the three withdrawal strategies defined in the main text, we define corresponding target trajectories, which are troop levels that remain constant until the beginning of withdrawal and then decrease linearly to 0 over the 13-month withdrawal process. We compute separate target trajectories for the Army (both active and reserves) and for the Marines by assuming that the fraction of servicemembers in the Army (Marines, respectively) from October 2008 until complete withdrawal is the same as the observed fraction between October 2007 and September 2008, which is 0.819 (0.181, respectively). In the remainder of this subsection, we describe how to construct withdrawal schedules to achieve these target trajectories.

To fit the withdrawal trajectories, we need to determine when brigades/battalions should deploy in the future. First, for any unit that is deployed during September 2008 and that does not have an estimated return date, we assume it deploys for the standard deployment length for its troop type (15 months for active Army before August 2008 and 12 months after August 2008, 12 months for reserve Army, and 7 months for Marines). Any unit deployed after September 2008 also deploys for the standard deployment length. Next, there are three types of units that could be deployed: combat and attached units, unattached units that deployed before October 2008 (these were estimated in section \[1.2\]), and unattached units that first deploy after October 2008. We will refer to unattached units that deployed before October 2008 as unattached 1 units and the unattached units that first deploy after
October 2008 as unattached 2 units. Unattached units will refer to both unattached 1 and unattached 2 units.

**Combat and Attached Units.** Because we are considering a withdrawal strategy, we allow combat and attached units to deploy only when replacing other combat and attached units that are finishing a tour of duty. We only replace an outgoing unit with a new unit if adding the unit will not make the total troop level greater than the target withdrawal trajectory at any point during the deployment.

To determine what order to deploy combat and attached units in the future, we form a queue based upon possible future deployment dates. Tables 1, 2, and 3 have estimates of past deployment dates as well as estimates of future deployment dates based upon published information. The queue is ordered with the units estimated to deploy closest to October 2008 at the front and those estimated to deploy farthest from October 2008 at the end. If units have the same estimated future deployment date, then the unit that is the most rested is the closest to the front of the queue. If units have the same estimated future deployment date and are equally rested, then they are ordered according to Tables 1, 2, and 3. Thus, in the Army queue, the 56th Brigade of the 36th Infantry Division of the National Guard is at the front of the queue and the 155th Brigade of the National Guard is at the end of the queue.

With this queue in place, we use the following procedure. In every month we see if any units finish a deployment. If they do then we deploy replacements according to the order of the queue, if adding them does not put the troop level greater than the target trajectory level. Because of modularity, it is possible that the unit at the front of the queue cannot be deployed but another in the queue can. In this case, we deploy the first unit in the queue that can be deployed.

It is possible that units in the queue never deploy, deploy at dates different than the
estimates in Tables 1, 2, and 3 or that all units in the queue deploy. If all the units in the queue have deployed then we consider candidates for future deployment in a manner related to using the most rested unit. Tables 1, 2, and 3 only list deployments for OIF. Units also deploy to Afghanistan or other expeditions, including humanitarian relief efforts. Thus, after the last recorded deployment of each combat and attached unit, we assume that the unit continues on a standard deployment cycle (e.g., for Marines there are 9 months of rest, then 7 months of deployment, then 9 months of rest, etc.). Based upon actual as well as these theoretical deployments, in each month we can define units that are eligible to deploy that month, and if a unit is needed to replace an outgoing unit we deploy the eligible unit that is most rested. A unit is eligible if the number of months since it last completed a deployment (real or theoretical) is within some range. The minimum of this range is the rest period for the standard deployment cycle (15 months for active Army, 52 months for reserve Army, and 9 months for Marines), and the maximum of the range is 24 months for active Army and 15 months for Marines, but we set no upper bound for reserve Army brigades. If it has been longer than this maximum value, we assume the unit would have gone on another theoretical deployment and thus would not be available to deploy. The eligible unit that is the most rested (from a real or theoretical deployment) is the next unit to deploy. If there are several most-rested eligible units, then they are ordered according to Tables 1, 2, and 3.

Unattached 1 Units. For each month from October 2008 until the end of the withdrawal period, we first determine if any combat and attached units have finished a deployment, and if so we replace them according to the method described above. Next we look at all the unattached 1 units and determine which are scheduled to deploy that month, assuming they stay on the standard deployment cycle. If an unattached 1 unit is scheduled to redeploy and deploying that unit does not put troop levels above the withdrawal trajectory, then we deploy this unattached 1 unit. Otherwise that unit is not deployed and we assume that this
unattached 1 unit is finished deploying and it will not redeploy at any future time. If there are several unattached 1 units that are scheduled for deployment during the same month, then we attempt to deploy them in a largest-unit-first manner.

**Unattached 2 Units.** After determining the deployment schedules for the combat and attached units and unattached 1 units for each month from October 2008 until the end of the withdrawal period, we have future troop levels that are by construction less than the target withdrawal trajectory. We add unattached 2 units to fit the withdrawal trajectory as well as possible. We determine their deployment schedule in an analogous fashion as described in section §1.2, i.e., embedding a system of equations, $Ba = n$, into a quadratic program. The vector $n$ is the target withdrawal trajectory minus the combat plus attached and unattached 1 troop levels, $a$ gives the number of unattached 2 servicemembers initially deploying in a given month, and the elements of the matrix $B$ are the fraction of servicemembers who initially deploy in a given month who are deployed during another month. The only difference between this method and the one in §1.2 is that unattached 2 units can only deploy if they can finish their entire deployment before the end of the withdrawal period, and thus there are no unattached 2 units that only deploy for a few months at the end of the withdrawal period.

Figure 4 displays the target withdrawal trajectories and the monthly troop levels for Army and Marines, broken down by combat plus attached troops vs. unattached 1 troops vs. unattached 2 troops, and Figure 5 gives the target withdrawal trajectories and the monthly troop levels for active Army ($j = 1$), reserve Army ($j = 2$), and Marines ($j = 3$).

## 2 PTSD Parameter Estimation

In this section, we estimate the PTSD model parameters, which are the mean initial stress $\alpha^{-1}$, the mean monthly stress exposures $\lambda_j(t)$, the batch size $b$, the recuperation parameter
θ, the mean stress threshold $\gamma^{-1}$, and the time lag parameters $\mu_i$ and $s_i, i = 1, 2$. We begin by estimating $\lambda_j(t)$ from monthly casualty data. We then estimate $\mu_i$ and $s_i$ from some limited longitudinal data, and finally jointly estimate $\alpha, b, \theta, \gamma$ from PTSD data.

**Average Monthly Stress.** For lack of disaggregated casualty data on active vs. reserve Army soldiers, we assume that $\lambda_1(t) = \lambda_2(t)$ and estimate $\lambda_1(t)$ and $\lambda_3(t)$ using monthly data on fatalities and wounded [228]. We computed the correlation between the number of fatalities and the number wounded in each month to be 0.75 for Army soldiers ($j = 1$ and 2 combined), 0.85 for Marines ($j = 3$), and 0.79 for the aggregated Army and Marines. The total number (from March 2003 to September 2008) wounded is 6.99 times as many as the total number of fatalities for Army and 8.51 for Marines. It is difficult to estimate the relative amount of stress caused by exposure to a fatality vs. exposure to a wounded servicemember, and for concreteness, we define the mean amount of stress to be $(6.99 \times \text{fatalities} + \text{wounded})$ divided by the number of troops deployed for Army and $(8.51 \times \text{fatalities} + \text{wounded})$ divided by the number of troops deployed for Marines, for each month $t$. That is, we divide the monthly $(6.99 \times \text{fatalities} + \text{wounded})$ quantity by $\hat{N}_1(t) + \tilde{N}_1(t) + \hat{N}_2(t) + \tilde{N}_2(t)$ for Army and we divide the monthly $(8.51 \times \text{fatalities} + \text{wounded})$ quantity by $\hat{N}_3(t) + \tilde{N}_3(t)$ for Marines to obtain our values for $\lambda_1(t)$ and $\lambda_3(t)$, where $\hat{N}_1(t) + \tilde{N}_1(t) + \hat{N}_2(t) + \tilde{N}_2(t)$ and $\hat{N}_3(t) + \tilde{N}_3(t)$ are linearly interpolated from the quarterly data in Table 5 from March 2003 to June 2008, and by assuming that the post-June 2008 monthly total (Army plus Marines) deployments in Table 6 are in the same proportion (0.822 Army and 0.178 Marines) as they were over the September 2007-June 2008 time period. The resulting values for $\lambda_1(t)$ and $\lambda_3(t)$ appear in Table 17 and Figure 6.

**Time Lag Parameters.** We estimate the four time lag parameters $\mu_i$ and $s_i, i = 1, 2$, from sparse longitudinal data from two separate studies. Our approach is to solve five equations for six unknowns, which are the four time lag parameters plus the unknown number of
servicemembers who had PTSD in each of the studies. The two studies, one from OIF and one from the Gulf War, screen servicemembers for PTSD at two time points, at the end of a deployment and then some months later. In each study, we assume that the PTSD rates are the same for active and reserve servicemembers (indeed, the rates are similar at the first time point in each study), but that the time lag depends upon whether the servicemember is physically with the military or has returned to civilian life.

