APPENDIX

This appendix formulates the mathematical model described in the main text. The Discrete Choice Submodel for aliens is given in §1, the Apprehension Submodel appears in §2, the Removal Submodel is depicted in §3, and the Illegal Wage Submodel is derived in §4. The submodels in these four sections correspond to relations (1)-(4) in the main text, respectively. Parameter estimation is carried out in §5 (parameters are listed in Tables 1-5), and supporting computational results are in Figures 1-9.

1 Discrete Choice Submodel

For Mexicans \((i = 1)\) and OTMs \((i = 2)\), we assume there are \(n_i\) aliens that are considering entering the U.S. illegally. In this section, we introduce two discrete choice models, one for Mexicans and one for OTMs, that specify the fraction of these potential border crossers that choose to illegally enter the U.S. \((j = 1)\) and the fraction that choose to stay at home \((j = 2)\). We need two models because apprehended Mexicans are typically offered voluntary departure, i.e., they are allowed to withdraw to Mexico without penalty rather than being detained, whereas OTMs cannot be returned to Mexico. We begin with the simpler model, which is for OTMs.

1.1 OTM Decision Model

We use the multinomial-logit model, which is the most widely used random utility model, particularly in the area of consumer choice (e.g., [1]). This model implicitly captures the heterogeneity in preferences, resources (e.g., money to buy fraudulent documents or hire a coyote, i.e., a human smuggler) and perceptions (e.g., about job opportunities or risk of being apprehended while crossing the border) among the population of potential border crossers. The model assumes that the utility for an OTM from choosing option \(j\) has a deterministic component \(u_{2j}\) plus a random component that is an independent and identically distributed (iid) logistic random variable with mean zero. According to the model, the probability \(P_{2j}\) that an OTM will choose option \(j\), in terms of the expected utility \(u_{2j}\) received from this option and the scale parameter \(\theta\), is

\[
P_{2j} = \frac{e^{\theta u_{2j}}}{e^{\theta u_{21}} + e^{\theta u_{22}}}
\]

for \(j = 1, 2\).

This model becomes deterministic as \(\theta \to \infty\) and becomes a pure random choice model as \(\theta \to 0\). Equation (1) is closely related to the logistic regression models used for analyzing the apprehension probability at the border, which replace \(u_{ij}\) by a linear function of independent variables.

Because the great majority of illegal immigrants are looking for a job in the U.S., the expected utilities are taken to be the expected undiscounted earnings over \(\tau\) years minus the cost of migration. Let \(w_u\) be the mean annual wage of an illegal immigrant in the U.S., which is computed in §4, and let \(w_o\) be the mean annual wage in the home country; in our
model, these two quantities do not depend on whether the alien is Mexican or an OTM. If the potential crosser decides to stay at home, his utility is

$$u_{22} = u_0 \tau.$$ (2)

Let $c_2$ denote the one-way cost of migration for an OTM. In the case where a crosser is apprehended, detained and removed, he does not pay his way home but incurs a cost $d_2$ due to lost wages in the home country and the physical and psychological toll of detention. We let $P_{ai}$ be the probability that an alien of type $i$ is apprehended while crossing the border; this quantity is computed in §2. Because OTMs are not offered voluntary departure to Mexico, apprehended OTMs are supposed to be detained and then removed to their home country. However, due to lack of beds in the detention and removal operations (DRO), an apprehended OTM will only be detained and removed with probability $P_r$, which is computed in §3. Otherwise, the apprehended OTM is released into the interior of the U.S., where he is free to (illegally) seek work; however, before being released, he incurs a cost of $f d_2$, where $f$ is the fraction of the total detention cost $d_2$ incurred before removal. Because $f$ varies with the number of DRO beds, which is a decision variable in our model, in a complex way, in §5.1 we approximate $f$ by means of a quadratic function of the removal probability $P_r$. Hence, the expected utility for an OTM if he decides to attempt an illegal crossing is

$$u_{21} = (1 - P_{a2})(u_0 \tau - c_2) + P_{a2}(1 - P_r)(u_0 \tau - c_2 - f d_2) + P_{a2}P_r(u_0 \tau - c_2 - d_2).$$ (3)

### 1.2 Mexican Decision Model

We assume that an unauthorized Mexican is offered voluntary departure to Mexico the first $a - 1$ times that he is apprehended, where $a$ is a decision variable for the U.S. Government. Upon being apprehended for the $a$th time, a Mexican alien is detained if there are DRO beds available and is offered voluntary departure to Mexico if there are no DRO beds available. Moreover, after the Mexican alien is detained, there is still the possibility for him to be forced out of DRO before his removal due to lack of bedspace, and in this case we assume that he is offered voluntary departure when he is forced out of DRO. We allow Mexicans who accept voluntary departure after apprehension (and perhaps some detention) to make the choice of whether to cross again or to stay in Mexico.