The data for the OIF study appears in Table 3 of [229] and generates 3 equations. Let \( n_1 \) be the unknown number of 88,235 (56,350 active and 31,885 reserve or National Guard) surveyed (at the end of a deployment between September 2004 and October 2006) servicemembers who eventually experienced PTSD due to combat exposure during OIF (i.e., some may not have experienced symptoms until after the second time point). We first need an estimate of the time in the deployment when the PTSD-generating event occurs. Using the deployment history vectors \( C_{kj}(t) \) (which are defined in the Model Overview section of the main text) for both unattached support troops and combat and attached support troops, we determine the average total OIF deployment for an Army soldier that finished a deployment between September 2004 and October 2006. We analyze soldiers who have completed only one deployment separately from those who have completed two deployments. For soldiers completing their first deployment, the median length of the deployment is 12 months. Let \( V_1 \) be a \([0,12]\) uniform random variable representing the time (during the 12 months of combat) of the PTSD-generating event. Even though we have a measure of the frequency of traumatic events over time, \( \lambda_1(t) \), for simplicity we assume that the PTSD-generating event occurs uniformly during the period of deployment. For soldiers who have completed their second deployment we have that the median length of the first deployment is 12 months, the median length between deployments is 15 months, and the median length of the second deployment is 12 months. Let \( V_2 \) be a uniform random variable over the
broken interval $[0, 12] \cup [27, 39]$, which represents the time (during the two deployments) of the PTSD-generating event. A fraction 0.79 of the soldiers have completed one deployment and we define $Z$, the time between the PTSD-generating event and the time of the first screening, to be a mixture of $12 - V_1$ with probability 0.79 and $39 - V_2$ with probability 0.21. Finally, let $U_1$ be a [3,9] month uniform random variable representing the time interval between the two screening time points in this study. Combining the data for active and reserve personnel for the first time points gives

$$P(T_1 < Z) = \frac{3474 + 2119}{n_1}. \quad (22)$$

Considering only the active soldiers at the second time point yields

$$P(T_1 < Z + U_1) = \frac{3474 + 3697}{\left(\frac{56,350}{88,235}\right)n_1}. \quad (23)$$

Considering the reserve soldiers in between the first and second time points leads to

$$P(T_2 < U_1) = \frac{3457}{\left(\frac{31,885}{88,235}\right)n_1 - 2119}. \quad (24)$$

The final two equations come from a study of Gulf War veterans [230], in which 2949 servicemembers were assessed for PTSD at two time points: 5 days post-return after an average of 4 months in combat, and 18-24 months later. The mix was 72% (i.e., 2123 servicemembers) reserve and 28% (i.e., 826 servicemembers) active. At the first time point, 3% (i.e., 88 servicemembers) screened positive for PTSD. Because the active vs. reserve did not play a significant role in the PTSD rate at the first time point, we assume that 72% of the 88, or 63, were reserve servicemembers and 25 were active. At the second time point, 8% (i.e., 236 servicemembers) screened positive for PTSD. Furthermore, 79% of those who screened positive for PTSD at the second time point (i.e., 186 servicemembers) did not screen positive at the first time point, and hence 274 (i.e., 88+186) servicemembers had screened positive for PTSD by the second time point. Let $n_2$ be the unknown number of
the 2949 servicemembers that eventually experienced PTSD due to the combat (i.e., some may not have experienced symptoms until after the second time point), let $U_2$ be a $[0,4]$ uniform random variable that represents the time (during the 4 months of combat) of the PTSD-generating event, and let $U_3$ be a $[18,24]$ uniform random variable representing the time interval between the two measurement points in this study. The equation emanating from the first time point is

$$P(T_1 < 4 - U_2) = \frac{88}{n_2}. \quad (25)$$

We also know that the odds ratio for screening positive for PTSD at the second time point was $2.0$ (unconditioned on what happened at the first time point). If we let $p_r$ and $p_a$ be the PTSD rates for reserve and active servicemembers at the second time point, then it follows that $p_r$ and $p_a$ satisfy the two equations

$$2123p_r + 826p_a = 236, \quad (26)$$

$$\frac{p_r(1 - p_a)}{(1 - p_r)p_a} = 2. \quad (27)$$

Solving (26)-(27) yields $p_r = 0.092$ and $p_a = 0.048$ and so 196 reserve servicemembers and 40 active servicemembers screened positive for PTSD at the second time point. Of the servicemembers who screened positive for PTSD at the first time point, $62\%$ also screened positive at the second time point, and thus 157 reserve servicemembers experienced symptoms between the two time points. Therefore our final equation arising from focusing on the reserve servicemembers in between the two time points is

$$P(T_2 < U_3) = \frac{157}{0.72n_2 - 63}. \quad (28)$$

Because we have five equations and six variables, we set $n_2 = 306$, which gives the right side of equation (28) to be unity, and thus we conservatively assume that all reserve servicemembers show symptoms within 2 years (to test the impact of this assumption, we
analyze a scenario with stochastically larger time lag distributions in \(3.3\). To solve for \(\mu_1, s_1, \mu_2, s_2,\) and \(n_1\), we minimize the sum of the squared deviations between the left side and right side of equations (22)-(25) and (28). Finally we have bounds on our least squares problem. We require \(s_1, s_2, n_1 > 0\), the right sides of equations (22)-(25) and (28) to be between 0 and 1, and because the left side of (25) is less than the left side of (22), which is less than the left side of (23), and the left side of (24) is less than the left side of (28), we require that the right sides of these equations follow the same relationships.

The solution to this problem is \(\mu_1 = 2.47, s_1 = 2.73, \mu_2 = 1.40, s_2 = 0.57, n_1 = 19448,\) and \(n_2 = 306\). The median military time lag is \(e^{\mu_1} = 11.78\) months, the mean time lag is \(e^{\mu_1 + \frac{s_1^2}{2}} = 40.87\) years, and the dispersion factor is \(e^{s_1} = 15.35\) (implying that, e.g., 95% of the time lags are between \(\frac{11.78}{15.35} = 0.05\) months and \(\frac{15.35^2(11.78)}{12} = 231.16\) years). The median civilian time lag is \(e^{\mu_2} = 4.05\) months, the mean time lag is \(e^{\mu_2 + \frac{s_2^2}{2}} = 4.77\) months, and the dispersion factor is \(e^{s_2} = 1.77\) (and hence 95% of the time lags are between 1.29 and 12.72 months).

**PTSD Parameters.** Finally, we jointly estimate \(\alpha, b, \theta,\) and \(\gamma\) by using a least squares approach, where the objective function is the sum of 18 squares. Seventeen of the 18 squares correspond to 17 reported (from Mental Health Advisory Team (MHAT) studies) vs. predicted (by our model) PTSD rates (Table 18). Table 18 presents the same information as Table 1 in the main text, but Table 18 contains several additional scenarios. The seventeen MHAT values in Table 18 are of the form \(P_j(t)\), which is the probability a type \(j\) service-member has symptomatic PTSD during month \(t\). These months all correspond to one of the first four MHAT studies. MHAT-I occurred in September 2003 \((t = 7)\) [227], MHAT-II occurred in October 2004 \((t = 20)\) [227], MHAT-III occurred in October 2005 \((t = 32)\) [227], and MHAT-IV occurred in September 2006 \((t = 43)\). For example, “\(P_1(32) > 1st\) deployment” is the probability that active Army servicemembers who were on at least their
second deployment during MHA T-III had symptomatic PTSD in month $t = 32$ (MHA T-III). Another example, “$P_{1+2}(43), \leq 6$ mo on cur. dep.” is the probability that active and reserve Army servicemembers who had been on their current deployment for less than 6 months during MHA T-IV had symptomatic PTSD in month $t = 43$ (MHA T-IV). The MHA T rates in Table 18 all appear in either the MHA T-III [227] or the MHA T-IV study [226]. The first three values in Table 18 appear in the figure on page 20 of the MHA T-III report [227]. The category on the left side of that figure (Acute Stress Symptoms) is what we use for the MHA T PTSD rates in Table 18. This rate is reported for OIF-I (MHA T-I, $t = 7$), OIF-II (MHA T-II, $t = 20$), and OIF-04-06 (MHA T-III, $t = 32$). The remaining MHA T values in Table 18 are reported in a similar manner on page 21 of [227] and on pages 20 through 24 of [226]. The category of interest is always “Acute Stress” and OIF-05-07 refers to MHA T-IV and month $t = 43$.

The final value in Table 18 (corresponding to the 18th term in our objective function) is “No combat exposure,” and this value does not come from a MHA T study. This value pertains to the fact that the probability that an Army soldier finishing a deployment between September 2004 and October 2006 was exposed to a potentially traumatic combat experience was 0.68 [229]. As noted earlier, 0.79 of servicemembers that finished a deployment during September 2004 and October 2006 completed their first deployment during that time and were deployed for a median of 12 months. From Table 17 we find that the average value of $\lambda_1(t)$ from October 2003 (which is 12 months prior to the start of the survey) to October 2006 was 0.0054. Because the batch size is $b$, the number of Poisson events during the 12 months of deployment was approximately $\frac{0.0054}{b}$. The remaining 0.21 of servicemembers completed their second deployment during September 2004 and October 2006, and their first deployment lasted a median of 12 months, their second deployment a median of 12 months, and the break between deployments 15 months. Since the total cycle of the two deployments
took 39 months to complete, the servicemembers could not have finished their deployment earlier than May 2006. From Table 17, we find that the average value of $\lambda_1(t)$ from the first deployment, which could occur between March 2003 and July 2004 (which is 27 months prior to the end of the survey), and the second deployment, which could occur between June 2005 (12 months prior to May 2006) and October 2006 was 0.0047. The number of Poisson events during the 24 noncontiguous months of deployment was approximately $0.113/\theta$. Thus the average number of Poisson events during deployments from both servicemembers completing one and two deployment is $\frac{0.075}{\theta}$. Hence, the 18th term in our least squares objective is $(e^{-0.075/\theta} - 0.32)^2$. Finally, 5% of servicemembers screen positive for PTSD pre-deployment [231, 232]. This value is similar to the PTSD rate in the general population [232], and therefore we take 5% to be the baseline PTSD symptomatic rate pre-deployment. We choose the parameter values to ensure this initial rate, and thus this is an equality constraint for our least squares problem. We assume everyone who develops PTSD before their initial deployment (i.e., their initial stress is greater than their threshold) is also symptomatic pre-deployment. For the Poisson model this equality constraint is equivalent to $\gamma = \frac{\alpha}{19}$.

We denote $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$, for $i = 1, \ldots, 17$, to be the model’s estimate of the $i$th MHAT probability in Table 18 given $\alpha = \tilde{\alpha}$, $b = \tilde{b}$, $\theta = \tilde{\theta}$, and $\gamma = \tilde{\gamma}$. For example, $f_1(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$ is the model’s estimate (for specific parameter values) of the PTSD rate of active and reserve Army servicemembers in month $t = 7$ ($P_{1+2}(7)$). To determine the optimal values of $\alpha$, $b$, $\theta$, and $\gamma$, we need to be able to evaluate the function $f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma})$. Once we are able to do this, we can compute for the optimal parameter values by solving the following least squares problem:

$$
(\alpha, b, \theta, \gamma) = \arg\min_{\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}} \sum_{i=1}^{17} (f_i(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) - \text{MHAT}_i)^2 + (e^{-0.075/\theta} - 0.32)^2,
$$

subject to $\tilde{\gamma} = \frac{\tilde{\alpha}}{19},$

where $\text{MHAT}_i$ is the $i$th reported MHAT PTSD rate in Table 18 and the constraint and the
last term in the objective function are described in the previous paragraph.

There does not appear to be any way to evaluate \( f_i(\bar{\alpha}, \bar{b}, \bar{\theta}, \bar{\gamma}) \) either analytically or numerically. The calculation involves manipulating and comparing many different types of random variables for servicemembers with different deployment schedules. Consequently, we use Monte Carlo methods to estimate \( f_i(\bar{\alpha}, \bar{b}, \bar{\theta}, \bar{\gamma}) \) as follows. From the deployment submodel in §1, we have the deployment history for each servicemember, given by \( C_{kj}(t) \).

For each servicemember we generate an initial stress level (an exponential random variable with mean \( \bar{\alpha}^{-1} \)) and a stress threshold \( \bar{D}_{kj} \) (an exponential random variable with mean \( \bar{\gamma}^{-1} \)). We then accumulate and decrease the servicemember’s monthly stress level, \( D_{kj}(t) \), according to his deployment history and equations (1) and (2) in the main text. This is a random process because we must generate the monthly stress, \( E_{kj}(t) \), which is a compound Poisson random variable with mean \( \lambda_j(t) \) and batch size \( b \). After computing \( D_{kj}(t) \) for each servicemember throughout OIF, we can determine if and when the servicemember develops PTSD (\( \bar{t}_{kj} \) in equation (3) in the main text) and his maximum stress level up until \( t = 43 \) (this quantity is used to compute \( f_i(\bar{\alpha}, \bar{b}, \bar{\theta}, \bar{\gamma}) \) for the MHAT-IV rates in Table 18 with low, medium, or high exposure to trauma). Finally, we generate a lognormal time lag (using the parameters estimated in the previous subsection) and compute when the servicemember reports symptoms. We only need the military time lag because the MHAT surveys occur during a deployment and no reserve servicemembers in our model deploy multiple times before MHAT-IV (\( t = 43 \)). We then determine \( f_i(\bar{\alpha}, \bar{b}, \bar{\theta}, \bar{\gamma}) \) by computing the fraction of servicemembers corresponding to the population in the \( i \)th row of Table 18 who report symptoms. For example, to compute \( f_{13}(\bar{\alpha}, \bar{b}, \bar{\theta}, \bar{\gamma}) \) (an estimate of \( P_3(43) \), high exposure to trauma) we run the procedure described in this paragraph and then rank the Marines who were deployed in month \( t = 43 \) (i.e., those Marines such that \( C_{k3}(43) = 1 \)) according to their maximum stress level up until \( t = 43 \) (i.e., \( \max_{t \leq 43} D_{k3}(t) \)). The Marines
ranked in the upper third (i.e., the one third of the deployed Marines with the highest maximum stress level) are the population used to compute \( f_{i3}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) \). We determine what fraction of this population has symptomatic PTSD by \( t = 43 \), and this is \( f_{i3}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) \). We have only described one iteration of this process, but in practice, to compute \( f_{i}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) \) we run this procedure several times to get a more precise point estimate of \( f_{i}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) \). Once we have this algorithm to evaluate \( f_{i}(\tilde{\alpha}, \tilde{b}, \tilde{\theta}, \tilde{\gamma}) \) for all \( i = 1, \ldots, 17 \), we can solve for \( \alpha, b, \theta, \) and \( \gamma \) in (29)–(30). Substituting these optimal parameters into \( f_{i}(\alpha, b, \theta, \gamma) \) yields the model estimates for the MHAT probabilities in Table 18. Because the objective function in (29) is complicated and has uncertainty, it is difficult to compute the global minimum. However, through various search and optimization methods we are confident that our calculated minimum is a reasonable estimate of the global minimum.

For the Poisson model, the minimum sum of squares of the 18 terms is 0.016, which gives a root mean square error of \( \sqrt{\frac{0.016}{18}} = 0.030 \). The average relative deviation for these 18 PTSD rates is 15.1%. The optimal solution is \( \alpha = 146.7, \gamma = 7.72, b = 0.0626, \) and \( \theta = 0 \). Because the amount of rest between deployments is usually at least 10 months (it is shorter for some Marine battalions), it is difficult to determine the optimal \( \theta \) when there is near full recuperation between deployments (e.g., \( 0.3^{10} = 6 \times 10^{-6} \)). Therefore to be conservative we set \( \theta = 0 \), although we perform a sensitivity analysis where we increase the value of \( \theta \) to 1 in §3.3. The mean stress pre-deployment, \( \alpha^{-1} \), is comparable to the average monthly stress from combat (0.0051 for Army, 0.0090 for Marines), and the mean threshold value, \( \gamma^{-1} \), is comparable though larger than average stress accumulated during a yearlong deployment. Table 18 compares the actual MHAT PTSD rates with the PTSD rates estimated using these parameter values (\( f_{i}(\alpha, b, \theta, \gamma) \)). We discuss the model fit to the MHAT rates in §2.3 of the main text. We also computed the optimal solution using a weighted least squares approach with weights \( (P_i(1-P_i))^{-1} \), where \( P_i \) are the actual MHAT PTSD rates in Table 18. The
solution using the weighted least squares objective function was very similar to the one given above using the standard least squares approach.

3 Results

The estimates for the future troop deployments from §1, the PTSD model from the main text, and the parameter estimates from §2 allow us to simulate our system and compute the cumulative number of symptomatic servicemembers over time. Because the difference between the civilian and military time lags is significant we need to determine when active Army soldiers and Marines separate from the military and return to civilian life. We assume that if a servicemember stops deploying before its unit’s final deployment then that servicemember enters civilian life as soon as the servicemember completes his last deployment. For the cohorts of servicemembers who complete their brigade’s final deployment, we assume that every year following that final deployment a fraction of those servicemembers enter civilian life according to the retention rate, which is given in §1.2.2. The base-case results appear in §3.1, several model modifications are considered in §3.2, and sensitivity analyses are performed in §3.3.