As explained in §5.1, we set $a = \infty$ in the base case, so that border patrol agents turn apprehended migrants back into Mexico. In this case, we assume that migrants keep crossing until they succeed, as in [2]. Because this case is much simpler than when $a$ is finite, we treat it separately. When $a = \infty$, the probability that a Mexican will choose option $j$ is

$$P_{1j} = \frac{e^{\theta u_{1j}}}{e^{\theta u_{11}} + e^{\theta u_{12}}} \quad \text{for} \quad j = 1, 2,$$ (4)

where the utility from choosing option 2 is

$$u_{12} = u_0 \tau.$$ (5)

If we let $\tilde{d}_1$ be the cost incurred each time an alien is apprehended at the border and returned to Mexico, then the expected utility from choosing option 1 is

$$u_{11} = w_0 \tau - c_1 - \frac{P_{a1}}{1 - P_{a1}} \tilde{d}_1.$$ (6)
Now we turn to the case where \( a \) is finite. For stage \( k = 1, 2, \ldots, \) let \( u_{ij}^{(k)} \) denote the expected utility from stage \( k \) onward for a Mexican immigrant if he chooses option \( j \) at stage \( k \), i.e., after being apprehended \( k - 1 \) times, and let \( P_{ij}^{(k)} \) be the probability of making this choice. As in equation (3), we use the multinomial-logit model and get

\[
P_{ij}^{(k)} = \frac{e^{\theta_{u_{ij}}^{(k)}}}{e^{\theta_{u_{ij}}^{(k)}} + e^{\theta_{u_{ij}}^{(k)}}} \quad \text{for} \quad j = 1, 2 \quad \text{and} \quad k = 1, \ldots, a.
\]  

(7)

If we view \( c_1 \) as the one-way cost between a Mexican’s hometown and the U.S.-Mexico border, then \( u_{11}^{(1)} = w_o \tau \) because the alien never travels to the border in this case, and \( u_{12}^{(k)} = w_o \tau - c_1 \) for \( k \geq 2 \) because the cost to get to the border is already sunk by the time he has been apprehended \( k - 1 \) times, but the cost to get home from the border still needs to be incurred for choice \( j = 2 \).

In the remainder of this subsection, we formulate and solve a recurrent relation between the \( P_{ij}^{(k)} \)’s and the \( u_{ij}^{(k)} \)’s. For stage \( k = 1, \ldots, a \), if the alien is apprehended then he is returned to Mexico and incurs the cost \( d_1 \), and his expected utility at stage \( k + 1 \) will be \( P_{11}^{(k+1)} u_{11}^{(k+1)} + P_{12}^{(k+1)} u_{12}^{(k+1)} \). Because the one-way travel cost is sunk after the alien arrives at the border, if we define \( I_{\{d\}} \) to be the indicator function of the event \( d \), then we have the relation

\[
u_{11}^{(k)} = (1 - P_{a1}) w_o \tau + P_{a1}(P_{11}^{(k+1)} u_{11}^{(k+1)} + P_{12}^{(k+1)} u_{12}^{(k+1)} - d_1) - c_1 I_{\{d=1\}} \quad \text{for} \quad k = 1, \ldots, a-1.
\]  

(8)

Beginning at stage \( a \), if the alien still chooses to cross, he risks being detained and removed. With probability \( P_{a1}P_d \), he will be detained and removed and have utility \( w_o \tau - c_1 I_{\{d=1\}} - d_1 \), where \( d_1 \) is the detection cost for Mexicans. With probability \( P_{a1}(1 - P_d) \), he will not be detained in DRO until removal and will be offered voluntary departure after incurring a loss of \( fd_1 \); he can then make a decision at stage \( a + 1 \). Hence, at stage \( a \) the relation is

\[
u_{11}^{(a)} = (1 - P_{a1}) w_o \tau + P_{a1}(1 - P_d)(P_{11}^{(a+1)} u_{11}^{(a+1)} + P_{12}^{(a+1)} u_{12}^{(a+1)} - fd_1) + P_{a1} P_d (w_o \tau - d_1) - c_1 I_{\{d=1\}}
\]  

(9)

Moreover, because aliens face the same situation at each stage \( k \geq a \), we have \( u_{11}^{(k)} = u_{11}^{(a)} \) and \( P_{11}^{(k)} = P_{11}^{(a)} \) for all \( k \geq a \), where

\[
u_{11}^{(a)} = (1 - P_{a1}) w_o \tau + P_{a1}(1 - P_d)(P_{11}^{(a)} u_{11}^{(a)} + P_{12}^{(a)} u_{12}^{(a)} - fd_1) + P_{a1} P_d (w_o \tau - d_1) - c_1 I_{\{d=1\}}
\]  

(10)

Because \( P_{12}^{(k)} = 1 - P_{11}^{(k)} \), equations (7), (8) and (10) are a system of \( 2a \) equations in terms of the \( 2a \) unknowns, \( u_{ij}^{(k)}, P_{ij}^{(k)} \) for \( k = 1, \ldots, a \). We solve these equations using backwards recursion in a fashion reminiscent of optimal stopping problems [3]; indeed, the decision problem faced by an individual Mexican is an optimal stopping problem, but we are solving this problem over the aggregate Mexican alien population using the multinomial-logit model. First, we jointly solve equation (7) with \( k = a \) and equation (10) for \( P_{11}^{(a)} \) and \( u_{11}^{(a)} \); the existence of a unique solution to these two equations, and an approach to numerically solve them, is shown in [4]. With \( P_{11}^{(a)} \) and \( u_{11}^{(a)} \) in hand, we use equation (8) for \( k = a - 1 \) to get \( u_{11}^{(a-1)} \), and then use equation (7) to get \( P_{11}^{(a-1)} \), and then recursively solve for \( u_{11}^{(a-2)}, P_{11}^{(a-2)}, \ldots \) in a similar manner until we obtain \( u_{11}^{(1)} \) and \( P_{11}^{(1)} \).
2 The Apprehension Submodel

The goal of this section is to compute the apprehension probability $P_{ai}$ at the border for type $i$ aliens. Our macro approach to the apprehension probability captures the impact of the alien flow and enforcement effort (both labor and technology), and ignores the micro-structural variables that provide insight into the personal and community characteristics of the types of people who are apt to be apprehended at the border; see [5] for an example of the latter approach. Espenshade and Acevedo [6] obtain a $R^2$ of 0.768 in a regression model that includes alien flow and enforcement effort. Our model also incorporates the fact that aliens adapt to the presence of enforcement effort by (often with the help of coyotes) choosing more remote routes [7].

Although the U.S. Government deploys a variety of technologies, including several helicopters and drones, for detecting border crossers, we focus on the Integrated Surveillance Intelligence Systems (ISIS), which are remote video surveillance systems consisting of a central command center and two infrared and two daytime cameras spanning a 3-5 mile radius [8]. As of 2005, these surveillance systems monitored approximately 15% of the U.S.-Mexico border [9], and hence are more prevalent than helicopters and drones. Although the Government has over 10,000 seismic, magnetic and thermal sensors on the U.S.-Mexico border [8], in areas without surveillance technology, border patrol agents need to travel – sometimes considerable distances – to investigate all sensor alarms, which are often nuisance alarms (e.g., animals); consequently, these sensor alarms are frequently ignored in these areas [8]. In areas with surveillance technology, the operator can use the video cameras to investigate the sensor alarms, which greatly enhances the effectiveness of these sensors [8].

We model the border as a straight line segment of length $L$. For $x \in [0, L]$, let the indicator function $I_x$ equal 1 if location $x$ is monitored by surveillance technology, and equal 0 otherwise. Let $n_b(x)$ be the density of border patrol agents at location $x \in [0, L]$ in units of miles$^{-1}$, where $n_b = \int_0^L n_b(x) \, dx$ is a decision variable that represents the total number of border patrol agents (working at one time). Similarly, for $x \in [0, L]$, let $\lambda_{bi}(x)$ be the arrival rate of type $i$ aliens ($i = 1$ is Mexican, $i = 2$ is OTM) at location $x$, which has units of miles$^{-1} \times$ time$^{-1}$, where $\lambda_{bi} = \int_0^L \lambda_{bi}(x) \, dx$ is the total number of type $i$ crossings per year. Due to the complexity of the model in this section, we do not capture the fact that arrivals to the border are seasonal; §3 incorporates seasonality of arrivals to DRO because it plays a bigger factor there than at the border (at least in the base case) due to the extreme lack of bedspsace.

We can determine $\lambda_{bi}$ from our analysis in §1. Recall that $n_i$ is the total annual number of potential border crossers of type $i$. Although the number of annual OTM border crossers is a binomial random variable with parameters $n_2$ and $P_{21}$, we simply assume that

$$\lambda_{b2} = n_2 P_{21}$$  \hspace{1cm} (11)

because the variability in the binomial random variable can be safely ignored: $n_i \approx 10^6$, $P_{11} \approx 0.9$, and hence three standard deviations divided by the mean $\approx \frac{3 \sqrt{10^5}}{10^6} \approx 10^{-3}$. Similarly, if $a = \infty$, then we assume that

$$\lambda_{b1} = \frac{n_1 P_{11}}{1 - P_{a1}}.$$  \hspace{1cm} (12)
If \( a \) is finite, the expected number of \( k \)th crossings by Mexicans is \( n_1 P_a^{k-1} \prod_{i=1}^{k} P_{11}^{(i)} \) for \( k = 1, \ldots, a \). For \( k > a \), there is the possibility of detention and removal, and the expected number of \( k \)th crossings is \( n_1 P_a^{k-1} \prod_{i=1}^{a} P_{11}^{(i)} (P_{a1}(1 - P_d) P_{11}^{(a)})^{k-a} \). Therefore, the arrival rate of illegal Mexican aliens to the border is (again, ignoring the variability in the binomial random variable)

\[
\lambda_{bi} = n_1 \sum_{k=1}^{a} P_a^{k-1} \prod_{i=1}^{k} P_{11}^{(i)} + \sum_{k=a+1}^{\infty} (P_a^{a-1} \prod_{i=1}^{a} P_{11}^{(i)} (P_{a1}(1 - P_d) P_{11}^{(a)})^{k-a}),
\]

(13)

\[
= n_1 \sum_{k=1}^{a-1} P_a^{k-1} \prod_{i=1}^{k} P_{11}^{(i)} + \sum_{k=a}^{\infty} (P_a^{a-1} \prod_{i=1}^{a} P_{11}^{(i)} (P_{a1}(1 - P_d) P_{11}^{(a)})^{k-a}],
\]

(14)

\[
= n_1 \left[ \sum_{k=1}^{a-1} P_a^{k-1} \prod_{i=1}^{k} P_{11}^{(i)} + \frac{P_a^{a-1} \prod_{i=1}^{a} P_{11}^{(i)}}{1 - P_{a1}(1 - P_d) P_{11}^{(a)}} \right].
\]

(15)

An alien arriving at location \( x \) will not necessarily cross there. We let \( \lambda_{ci}(x) \) be the total crossing rate at location \( x \) (in units of miles\(^{-1}\times\)time\(^{-1}\)), where \( \int_0^L \lambda_{ci}(x) \, dx = \lambda_{bi} \).