3.1 Base-case Results

Figure 1 in the main text shows the cumulative number of troops who are symptomatic for the three withdrawal scenarios described in the main text. Figures 7 and 8 also illustrate the difference between the civilian and military time lags. Because the reserve Army servicemembers switch to the civilian time lag as soon as they return from a deployment, the gap between the number who develop PTSD and the number of symptomatic cases closes quickly for the reserve Army at the end of the withdrawal. In contrast, the active Army and Marine servicemembers switch only once they separate from the military, and so the
gap between the number who develop PTSD and the number of symptomatics closes more slowly for active Army and Marines. The average time lag for active servicemembers (active Army and Marines) is 24.6 months and the average time lag for reserve servicemembers is 7.5 months. The 90th percentile for active service members is 68 months and the 90th percentile for reserve servicemembers is 15 months, implying that it could be years before all of the active servicemembers with PTSD exhibit symptoms.

Table 19 gives the probability mass function for the number of stressful events (i.e., the number of Poisson events in our model) for servicemembers in withdrawal scenario 2. For these same servicemembers, Figure 9 displays a histogram of the maximum cumulative stress strength threshold ratio. These data reveal the heterogeneity of stressful experiences and PTSD severity among servicemembers.

### 3.2 Model Modifications

In this section we modify our model under three different scenarios. In the first scenario we replace the Poisson dose-response model with the probit model, in the second we assume each servicemember only deploys once, and finally we analyze the situation where the servicemembers leave the military and return to civilian status based on how resilient they are to the stress they have faced. The first and third scenarios require the re-estimation of the PTSD parameters. Table 18 gives the estimated parameter values and the predicted PTSD probabilities for these two cases. Figure 10 shows the cumulative number of symptomatics over time for all three scenarios.

**Probit Model.** For the probit model, in which $\bar{D}_{jk} = ID_{50} \exp \left( \frac{\Phi^{-1}(u_{jk})}{\beta} \right)$ (see the §2.2 of the main text), we need to solve for $\alpha$, $ID_{50}$, $\beta$, $b$, and $\theta$ by minimizing the sum of squared deviations between the actual PTSD rates in Table 18 and the predicted rates from the model. To ensure that 5% of servicemembers have PTSD pre-deployment, for a given $ID_{50}$
and $\alpha$ we compute the $\beta$ that yields this rate. The resulting root mean square error is 0.028; as expected, this is smaller than in the base-case, which has one less parameter to optimize over. Our analysis yields what appears to be a one-dimensional subspace of parameters that achieve close to the minimum sum of squares, thereby making it difficult to pinpoint the exact optimal parameter values. For all near-optimal solutions, $\theta = 1$ and $b \approx 0.065$; i.e., in contrast to the Poisson base-case model, there is no recuperation in the probit case. By increasing $\alpha$ and decreasing $ID_{50}$ at the proper relative rates ($\beta$ is just a function of $\alpha$ and $ID_{50}$ to ensure the initial PTSD rate is 0.05), the sum of squares can be maintained near its minimum value. We have found solutions that have $\alpha$ ranging from 500 to $10^7$, $\beta$ ranging from 0.1 to 0.4, and $ID_{50}$ ranging from 0.03 to 0.10. We compared several solutions with widely varying $\alpha$’s and they all yield a similar fit to the MHAT probabilities in Table 18 and predict a similar number of symptomatics. The parameter values $\alpha = 8.495 \times 10^6$, $ID_{50} = 0.0447$, $\beta = 0.124$, $b = 0.0682$, and $\theta = 1$ achieved the minimum sum of squares, and so we use these parameters to compute the values in column A of Table 18 and Figure 10(a).

The probit model provides a better fit to, although still underestimates, the PTSD rate for Army soldiers receiving the lowest level of stress ($P_{1+2}(43)$, low exposure to trauma), but it underestimates the PTSD rate of Marines receiving the highest level of stress ($P_3(43)$, high exposure to trauma).

In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.39 for active Army, 0.31 for reserve Army, and 0.37 for Marines. Overall there are $\approx 5\%$ fewer servicemembers who develop PTSD in the probit model compared to the Poisson model.

**No Multiple Deployments.** To isolate the effects of multiple deployments, we look at the extreme hypothetical case in which there are no multiple deployments. That is, we use the base-case PTSD parameters and the base-case brigade/battalion rotation schedule, except
that each time a unit is deployed each servicemember is new; presumably, this state of affairs could only be achieved with an involuntary draft. Figure 10(b) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.33 for active Army, 0.30 for reserve Army, and 0.28 Marines.

The fraction of troops who develop symptomatic PTSD does drop substantially, especially for Marines because of their frequent deployments and short deployment length. However $\approx 30\%$ more servicemembers develop symptomatic PTSD than in the base-case. To maintain troop levels while simultaneously disallowing multiple deployments would certainly require a draft, and it is possible that drafted servicemembers would be less prepared to handle the stress than volunteer servicemembers and would be more susceptible to PTSD; this aspect is not taken into account by our model. Disallowing multiple deployments in the probit model leads to a somewhat lower PTSD rate for individual servicemembers (0.32 for active Army, 0.30 for reserve Army, and 0.26 for Marines), and an overall increase in symptomatic PTSD cases of $\approx 30\%$ over the probit model with multiple deployments allowed.

**Stress-based Attrition.** It is possible that servicemembers choose when to leave the military and return to civilian status based partially upon the amount of stress they have been exposed to and their ability to handle that stress. Therefore, in this analysis we use the same cohort and deployment schedule; however, after each deployment, the servicemembers who return to civilian life are the servicemembers with the highest $\frac{\text{stress}}{\text{threshold}}$ value at the end of that deployment. Thus the servicemembers who serve on multiple tours of duty should be better equipped to handle the stress than in the base-case model. While servicemembers who deploy several times are more resilient to stress, there are more vulnerable servicemembers exposed to stress in this scenario because the servicemembers who develop PTSD leave the military, and hence must be replaced sooner than in the base-case model. Figure 10(c) gives
the predicted number of symptomatic servicemembers for this situation using the base-case parameter values. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.44 for active Army, 0.32 for reserve Army, and 0.44 Marines. The total number of servicemembers who develop symptomatic PTSD is \(\approx 5\%\) more than the base-case. Using the probit dose-response for this scenario yields a similar increase over the base-case probit model.

Column B of Table 18 shows that the estimated MHAT probabilities under this modeling assumption are much worse than the other scenarios; the resulting root mean square error is 0.078. Because of the time lag before the onset of symptoms, many of the servicemembers in our model who screen for PTSD during the MHAT surveys developed PTSD on a prior deployment. However, for this modeling scenario most servicemembers who develop PTSD return to civilian life rather than another deployment, and therefore almost all of the estimated MHAT probabilities are smaller in this scenario than in the base-case. The difference is most evident for \(P_1(32)\), >1st deployment and \(P_{1+2}(43)\), >1st deployment because in the base-case over 75\% of these troops who screened positive for PTSD developed PTSD during an earlier deployment. Because the base-case parameters yield such a poor fit, we recompute the optimal parameters for this scenario. The optimal parameter values are \(\alpha = 306.6\), \(\gamma = 16.14\), \(b = 0.0588\), and \(\theta = 1\) and the estimated MHAT probabilities are in column C of Table 18. The root mean square error is 0.049, which is much better than using the base-case parameters, but it is still worse than most of the other scenarios in Table 18. To achieve the MHAT PTSD rates in Table 18, more servicemembers need to develop PTSD to compensate for both the larger number of servicemembers with PTSD who leave the military before the later MHAT surveys occur and the increased resiliency of those servicemembers who stay in the military and take part in the later MHAT surveys. This decreases the mean threshold level \(\gamma^{-1}\), which is why \(\gamma\) and \(\alpha\) increase (\(\alpha\) is a function of \(\gamma\) to ensure the initial PTSD rate
is 0.05), and causes there to be no recuperation. Figure 10(d) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.69 for active Army, 0.47 for reserve Army, and 0.65 Marines. The total number of servicemembers who develop symptomatic PTSD is \( \approx 60\% \) more than the base-case. Using the probit dose-response for this scenario yields a similar increase over the base-case probit model. The decision to stay in the military or return to civilian life is based on many factors related to family, health, finances, morale, and camaraderie. The poor fit of this model to the MHAT probabilities (column C of Table 18) suggests that a servicemember’s ability to handle the stress he is exposed to may be a second-order consideration in this decision.

### 3.3 Sensitivity Analyses

This subsection reports on the results of three sensitivity analyses, each of which test how the results change when we adjust the parameter values. We first set \( \theta = 1 \) and assume there is no recuperation, we next analyze different time lag parameters, and finally we adjust the future monthly stress level. The first two analyses require the re-estimation of the PTSD parameters. Table 18 gives the estimated parameter values and the predicted PTSD probabilities for these two cases, and Figure 11 shows the cumulative number of symptomatics over time. The results for the final analysis are shown in Figure 12.

**No Recuperation.** The base-case model has \( \theta = 0 \) and thus predicts no cumulative effects from multiple deployments. We investigate how our model changes if we set \( \theta = 1 \), thereby disallowing recuperation between deployments. First, we keep the same parameter values as in the base-case (\( \alpha = 146.7, \gamma = 7.72, \) and \( b = 0.0626 \)), but set \( \theta = 1 \). The root mean square error is 0.031. Column D of Table 18 and Figure 11(a) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for
this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.45 for active Army, 0.32 for reserve Army, and 0.45 for Marines. Increasing $\theta$ from 0 to 1 does not alter the fit of the MHAT probabilities significantly, and < 10% more servicemembers develop PTSD in this scenario compared to the base-case.

Next we assume $\theta = 1$ and recompute the optimal values for the remaining parameters, which yield $\alpha = 132.9, \gamma = 6.99, b = 0.0625$. Because there is no recuperation, servicemembers on average will have higher levels of stress. Consequently, to achieve the same PTSD rates in the MHAT studies, there needs to be a larger mean threshold $(\gamma^{-1})$ and a smaller $\alpha$ ($\alpha$ is a function of $\gamma$ to ensure the initial PTSD rate is 0.05). The root mean square error is 0.031. Column E of Table 18 and Figure 11(b) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.43 for active Army, 0.30 for reserve Army, and 0.44 Marines. The total number of servicemembers who develop symptomatic PTSD is < 5% more than the base-case.

**Time Lag Parameters.** We investigate the impact of varying the time lag parameters $\mu_1$ and $\mu_2$, while maintaining the same dispersion factors. First, we increase $\mu_1$ and $\mu_2$ so that the median for both the civilian and military time lags increases by a factor of 2, which is equivalent to increasing $\mu_1$ and $\mu_2$ by ln(2). The optimal parameter values are $\alpha = 197.0, \gamma = 10.37, b = 0.0624$, and $\theta = 1$, with a root mean square error of 0.030. To achieve the MHAT PTSD rates in Table 18 when the time lags are longer, more servicemembers need to develop PTSD during the time period of those studies because a smaller fraction of those who develop PTSD will be symptomatic. This decreases the mean threshold level $\gamma^{-1}$, which is why $\gamma$ and $\alpha$ increase ($\alpha$ is a function of $\gamma$ to ensure the initial PTSD rate
is 0.05. Column F of Table 18 and Figure 11(c) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.51 for active Army, 0.37 for reserve Army, and 0.50 for Marines, which is \( \approx 20\% \) higher than in the base-case. Because the threshold is lower, more servicemembers will develop PTSD although it will take longer for the symptoms to appear.

Next we decrease \( \mu_1 \) and \( \mu_2 \) so that the median for both the civilian and military time lags decrease by a factor of 2, which is equivalent to decreasing \( \mu_1 \) and \( \mu_2 \) by \( \ln(2) \). The optimal parameter values change to \( \alpha = 110.9 \), \( \gamma = 5.83 \), \( b = 0.0626 \), and \( \theta = 0 \), with a root mean square error of 0.030. Because the median time lags have decreased, to achieve the same PTSD rates in the MHAT studies, fewer servicemembers need to develop PTSD during the time period of those studies because a larger fraction of those who develop PTSD will be symptomatic, which leads to a larger mean threshold level and full recuperation \( (\theta = 0) \). Column G of Table 18 and Figure 11(d) give the estimated MHAT probabilities and the predicted number of symptomatic servicemembers, respectively, for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.35 for active Army, 0.27 for reserve Army, and 0.35 for Marines, which is \( \approx 15\% \) fewer PTSD cases than in the base-case.

We also analyze the impact of varying the time lag parameters \( s_1 \) and \( s_2 \), while maintaining the same medians. This change has less of an impact than varying the medians, so we just state the results here. When we increase the dispersion parameters by \( \ln(5) \) (which is a more significant modification than we analyzed for the median parameters) the optimal parameter values are \( \alpha = 137.3 \), \( \gamma = 7.22 \), \( b = 0.0626 \), and \( \theta = 0 \), with a root mean square error of 0.030. In withdrawal scenario 2, the fraction of servicemembers who served during
OIF that are symptomatic with PTSD by February 2023 is 0.39 for active Army, 0.30 for reserve Army, and 0.39 for Marines, which is \approx 5\% less than in the base-case. When we decrease the dispersion parameters by ln(5) (we set \( s_2 = 0.01 \)) the optimal parameter values are \( \alpha = 150.3, \gamma = 7.90, b = 0.0633, \) and \( \theta = 1 \), with a root mean square error of 0.035.

In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.45 for active Army, 0.32 for reserve Army, and 0.46 for Marines, which is \approx 10\% higher than in the base case.

**Future Stress Level.** In the base-case we define the average monthly stress process from October 2008 until the end of the withdrawal, \( \lambda_j(t) \) for \( t \geq 68 \), to equal the average of \( \lambda_j(t) \) between October 2007 – September 2008 (calculated separately for Army and Marines). This value is 0.0030 for Army and 0.0013 for Marines. We consider several other possible values for the future mean monthly stress because this value is used for up to three and half years of deployments.

Because the stress process is close to its lowest values between October 2007 – September 2008 (see Figure 5), we set \( \lambda_1(t) = \lambda_3(t) = 0 \) for \( t \geq 68 \) to model the best-case scenario where violence, and hence stress, continues to decrease. Figure 12(a) gives the predicted number of symptomatic servicemembers for this situation. These curves are not exactly the same because even though no future servicemembers develop PTSD via exposure to stress during OIF, servicemembers can still develop PTSD from pre-deployment stress. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.37 for active Army, 0.27 for reserve Army, and 0.39 for Marines, which is \approx 10\% fewer PTSD cases than in the base-case.

We next assume the future average monthly stress is equal to the median value between March 2003 and September 2008. This value is 0.0049 for Army and 0.0084 for Marines, which is a significant increase over the base-case estimates, especially for the Marines. Fig-
Figure 12(b) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.43 for active Army, 0.34 for reserve Army, and 0.46 for Marines, which is $\approx 8\%$ more PTSD cases than in the base-case.

Finally, to model the worst-case scenario, we set the future average monthly stress equal to the 90$^{th}$ percentile of stress between March 2003 and September 2008. This value is 0.0081 for Army and 0.00164 for Marines. Figure 12(c) gives the predicted number of symptomatic servicemembers for this situation. In withdrawal scenario 2, the fraction of servicemembers who served during OIF that are symptomatic with PTSD by February 2023 is 0.46 for active Army, 0.38 for reserve Army, and 0.51 for Marines, which is $\approx 18\%$ more PTSD cases than in the base-case.

In all three scenarios, the deviation from the base case becomes more significant the farther in the future the withdrawal occurs.
References


[40] 3rd Brigade, 1st Armored webpage. 2007. 


[52] 2nd Brigade, 1st Infantry webpage. 2007.  
[http://www.2bct.1id.army.mil/Primary%20Sites/history/history_oif2.htm](http://www.2bct.1id.army.mil/Primary%20Sites/history/history_oif2.htm)


[77] Less fanfare this time as original Stryker brigade gears up for return to Iraq. *The Associated Press State & Local Wire*, October 15, 2005.


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Figure Legends

**Fig. 1.** Attached and unattached computed monthly troop levels for OIF from February 2003 until September 2008.

**Fig. 2.** Computed monthly troop levels for OIF by service branch from February 2003 until September 2008.

**Fig. 3.** Troop level comparisons from February 2003 until September 2008. (a) Comparison of computed Army troop levels vs. official DOD troop numbers [224]; (b) comparison of computed Marines troop levels vs. official DOD troop numbers [224]; (c) comparison of computed troop levels vs. Post-surge projected troop numbers [225].

**Fig. 4.** Attached and unattached predicted monthly troop levels for OIF from February 2003 until the end of the withdrawal. (a) Withdrawal scenario 1, withdrawal begins February 2009; (b) withdrawal scenario 2, withdrawal begins February 2010; (c) withdrawal scenario 3, withdrawal begins February 2011.

**Fig. 5.** Predicted monthly troop levels for OIF by service branch from February 2003 until the end of the withdrawal. (a) Withdrawal scenario 1, withdrawal begins February 2009; (b) withdrawal scenario 2, withdrawal begins February 2010; (c) withdrawal scenario 3, withdrawal begins February 2011.

**Fig. 6.** Average monthly stress for Army soldiers, $\lambda_1(t)$, (—) and Marines, $\lambda_3(t)$, (- - -) between March 2003 and September 2008.

**Fig. 7.** Predicted cumulative number of servicemembers who develop PTSD (—) and are symptomatic (- - -) by service branch from March 2003 until February 2023. (a) Withdrawal scenario 1, withdrawal begins February 2009; (b) withdrawal scenario 2, withdrawal begins February 2010; (c) withdrawal scenario 3, withdrawal begins February 2011.

**Fig. 8.** Predicted cumulative fraction of servicemembers (out of total deployed) who develop PTSD (—) and are symptomatic (- - -) by service branch from March 2003 until February
Withdrawal scenario 1, withdrawal begins February 2009; (b) withdrawal scenario 2, withdrawal begins February 2010; (c) withdrawal scenario 3, withdrawal begins February 2011.

**Fig. 9.** Histogram of the ratio $\frac{\text{maximum cumulative stress}}{\text{strength threshold}}$ for servicemembers in withdrawal scenario 2. The red dashed line at 1 partitions the servicemembers into whether they develop PTSD or not. The extreme bins of the histogram contain the remaining mass for the corresponding tail of the distribution.