We are essentially modeling this process as a Stackelberg game with the U.S. Government as leader (choosing \( n_b(x) \)) and the aliens as followers (choosing where to cross, i.e., \( \lambda_{ci}(x) \)); as in §1.2, the aliens are not behaving in a coordinated manner, but rather their aggregate behavior is modeled using a multinomial logit model (see (24) below). However, we can think of this game as being part of an ongoing sequence of Stackelberg games. If we think in terms of discrete time periods, in each time period there are three decisions: first, the government chooses the spatial allocation of agents \( n_b(x) \), then the aliens choose where to arrive \( (\lambda_{bi}(x)) \), and finally the aliens choose where to cross \( (\lambda_{ci}(x)) \). It is natural to assume that the government adapts in period \( t \) by choosing \( n_b(x) \) to be proportional to the \( \lambda_{ci}(x) \) + \( \lambda_{ci}(x) \) from period \( t - 1 \) (i.e., \( \frac{n_b(x)}{\lambda_{ci}(x) + \lambda_{ci}(x)} \) is a constant for all \( x \)). Similarly, it is not unreasonable to assume that the aliens choose \( \lambda_{bi}(x) \) in period \( t \) to be proportional to the \( \lambda_{ci}(x) \) from period \( t - 1 \). That is, our model assumes that the U.S. Government places its border patrol agents in regions that have experienced the most crossings recently, and that crossers (via word-of-mouth or coyotes) arrive at the border at locations where the recent crossings have occurred.

There are very busy areas and very quiet areas along the border. To capture the spatial heterogeneity in the simplest possible way, we assume that both \( \lambda_{bi}(x) \) and \( n_b(x) \) are sinusoidal functions with the same frequency \( \omega_b \) (where \( \omega_b L \) is an integer) and relative amplitudes \( \alpha_b \in [0, 1] \) and \( \tilde{\alpha}_b \in [0, 1] \):

\[
\lambda_{bi}(x) = \frac{\lambda_{bi}}{L} + \frac{\lambda_{bi}}{L} \alpha_b \sin(2\pi \omega_b x) \quad \text{for} \quad x \in [0, L],
\]

(16)

\[
n_b(x) = \frac{n_b}{L} + \frac{n_b}{L} \tilde{\alpha}_b \sin(2\pi \omega_b x) \quad \text{for} \quad x \in [0, L].
\]

(17)

For simplicity, we assume that \( \lambda_{bi}(x) \) and \( n_b(x) \) are proportional to each other, which would be the case if \( \lambda_{ci}(x) \) and \( \lambda_{ci}(x) \) were proportional to each other (although they are not). These assumptions enable us to optimize \( \lambda_{bi}(x) \) and \( n_b(x) \) in a computationally tractable manner, via the parameters \( \alpha_b \) and \( \tilde{\alpha}_b \).
We introduce the decision variable $s_b$, which is the number of miles along the border that are monitored by surveillance technology. We assume that surveillance technology is implemented at the busiest parts of the border, so that

$$I_i(x) = \begin{cases} 
1 & \text{if } x \in \left[\left(\frac{i}{\omega_b} + \frac{1}{4\omega_b}\right) - \frac{1}{2\omega_b}\left(\frac{m_i}{L}\right), \left(\frac{i}{\omega_b} + \frac{1}{4\omega_b}\right) + \frac{1}{2\omega_b}\left(\frac{m_i}{L}\right)\right] \\
0 & \text{otherwise.}
\end{cases} \quad (18)$$

The main modeling challenge is to determine $\lambda_{ci}(x)$ given both $\lambda_b(x)$ and $n_b(x)$. Let $u_i(x,y)$ be the utility of a type $i$ alien who arrives at location $x$ (we assume the cost to get to location $x$ on the border is sunk) and crosses at location $y$. Let $P_a(y)$ be the apprehension probability at location $y \in [0,L]$ and let $k$ be the cost of traveling (in units of miles$^{-1}$) for aliens of type $i = 1, 2$. Then the utility for OTMs is given by

$$u_2(x,y) = (1 - P_a(y))w_a\tau + P_a(y)((1 - P_r)(w_a\tau - fd_2) + P_r(w_a\tau - d_2)) - k|x - y|. \quad (19)$$