**Fig. 10.** Predicted cumulative number of servicemembers symptomatic with PTSD for the model modifications described in §3.2 (a) Probit model; (b) Poisson base-case model, servicemembers only deploy once; (c) Poisson base-case model, servicemembers return to civilian status according to their stress-to-threshold ratio; (d) Poisson model, parameters reestimated, servicemembers return to civilian status according to their stress-to-threshold ratio.

**Fig. 11.** Predicted cumulative number of servicemembers symptomatic with PTSD for the sensitivity analyses described in §3.3 (a) Poisson base-case model with $\theta = 1$; (b) Poisson model, $\theta$ constrained to 1, other parameters reestimated; (c) Poisson model, $\mu_1$ and $\mu_2$ increase by ln(2), other parameters reestimated; (d) Poisson model, $\mu_1$ and $\mu_2$ decrease by ln(2), other parameters reestimated.

**Fig. 12.** Predicted cumulative number of servicemembers symptomatic with PTSD for various values of the future average monthly stress. The Poisson base-case parameters are used for all scenarios. (a) Future monthly stress = 0; (b) Future monthly stress = Median stress between March 2003 and September 2008; (c) Future monthly stress = 90th percentile stress between March 2003 and September 2008.
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<td>08/01/06</td>
<td>09/01/07</td>
<td>12/01/08</td>
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<td></td>
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<tr>
<td>10th Mountain</td>
<td>2nd BCT</td>
<td>Light</td>
<td>d1</td>
<td>06/01/04</td>
<td>06/01/05</td>
<td>08/01/06</td>
<td>11/01/07</td>
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<tr>
<td>10th Mountain</td>
<td>4th BCT</td>
<td>Light</td>
<td>d1</td>
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</table>

Table 1: Active Army deployment dataset. For each brigade, the start and end dates of each deployment, and the time each brigade was modularized. The notation di stands for the ith deployment for i=1,2,3.
<table>
<thead>
<tr>
<th>Division</th>
<th>Brigade</th>
<th>Type</th>
<th>Refs</th>
<th>d1 start</th>
<th>d1 end</th>
<th>d2 start</th>
<th>d2 end</th>
<th>modular date</th>
</tr>
</thead>
<tbody>
<tr>
<td>–</td>
<td>29th IBCT</td>
<td>Light</td>
<td>d1 91 d2 91</td>
<td>03/01/05</td>
<td>03/01/06</td>
<td>11/01/08</td>
<td>11/01/09</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>30th IBCT</td>
<td>Light</td>
<td>d1 91 d2 91</td>
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<td>01/01/05</td>
<td>04/01/09</td>
<td>04/01/10</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>32th IBCT</td>
<td>Light</td>
<td>92 93</td>
<td>05/01/09</td>
<td>03/01/10</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
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<td>–</td>
<td>37th IBCT</td>
<td>Light</td>
<td>94</td>
<td>05/01/08</td>
<td>02/01/09</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>39th BCT</td>
<td>Light</td>
<td>d1 95 d2 95 96</td>
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<td>04/01/05</td>
<td>04/01/08</td>
<td>02/01/09</td>
<td>–</td>
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<tr>
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<td>Light</td>
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<td>–</td>
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<td>Light</td>
<td>97</td>
<td>03/01/08</td>
<td>11/01/08</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>50th IBCT</td>
<td>Light</td>
<td>98 99</td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
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<td>58th IBCT</td>
<td>Light</td>
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<td>07/01/08</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
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<td>–</td>
<td>72th IBCT</td>
<td>Light</td>
<td>29</td>
<td>08/01/09</td>
<td>08/01/10</td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>76th IBCT</td>
<td>Light</td>
<td>95 101</td>
<td>04/01/08</td>
<td>11/01/08</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>81st ABCT</td>
<td>Armored</td>
<td>d1 91 d2 91 102</td>
<td>03/01/04</td>
<td>03/01/05</td>
<td>11/01/08</td>
<td>11/01/09</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>116th Cav B</td>
<td>Heavy</td>
<td>103</td>
<td>12/01/04</td>
<td>12/01/05</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>155th ABCT</td>
<td>Heavy</td>
<td>d1 104 d2 105</td>
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<td>01/01/06</td>
<td>11/01/09</td>
<td>11/01/10</td>
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</tr>
<tr>
<td>–</td>
<td>256th BCT</td>
<td>Heavy</td>
<td>d1 106 d2 29</td>
<td>10/01/04</td>
<td>10/01/05</td>
<td>07/01/09</td>
<td>07/01/10</td>
<td>–</td>
</tr>
<tr>
<td>–</td>
<td>278th ACR</td>
<td>ACR</td>
<td>d1 107 d2 29</td>
<td>11/01/04</td>
<td>11/01/05</td>
<td>10/01/09</td>
<td>10/01/10</td>
<td>–</td>
</tr>
<tr>
<td>28th Infantry</td>
<td>2nd BCT</td>
<td>Heavy</td>
<td>d1 108 d2 29</td>
<td>06/01/05</td>
<td>06/01/06</td>
<td>08/01/09</td>
<td>08/01/10</td>
<td>–</td>
</tr>
<tr>
<td>28th Infantry</td>
<td>56th SBCT</td>
<td>Heavy</td>
<td>91</td>
<td>02/01/09</td>
<td>02/01/10</td>
<td>–</td>
<td>–</td>
<td>06/01/06</td>
</tr>
<tr>
<td>34th Infantry</td>
<td>1st BCT</td>
<td>Heavy</td>
<td>109 110</td>
<td>04/01/06</td>
<td>08/01/07</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>35th Infantry</td>
<td>48th IB</td>
<td>Heavy</td>
<td>111</td>
<td>05/01/05</td>
<td>05/01/06</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>36th Infantry</td>
<td>56th BCT</td>
<td>Heavy</td>
<td>d1 91 d2 91</td>
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<td>01/01/06</td>
<td>10/01/08</td>
<td>10/01/09</td>
<td>–</td>
</tr>
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</table>

Table 2: National Guard deployment dataset. For each brigade, the start and end dates of each deployment, and the time each brigade was modularized. The notation di stands for the i\textsuperscript{th} deployment for i=1,2.
<table>
<thead>
<tr>
<th>Regiment</th>
<th>Battalion</th>
<th>Reserve/Active</th>
<th>Refs</th>
<th>d1 start</th>
<th>d1 end</th>
<th>d2 start</th>
<th>d2 end</th>
<th>d3 start</th>
<th>d3 end</th>
<th>d4 start</th>
<th>d4 end</th>
<th>d5 start</th>
<th>d5 end</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st RCT</td>
<td>1-1</td>
<td>A</td>
<td></td>
<td>01/01/05</td>
<td>08/01/05</td>
<td>01/01/06</td>
<td>09/01/06</td>
<td>07/01/07</td>
<td>02/01/08</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1st RCT</td>
<td>1-2</td>
<td>A</td>
<td></td>
<td>02/01/04</td>
<td>09/01/04</td>
<td>10/01/05</td>
<td>01/01/06</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st RCT</td>
<td>1-3</td>
<td>A</td>
<td></td>
<td>03/01/04</td>
<td>07/01/04</td>
<td>02/01/05</td>
<td>09/01/05</td>
<td>04/01/06</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>1st RCT</td>
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<td>A</td>
<td></td>
<td>04/01/03</td>
<td>07/01/03</td>
<td>07/01/04</td>
<td>02/01/05</td>
<td>11/01/05</td>
<td>06/01/06</td>
<td>10/01/07</td>
<td>10/01/07</td>
<td>08/01/08</td>
<td>03/01/09</td>
</tr>
<tr>
<td>1st RCT</td>
<td>1-5</td>
<td>A</td>
<td></td>
<td>05/01/03</td>
<td>09/01/03</td>
<td>10/01/04</td>
<td>07/01/05</td>
<td>02/01/06</td>
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<td></td>
<td></td>
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<td>1st RCT</td>
<td>1-6</td>
<td>A</td>
<td></td>
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<td>09/01/03</td>
<td>09/01/04</td>
<td>08/01/05</td>
<td>09/01/06</td>
<td>07/01/07</td>
<td>03/01/08</td>
<td>10/01/08</td>
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<tr>
<td>1st RCT</td>
<td>1-7</td>
<td>A</td>
<td></td>
<td>07/01/03</td>
<td>09/01/03</td>
<td>09/01/04</td>
<td>10/01/04</td>
<td>03/01/05</td>
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<tr>
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<td>1-8</td>
<td>A</td>
<td></td>
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<td>09/01/03</td>
<td>09/01/04</td>
<td>08/01/05</td>
<td>09/01/06</td>
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<td></td>
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<tr>
<td>1st RCT</td>
<td>1-9</td>
<td>A</td>
<td></td>
<td>09/01/03</td>
<td>09/01/03</td>
<td>09/01/04</td>
<td>10/01/04</td>
<td>03/01/05</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st RCT</td>
<td>1-10</td>
<td>A</td>
<td></td>
<td>10/01/03</td>
<td>09/01/03</td>
<td>09/01/04</td>
<td>08/01/05</td>
<td>09/01/06</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1st RCT</td>
<td>1-11</td>
<td>A</td>
<td></td>
<td>11/01/03</td>
<td>09/01/03</td>
<td>09/01/04</td>
<td>08/01/05</td>
<td>09/01/06</td>
<td>07/01/07</td>
<td>05/01/08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st RCT</td>
<td>1-12</td>
<td>A</td>
<td></td>
<td>12/01/03</td>
<td>09/01/03</td>
<td>09/01/04</td>
<td>08/01/05</td>
<td>09/01/06</td>
<td>07/01/07</td>
<td>05/01/08</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Marine deployment dataset. For each battalion, the start and end dates of each deployment. The notation di stands for the i\(^{th}\) deployment for i=1,2,3,4,5.
<table>
<thead>
<tr>
<th>Type of brigade</th>
<th>pre-modular</th>
<th>modular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy</td>
<td>3500</td>
<td>3700</td>
</tr>
<tr>
<td>Light</td>
<td>2500</td>
<td>3200</td>
</tr>
<tr>
<td>Stryker</td>
<td>3900</td>
<td>3900</td>
</tr>
<tr>
<td>ACR</td>
<td>4800</td>
<td>4800</td>
</tr>
</tbody>
</table>

Table 4: Estimated number of personnel per brigade [1].

<table>
<thead>
<tr>
<th>Date</th>
<th>t</th>
<th>Army</th>
<th>Marines</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/2003</td>
<td>1</td>
<td>99,664</td>
<td>66,166</td>
</tr>
<tr>
<td>06/2003</td>
<td>4</td>
<td>179,320</td>
<td>22,885</td>
</tr>
<tr>
<td>09/2003</td>
<td>7</td>
<td>152,815</td>
<td>6545</td>
</tr>
<tr>
<td>12/2003</td>
<td>10</td>
<td>138,120</td>
<td>2557</td>
</tr>
<tr>
<td>03/2004</td>
<td>13</td>
<td>155,291</td>
<td>25,568</td>
</tr>
<tr>
<td>06/2004</td>
<td>16</td>
<td>120,703</td>
<td>32,636</td>
</tr>
<tr>
<td>09/2004</td>
<td>19</td>
<td>101,932</td>
<td>35,216</td>
</tr>
<tr>
<td>12/2004</td>
<td>22</td>
<td>135,700</td>
<td>30,500</td>
</tr>
<tr>
<td>03/2005</td>
<td>25</td>
<td>121,400</td>
<td>30,500</td>
</tr>
<tr>
<td>06/2005</td>
<td>28</td>
<td>113,600</td>
<td>23,100</td>
</tr>
<tr>
<td>09/2005</td>
<td>31</td>
<td>132,400</td>
<td>25,900</td>
</tr>
<tr>
<td>12/2005</td>
<td>34</td>
<td>137,600</td>
<td>27,400</td>
</tr>
<tr>
<td>03/2006</td>
<td>37</td>
<td>105,100</td>
<td>26,700</td>
</tr>
<tr>
<td>06/2006</td>
<td>40</td>
<td>103,300</td>
<td>23,300</td>
</tr>
<tr>
<td>09/2006</td>
<td>43</td>
<td>119,500</td>
<td>25,600</td>
</tr>
<tr>
<td>12/2006</td>
<td>46</td>
<td>100,200</td>
<td>23,200</td>
</tr>
<tr>
<td>06/2007</td>
<td>52</td>
<td>125,300</td>
<td>26,700</td>
</tr>
<tr>
<td>09/2007</td>
<td>55</td>
<td>138,500</td>
<td>31,300</td>
</tr>
<tr>
<td>12/2007</td>
<td>58</td>
<td>125,800</td>
<td>26,900</td>
</tr>
<tr>
<td>03/2008</td>
<td>61</td>
<td>126,000</td>
<td>27,100</td>
</tr>
<tr>
<td>06/2008</td>
<td>64</td>
<td>117,100</td>
<td>24,500</td>
</tr>
</tbody>
</table>

Table 5: Monthly OIF troop deployments for Army and Marines [224].

<table>
<thead>
<tr>
<th>Date</th>
<th>t</th>
<th>Troops</th>
</tr>
</thead>
<tbody>
<tr>
<td>07/2008</td>
<td>65</td>
<td>140,000</td>
</tr>
<tr>
<td>08/2008</td>
<td>66</td>
<td>140,000</td>
</tr>
<tr>
<td>09/2008</td>
<td>67</td>
<td>140,000</td>
</tr>
</tbody>
</table>

Table 6: Recent monthly OIF troop deployments (Army plus Marines) [225].
<table>
<thead>
<tr>
<th>Date</th>
<th>( t )</th>
<th>Fraction Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/2003</td>
<td>01</td>
<td>0.82</td>
</tr>
<tr>
<td>09/2003</td>
<td>07</td>
<td>0.88</td>
</tr>
<tr>
<td>10/2004</td>
<td>20</td>
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<td>10/2005</td>
<td>32</td>
<td>0.69</td>
</tr>
<tr>
<td>09/2006</td>
<td>43</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 7: Fraction of Army soldiers that are active \([224, 226]\).

<table>
<thead>
<tr>
<th>Date</th>
<th>( t )</th>
<th>Army</th>
<th>Marines</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/2005</td>
<td>32</td>
<td>0.55</td>
<td>–</td>
</tr>
<tr>
<td>09/2006</td>
<td>43</td>
<td>0.71</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 8: Fraction of troops on first deployment \([226]\).

<table>
<thead>
<tr>
<th>Date</th>
<th>( t )</th>
<th>Army</th>
<th>Marines</th>
</tr>
</thead>
<tbody>
<tr>
<td>09/2006</td>
<td>43</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 9: Median number of months deployed during current deployment \([226]\).

<table>
<thead>
<tr>
<th>Date</th>
<th>( t )</th>
<th>First Deployment</th>
<th>Second Deployment</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/2005</td>
<td>32</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 10: Mean total number of months deployed (over all OIF deployments) \([227]\).

<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Assigned Weight</th>
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</thead>
<tbody>
<tr>
<td>DOD troop totals</td>
<td>5</td>
</tr>
<tr>
<td>DOD post-surge troop level estimates</td>
<td>1</td>
</tr>
<tr>
<td>Fraction of Army soldiers active</td>
<td>20</td>
</tr>
<tr>
<td>Mean service time</td>
<td>1</td>
</tr>
<tr>
<td>Fraction on first deployment</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 11: Constraint weights for unattached support troop estimation. DOD troop totals occur with lower frequency and are presumably more reliable than the post-surge estimates, so they have a higher weight in our model. Other weights were chosen such as to keep violation of any constraint less than 10%.
<table>
<thead>
<tr>
<th>Month $t$</th>
<th>$A_1(t)$</th>
<th>$A_2(t)$</th>
<th>$A_3(t)$</th>
<th>Month $t$</th>
<th>$A_1(t)$</th>
<th>$A_2(t)$</th>
<th>$A_3(t)$</th>
<th>Month $t$</th>
<th>$A_1(t)$</th>
<th>$A_2(t)$</th>
<th>$A_3(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>25089</td>
<td>5123</td>
<td>14448</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>7336</td>
<td>44</td>
<td>1724</td>
<td>1040</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
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<td>5123</td>
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<td>21</td>
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<td>2922</td>
<td>0</td>
<td>45</td>
<td>1724</td>
<td>1040</td>
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<td>5977</td>
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<td>1062</td>
<td>66</td>
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<td>43</td>
<td>5977</td>
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<td>1323</td>
<td>67</td>
<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

Table 12: The solution $A_j(t)$ to (19)-(21), where month $t = 1$ corresponds to March 2003.
<table>
<thead>
<tr>
<th>Date</th>
<th>Month</th>
<th>Computed Fraction Active</th>
<th>Reported Fraction Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/2003</td>
<td>01</td>
<td>0.770</td>
<td>0.820</td>
</tr>
<tr>
<td>9/2003</td>
<td>07</td>
<td>0.850</td>
<td>0.880</td>
</tr>
<tr>
<td>10/2004</td>
<td>20</td>
<td>0.610</td>
<td>0.540</td>
</tr>
<tr>
<td>10/2005</td>
<td>32</td>
<td>0.616</td>
<td>0.690</td>
</tr>
<tr>
<td>09/2006</td>
<td>43</td>
<td>0.799</td>
<td>0.790</td>
</tr>
</tbody>
</table>

Table 13: Computed vs. reported fraction of Army soldiers that are active.

<table>
<thead>
<tr>
<th>Type</th>
<th>Computed</th>
<th>Reported</th>
<th>Computed</th>
<th>Computed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combat + Attached</td>
<td>Unattached</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Army</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Marines</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 14: Computed vs. reported median number of months deployed during current deployment in September 2006 \((t = 43)\). The computed median number of months is also broken down for combat plus attached troops vs. unattached troops.