For simplicity, we assume that Mexicans are not detained and will keep attempting to cross (at their chosen location) until they succeed, and that each time they are apprehended they are immediately returned to Mexico and suffer a cost $\tilde{d}_1$. In essence, in this submodel we assume that if the U.S. Government became more aggressive about detaining Mexicans (e.g., reduced $a$ to 1 or 2), then this would deter Mexicans at the initial decision stage of whether or not to travel to the border, and those who traveled to the border would be undeterred. Hence, we have

$$u_1(x,y) = w_a\tau - \frac{P_a(y)}{1 - P_a(y)}\tilde{d}_1 - k|x - y|. \quad (20)$$

At location $y$, there is a border patrol agent every $\frac{1}{n_b(y)}$ miles. Each agent can be viewed as the server in a single-server loss queueing system with (at this point unknown) arrival rate $\frac{P_a(y)\lambda_{ci}(y) + \lambda_{ci}(y)}{n_b(y)}$ and mean apprehension time $m_b$, which is only incurred when aliens are actually apprehended (i.e., aliens that go undetected, or are detected but not apprehended, experience no service time, because most of the apprehension time is devoted to transporting – or waiting for transportation for – the apprehended alien). It follows that the probability that the agent is busy with an apprehension is $\rho_b(y) = \frac{P_a(y)[\lambda_{c1}(y) + \lambda_{c2}(y)]m_b}{n_b(y)}$, where

$$\rho_b(y) = \frac{P_a(y)[\lambda_{c1}(y) + \lambda_{c2}(y)]m_b}{n_b(y)}. \quad (21)$$

We now derive an expression for the apprehension probability at location $y$, $P_a(y)$. If a crossing occurs while the agent at this location is busy with another apprehension, then the agent will not apprehend the crosser. We assume that the arrivals near location $y$ follow a spatially homogeneous Poisson process and the location of the agent is random (i.e., he spends his time driving back and forth along the $\frac{1}{n_b(y)}$ miles), so that the distance between the agent and the alien at the time of crossing is the random distance between any two points that are uniformly distributed on $[0, \frac{1}{n_b(y)}]$, which has pdf

$$f(x) = \frac{2\left(\frac{1}{n_b(y)} - x\right)}{\left(\frac{1}{n_b(y)}\right)^2}. \quad (22)$$
If there is no technology deployed at location \( y \) (i.e., \( I\{y\} = 0 \)), then we assume that an idle agent will detect and apprehend the crosser with probability \( e^{-\alpha_1 x} \) if the agent and alien are at a distance \( x \) apart at the time of crossing (i.e., apprehension occurs if the distance between them is less than an exponential random variable with parameter \( \alpha_1 \)). Similarly, if there is technology deployed at location \( y \) (i.e., \( I\{y\} = 1 \)), then we assume that an idle agent will detect and apprehend the crosser with probability \( e^{-\alpha_2 x} \) if the agent and alien are at a distance \( x \) apart at the time of crossing, where it is natural to expect that \( \alpha_2 < \alpha_1 \) (i.e., technology aids in detection and apprehension). Taken together, the probability of apprehension at location \( y \) is

\[
P_a(y) = \frac{1}{1 + \rho_b(y)} \left[ I\{y\} \int_0^{\frac{1}{\alpha_2(y)}} f(x)e^{-\alpha_2 x} \, dx + (1 - I\{y\}) \int_0^{\frac{1}{\alpha_1(y)}} f(x)e^{-\alpha_1 x} \, dx \right].
\] (23)

We now use a multinomial logit model with a continuum of choices to compute \( P_{ci}(x,y) \), which is the probability that an alien arriving at location \( x \) will cross at location \( y \). This model is given by

\[
P_{ci}(x,y) = \frac{e^{\theta_{ui}(x,y)}}{\int_0^L e^{\theta_{ui}(x,y)} \, dy} \quad \text{for} \quad i = 1, 2.
\] (24)

Hence, the crossing rate of a type \( i \) alien at location \( y \) is given by

\[
\lambda_{ci}(y) = \int_0^L \lambda_{bi}(x)P_{ci}(x,y) \, dx.
\] (25)

Equation (25) is a fixed-point equation for \( \lambda_{ci}(y) \) because \( P_{ci}(x,y) \) on the right side of (25) is a function of \( u_i(x,y) \) in equation (24), which is a function of \( P_a(y) \) in equations (19)-(20), which is a function of \( \rho_b(y) \) in (23), which is a function of \( \lambda_{ci}(y) \) in (21).

Finally, we obtain the apprehension probability \( P_{ai} \) for type \( i \) aliens, which is

\[
P_{ai} = \frac{\int_0^L \lambda_{ci}(y)P_a(y) \, dy}{\lambda_{bi}}.
\] (26)

Equation (26) corresponds to equation (2) in the main text.

### 3 The Removal Submodel

The goal of this section is to compute \( P_r \), the probability that an illegal alien apprehended at the border is detained and removed, given that the U.S. Government would like to detain and remove him (i.e., given that he is either Mexican or at least his \( a \)th apprehension or an OTM). The DRO facilities, which consist of eight large facilities in the U.S. plus contracted beds at a variety of other facilities, house aliens while they are waiting for removal proceedings to be completed. Most detained OTMs are ready to be voluntarily removed, but need to be detained until the home country verifies their identification (most come with no identification). There are mandatory detainees and non-mandatory detainees. Nearly all criminals are mandatory detainees, and some noncriminals are mandatory [10]. We model this facility as the following 2-class, infinite-server, partial-loss queueing system that has been analyzed
elsewhere [11]. Because nearby DRO facilities are used if the closest facility is full, we model
this system by a single pooled queue, which should be an accurate approximation [12, 13].
We let aliens of class \(i (i = m\) is mandatory, \(i = n\) is nonmandatory\) arrive according to a
sinusoidal Poisson arrival process with average rate \(\bar{\lambda}_i\), frequency \(\omega_d\) and relative amplitude
\(\alpha_d\); i.e., the arrival rate for class \(i\) aliens at time \(t\) is \(\bar{\lambda}_i + \bar{\lambda}_i \alpha_d \sin(2\pi \omega_d t)\).

Let \(\lambda_{d1}\) be the rate at which Mexicans are apprehended at the border on at least their
\(a^{th}\) apprehension and let \(\lambda_{d2}\) be the rate at which OTMs are apprehended at the border. We
know from §1.1 that

\[
\lambda_{d2} = n_2 P_{21} P_{a2}. \tag{27}
\]

Similarly, \(\lambda_d\) equals the expected number of \(a^{th}\) and later crossings multiplied by apprehension
probability, which by (15) is

\[
\lambda_{d1} = n_1 \frac{P_{a1}^{a-1} \Pi_{i=1}^{a} P_{11}^{(i)}}{1 - P_{a1} (1 - P_d) P_{11}^{(a)}} P_{a1} \tag{28}
\]

if \(a\) is finite, and \(\lambda_{d1} = 0\) if \(a = \infty\).

Aliens arriving to DRO are apprehended not only at the U.S.-Mexico border, but
also at the other borders, at the ports-of-entry, in the interior, and from jails (i.e., they
have finished serving jail time but are yet to be removed). Hence, we need to determine a
relationship between the mean arrival rates of mandatory and nonmandatory aliens to DRO,
\(\lambda_m\) and \(\lambda_n\), and the U.S.-Mexico border rates in equations (27) and (28). Because most of
the criminals entering DRO come directly from jails, not from the border, we assume that
\(\lambda_m\) is an exogenous parameter that is independent of \(\lambda_{di}\). We assume that

\[
\bar{\lambda}_n = a_n + b_n [\lambda_{d1} + \lambda_{d2}], \tag{29}
\]

where the constants \(a_n\) and \(b_n\) are computed in §5.3 using DRO data. Equation (29) implicitly
assumes that the nonmandatory apprehensions away from the U.S.-Mexico border are
roughly proportional to the nonmandatory apprehensions at the U.S.-Mexico border, as they
were before and after the Hold The Line operation [6]. The rationale is that as the number
of aliens apprehended at the border increases, the number that successfully cross the border
also increases, which should increase the number of apprehensions that occur in the interior.

The residence time for class \(i\) aliens is an exponential random variable with mean \(m_i\)
for \(i = \{m, n\}\). When a mandatory detainee arrives to the system he is detained. If there
is not sufficient bed space, then one is rented and he is moved into a DRO bed later if
room becomes available [8]. When a nonmandatory detainee arrives to the system, he is
blocked from entering the system if no beds are available. As explained in §1, a blocked
Mexican is returned to Mexico and a blocked OTM is released into the interior of the U.S.
If all DRO beds are filled but at least one nonmandatory detainee is being detained, then a
nonmandatory detainee is released into the interior of the U.S. and the bed is given to the
arriving mandatory detainee. In this case, we say that the mandatory detainee is preempted.

An equation for the mean total number of nonmandatory aliens released (i.e, blocked
plus preempted) per year is derived in [11], which allows us to compute the detection prob-
ability \(P_r\) in terms of the seven DRO parameters: the number of DRO beds \((s)\), the mean
arrival rate of mandatory \((\lambda_m)\) and nonmandatory \((\lambda_n)\) detainees, the mean residence time
of mandatory \((m_m)\) and nonmandatory \((m_n)\) detainees, and the frequency \((\omega_d)\) and relative amplitude \((\alpha_d)\) of the sinusoidal arrival rates. To this end, for \(i = \{m, n\}\), let \(f_{it}\) and \(F_{it}\) be the probability density function (pdf) and cumulative distribution function (cdf) of a normal random variable with mean and variance

\[
\lambda_i m_i \left(1 + \frac{\alpha_d}{1 + (2\pi m_i \omega_d)^2} \left[\sin(2\pi \omega_d t) - 2\pi m_i \omega_d \cos(2\pi \omega_d t)\right]\right).
\]

(30)

Then the mean number of detainees released per year (the frequency is \(\omega_d = 1/yr\)) is

\[
R = \int_0^{\omega_d^{-1}} R(t) \, dt,
\]

(31)

where

\[
R(t) = \lambda_n(t) \left(\int_0^s \frac{f_{mt}(x) f_{nt}(s-x)}{F_{mt}(s-x)} \, dx + 1 - F_{mt}(s+1)\right),
\]

(32)

and the probability of removal is

\[
P_r = 1 - \frac{R}{\lambda_n},
\]

(33)

which is the mathematical version of relation (3) in the main text.

Our main performance measure, denoted by \(P_T\), is the probability that an OTM terrorist successfully enters the country, which is given by

\[
P_T = 1 - P_{a2} + P_{a2}(1-P_r).
\]

(34)

Preempted nonmandatory aliens are released (on bond) from the DRO facilities but are still processed, and approximately 1% of these overflow cases are released on order of supervision. However, more than 90% of nondetained aliens do not appear in court, and only 13% of nondetained aliens (and only 6% from countries believed to support terrorist activities) with final removal orders are actually removed [14]. Electronic home-monitoring of low-risk nondetained aliens has been tested and appears to be moderately successful [15]. From a homeland security viewpoint, we assume in equation (34) that a terrorist who is released on bond will not appear in court.

### 4 The Illegal Wage Submodel

The goal here is to construct a model that quantifies how changes in worksite enforcement, including fines for hiring illegal aliens and the number of worksite inspectors, as well as guest-worker program that brings new aliens into the country as legal workers and an amnesty program that would legalize illegal workers currently in the U.S., would impact the wages of illegal aliens that sneak across the border.

It appears that the 1986 Immigration Reform and Control ACT (IRCA) sanctions did not work because of the widespread use of illegal documents and contracting firms, the easy way out for employers (they were protected as long as they filled out I-9 forms and looked at workers’ identification cards), and the low level of sanctions enforcement [16]. We assume an idealized system that works according to plan (e.g., perfect document security for
employment verification, employers cannot pass the risk along to subcontractors), with five controllable variables: the number of currently illegal workers that are legalized, the number of newly legal workers who participate in a guest-worker program, the number of worksite inspectors, the extent to which the inspections are targeted (characterized by the parameter $r_w$ in equation (35)), and the size of the fine (we focus on a hiring fine, even though the IRCA enforcements relied mostly on paperwork fines).

Our model is described in three subsections: how the Government deploys its worksite enforcement resources and what the employers do in the face of this enforcement, the labor demand, and the labor supply.

4.1 Work Site Enforcement

Suppose $m_w$ worksite enforcement agents are hired, each of which performs inspections at $\mu_w$ firms per year. Let $n_w = m_w\mu_w$ be the number of firms inspected per year, $N_w$ be the total number of firms in the U.S. that hire illegal aliens, and $N_i$ be the total number of illegal workers in the U.S. For mathematical simplicity, we assume that the number of illegal workers in a firm is an exponential random variable with mean $\frac{N_i}{N_w}$, which succinctly captures the phenomenon that many illegal workers are concentrated in a handful of industries [17].

The only study to explicitly address how the Government should allocate enforcement resources across firms is [18], which solves a constrained optimization problem and finds that only firms in industries with the largest number of illegal employees should be inspected, and that among inspected industries, the fraction of firms in an industry that should be inspected is linearly increasing with the number of illegal workers per firm in that industry. However, in contrast to this policy, approximately 60% of the IRCA investigations were aimed at suspected companies/industries, and the remaining investigations were essentially random [19].

We follow what was used during IRCA and assume a hybrid targeted-random strategy, where a fraction $r_w$ of inspections are targeted and the remaining inspections are random. We assume that the Government knows which firms have the most illegal workers, and the targeted inspections are aimed at these firms. That is, the firms in the highest $\frac{m_w n_w}{N_w}$ fractile of illegal workers are targeted, which corresponds to the firms with more than $\frac{N_i}{N_w} \ln \left(\frac{N_w}{r_w n_w}\right)$ illegal workers. The remaining fraction $1 - r_w$ of inspections are randomly sampled from the untargeted industries, so that the annual probability of inspecting a firm that hires $x$ illegal aliens is

$$p_w(x) = \begin{cases} 
\frac{(1-r_w)n_w}{N_w-r_w n_w} & \text{if } x < \frac{N_i}{N_w} \ln \left(\frac{N_w}{r_w n_w}\right) \\
1 & \text{if } x \geq \frac{N_i}{N_w} \ln \left(\frac{N_w}{r_w n_w}\right).
\end{cases} \quad (35)$$

In the face of sanctions, we assume that employers pass the expected penalties on to the illegal workers in the form of lower wages [18, 20]. Due to our assumption about perfect document security, employers can fully discern the legal status of workers. Therefore, illegal laborers at a firm with $x$ illegal laborers are paid, assuming that employment decisions are made on an annual basis so that the expected fine is allocated over one year,

$$w_i(x) = \max\{w - p_w(x)f_w, 0\}, \quad (36)$$
where $w$ is the legal wage that is determined in §4.3 and $f_w$ is the fine per illegal worker per hour of work.

### 4.2 Labor Demand

We take the view that immigrants tend to fill jobs that native workers do not want, and that in the absence of these workers, many of these jobs would be replaced by capital or move offshore [21]. Hence, we ignore high-skilled labor in our model, and assume that each of the $N_w$ firms has a Cobb-Douglas production function with two factors, unskilled workers and capital [20], i.e., the output equals $Y = L^{\alpha_w}K^{1-\alpha_w}$, where $r$ is the cost of capital and $w$ is the cost of labor, which includes wages and expected sanctions; because employers pass the expected penalties on to the workers, this $w$ is the same as in equation (36). The cost-minimizing demand for labor is

$$L_d = A_w \left( \frac{1}{w} \right)^{1-\alpha_w} Y,$$

(37)

where $A_w = \left( \frac{\alpha_w r}{1-\alpha_w} \right)^{1-\alpha_w} N_w Y$.

### 4.3 Labor Supply

The unskilled labor is supplied by both legal and illegal workers currently in the U.S., which are assumed to have equal skill levels. Let the total legal labor supply be initially fixed at $N_l$ and recall that $N_i$ be the total number of illegal workers currently in the U.S. We model legalization by the decision variable $\Delta_l \in [0, N_i]$, which is the number of illegal workers that are legalized, and we model a guest-worker program by the decision variable $N_g$, which is the number of new aliens that are legally brought into the U.S. to work. We assume that the guest-worker slots are allocated to Mexicans and OTMs in proportion to $n_i$, so that the number of potential border crossers that are Mexican and OTM is reduced to

$$n_1 - \frac{n_1}{n_1 + n_2} N_g,$$

(38)

$$n_2 = \frac{n_2}{n_1 + n_2} N_g.$$

(39)

Hence, we have four pools of labor supply: $N_l$ originally legal, $N_i - \Delta_l$ illegal, $\Delta_l$ newly legal and $N_g$ new guest workers. The legal laborers (i.e., all but the $N_i - \Delta_l$ illegal workers) are paid the equilibrium legal wage $w$, while an illegal laborer at a firm with $x$ illegal laborers is paid $w_i(x)$ in equation (36).

As in [20], we use the neo-classical labor supply function to compute the number of legal labor hours the $N_l$ originally legal people are willing to work at wage $w$. This model (§4.1 of [23]) assumes each person has the utility $\alpha_0 \log(q_0 - \gamma_0) + (1 - \alpha_0) \log(q - \gamma)$, where $q_0$ is the
number of waking hours minus the number of labor hours, $q$ is the quantity of goods bought at price $p$, $\alpha_0$ is a weighting factor, and $\gamma_0$ and $\gamma$ are committed leisure and consumption, respectively. The utility-maximizing amount of legal labor supplied per person, subject to the budget constraint $wq_0 + pq = wT + \mu$, where $T$ is 24 hr, $\mu$ is nonlabor income, and $p$ is an index of consumption good prices, is ($\S$4.1 of [23])

$$(1 - \alpha_0)(T - \gamma_0) + \frac{\alpha_0 \gamma p}{w} - \frac{\alpha_0 \mu}{w}. \quad (40)$$

To reduce the number of parameters to estimate, we multiply (40) by $365N_l$ and express the annual legal labor hours supplied by

$$N_l \left( \kappa_1 - \kappa_2 \right), \quad (41)$$

where $\kappa_1 = 365(1 - \alpha_0)(T - \gamma_0)$ and $\kappa_2 = 365\alpha_0(\mu - \gamma p)$.

Because the base-case amount of worksite enforcement is negligible, and because we are interested in the impact of greater levels of enforcement, we implicitly assume that most illegal workers are already employed before enforcement. In our model, illegal aliens who have their wages lowered due to worksite enforcement have the option of staying in their present job, quitting and looking for a job with a U.S. firm that has a lower probability of being inspected, or returning home. Because there are only two different illegal wages offered (see (35)-(36)), we assume that illegal workers who quit a job at an untargeted firm will return to their home country because the other untargeted firms pay the same wage and the targeted firms pay a lower wage. In contrast, we consider the possibility that illegal workers who quit a job at a targeted firm may prefer to fill a vacated job at an untargeted firm rather than go home. Hence, we consider a matching process between the illegal workers who quit a job at a targeted firm, and the jobs at untargeted firms that were vacated by illegal aliens; illegal workers who are not matched return to their home country. This matching process takes place before the firms decide to raise the legal wage and/or increase the amount of capital in response to the vacated jobs.

We again use a multinomial logit model similar to equation (1) to compute how many people stay in their reduced-wage job. The expected utility if an illegal alien returns home is $w_0 \tau - c_1$ where $w_0$ is the annual home wage and $c_1$ is the cost to return home (we use $c_1$ because most illegal workers are Mexican). The expected utility if he stays in the U.S. is $hx\tau$ if he is offered the hourly wage $x$, where $h$ is the number of hours worked per worker per year.

By equation (35), the probability that an illegal worker works in a targeted firm is

$$P_t = \frac{N_w \int_{N_w}^{\infty} \ln \left( \frac{x}{r_w n_w} \right) x^{N_w} e^{-N_w x / N_i} dx}{N_i}, \quad (42)$$

$$= \frac{r_w n_w}{N_w} \left( 1 + \ln \left( \frac{N_w}{r_w n_w} \right) \right). \quad (43)$$

Equation (36) implies that an illegal worker will be offered the hourly wage

$$w_t = \max(w - f_w, 0) \quad (44)$$
with probability $P_t$ (i.e., if he works at a targeted firm), and the hourly wage
\[
w_r = \max(w - \frac{f_w(1 - r_w)n