<table>
<thead>
<tr>
<th>Type</th>
<th>Computed</th>
<th>Reported</th>
<th>Computed</th>
<th>Computed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combat + Attached</td>
<td>Unattached</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Deployers</td>
<td>8.48</td>
<td>10</td>
<td>5.45</td>
<td>12.00</td>
</tr>
<tr>
<td>Multiple Deployers</td>
<td>17.89</td>
<td>20</td>
<td>15.79</td>
<td>20.03</td>
</tr>
</tbody>
</table>

Table 15: Computed vs. reported mean number of months deployed over all OIF deployments in October 2005 \((t = 32)\). The computed mean is also broken down for combat plus attached troops vs. unattached troops.

<table>
<thead>
<tr>
<th>Type</th>
<th>Date</th>
<th>Month</th>
<th>Computed</th>
<th>Reported</th>
<th>Computed</th>
<th>Computed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Combat + Attached</td>
<td>Unattached</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Army</td>
<td>10/2005</td>
<td>32</td>
<td>0.50</td>
<td>0.55</td>
<td>0.52</td>
<td>0.48</td>
</tr>
<tr>
<td>Army</td>
<td>09/2006</td>
<td>43</td>
<td>0.69</td>
<td>0.71</td>
<td>0.49</td>
<td>0.98</td>
</tr>
<tr>
<td>Marines</td>
<td>09/2006</td>
<td>43</td>
<td>0.66</td>
<td>0.67</td>
<td>0.56</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 16: Computed vs. reported fraction of troops on first deployment. The computed fraction is also broken down for combat plus attached troops vs. unattached troops.
Table 17: Monthly mean stress levels for Army ($\lambda_1(t)$) and Marines ($\lambda_3(t)$), where month $t = 1$ corresponds to March 2003.
<table>
<thead>
<tr>
<th>PTSD Prevalence Rate</th>
<th>Reported</th>
<th>Base</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{t+1+2}(7)$</td>
<td>0.158</td>
<td>0.072</td>
<td>0.075</td>
<td>0.072</td>
<td>0.086</td>
<td>0.072</td>
<td>0.070</td>
<td>0.070</td>
<td>0.073</td>
<td>Pg. 20, [227]</td>
</tr>
<tr>
<td>$P_{t+1+2}(20)$</td>
<td>0.113</td>
<td>0.109</td>
<td>0.112</td>
<td>0.110</td>
<td>0.144</td>
<td>0.109</td>
<td>0.106</td>
<td>0.104</td>
<td>0.112</td>
<td>Pg. 20, [227]</td>
</tr>
<tr>
<td>$P_{t+1+2}(32)$</td>
<td>0.136</td>
<td>0.133</td>
<td>0.137</td>
<td>0.092</td>
<td>0.134</td>
<td>0.136</td>
<td>0.130</td>
<td>0.131</td>
<td>0.133</td>
<td>Pg. 20, [227]</td>
</tr>
<tr>
<td>$P_t(32)$, 1st deployment</td>
<td>0.125</td>
<td>0.127</td>
<td>0.127</td>
<td>0.126</td>
<td>0.168</td>
<td>0.126</td>
<td>0.122</td>
<td>0.120</td>
<td>0.129</td>
<td>Pg. 21, [227]</td>
</tr>
<tr>
<td>$P_t(32)$, &gt;1st deployment</td>
<td>0.184</td>
<td>0.178</td>
<td>0.189</td>
<td>0.046</td>
<td>0.106</td>
<td>0.189</td>
<td>0.180</td>
<td>0.185</td>
<td>0.174</td>
<td>Pg. 21, [227]</td>
</tr>
<tr>
<td>$P_{t+1+2}(43)$</td>
<td>0.170</td>
<td>0.148</td>
<td>0.151</td>
<td>0.090</td>
<td>0.146</td>
<td>0.152</td>
<td>0.146</td>
<td>0.148</td>
<td>0.147</td>
<td>Pg. 20, [227]</td>
</tr>
</tbody>
</table>
| $P_t(43)$              | 0.140   | 0.166| 0.163| 0.119| 0.195| 0.174| 0.167| 0.164| 0.168| Pg. 6, [226]  
| $P_{t+1+2}(43)$, low exposure to trauma | 0.080 | 0.026| 0.050| 0.018| 0.020| 0.026| 0.026| 0.026| 0.026| Pg. 7a, [226] |
| $P_{t+1+2}(43)$, medium exposure to trauma | 0.140 | 0.139| 0.136| 0.083| 0.135| 0.135| 0.129| 0.133| 0.136| Pg. 7a, [226] |
| $P_{t+1+2}(43)$, high exposure to trauma | 0.280 | 0.281| 0.267| 0.168| 0.283| 0.296| 0.283| 0.284| 0.280| Pg. 7a, [226] |
| $P_t(43)$, low exposure to trauma | 0.060 | 0.050| 0.050| 0.037| 0.045| 0.050| 0.050| 0.050| 0.050| Pg. 7b, [226] |
| $P_t(43)$, medium exposure to trauma | 0.110 | 0.159| 0.167| 0.105| 0.185| 0.152| 0.144| 0.146| 0.156| Pg. 7b, [226] |
| $P_t(43)$, high exposure to trauma | 0.280 | 0.291| 0.272| 0.214| 0.355| 0.321| 0.308| 0.297| 0.299| Pg. 7b, [226] |
| $P_{t+1+2}(43)$, 1st deployment | 0.150 | 0.110| 0.112| 0.110| 0.143| 0.110| 0.106| 0.105| 0.112| Pg. 8, [226]  
| $P_{t+1+2}(43)$, >1st deployment | 0.240 | 0.236| 0.239| 0.045| 0.152| 0.249| 0.237| 0.246| 0.227| Pg. 8, [226] |
| $P_{t+1+2}(43)$, ≤6 mo on cur. dep. | 0.120 | 0.118| 0.119| 0.054| 0.099| 0.119| 0.115| 0.118| 0.117| Pg. 9, [226] |
| $P_{t+1+2}(43)$, >6 mo on cur. dep. | 0.190 | 0.174| 0.178| 0.121| 0.187| 0.181| 0.173| 0.174| 0.174| Pg. 9, [226] |
| No combat exposure | 0.320 | 0.304| 0.335| 0.304| 0.282| 0.304| 0.303| 0.304| 0.304| [229]  
| Root mean square error | 0.030 | 0.028| 0.078| 0.049| 0.031| 0.031| 0.030| 0.030|  

Table 18: Reported PTSD prevalence rates and estimated PTSD prevalence rates for the base-case Poisson model ($\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$, and $\theta = 0$) as well as several variations described in §8.2 and §8.3. $P_j(t)$ represents the probability of a type $j$ servicemember having symptomatic PTSD in month $t$, where $j = 1 + 2$ represents active and reserve Army soldiers. (A) Probit model, $\alpha = 8.495 \times 10^6$, $\text{ID}_50 = 0.0447\beta = 0.124$, $b = 0.0682$, and $\theta = 1$; (B) Poisson model with servicemembers returning to civilian status according to their stress-to-threshold ratio, base-case parameters, $\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$, and $\theta = 0$; (C) Poisson model with servicemembers returning to civilian status according to their stress-to-threshold ratio, parameters reestimated, $\alpha = 306.6$, $\gamma = 16.14$, $b = 0.0588$, and $\theta = 1$; (D) Poisson base-case model with $\theta = 1$, $\alpha = 146.7$, $\gamma = 7.72$, $b = 0.0626$; (E) Poisson model, $\theta$ constrained to 1, other parameters reestimated, $\alpha = 132.9$, $\gamma = 6.99$, $b = 0.0625$, and $\theta = 1$; (F) Poisson model, $\mu_1$ and $\mu_2$ increased by ln(2), $\alpha = 197.0$, $\gamma = 10.37$, $b = 0.0624$, and $\theta = 1$; (G) Poisson model, $\mu_1$ and $\mu_2$ decreased by ln(2), $\alpha = 110.9$, $\gamma = 5.83$, $b = 0.0626$, and $\theta = 0$. 
<table>
<thead>
<tr>
<th>Number of stressful events</th>
<th>Fraction of servicemenbers</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>0.281</td>
</tr>
<tr>
<td>2</td>
<td>0.172</td>
</tr>
<tr>
<td>3</td>
<td>0.097</td>
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<tr>
<td>4</td>
<td>0.052</td>
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<tr>
<td>5</td>
<td>0.027</td>
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<tr>
<td>6</td>
<td>0.013</td>
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<tr>
<td>7</td>
<td>0.006</td>
</tr>
<tr>
<td>8</td>
<td>0.003</td>
</tr>
<tr>
<td>9</td>
<td>0.001</td>
</tr>
<tr>
<td>≥ 10</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 19: Probability mass function for the number of stressful events servicemenbers are exposed to during withdrawal scenario 2
Figure 1:
Figure 2:
Figure 3:
Figure 4:
Figure 5:

(a)

(b)

(c)

73
Figure 6:
Figure 7:
Figure 8:
Figure 9:
Figure 10:
Figure 11:
Figure 12: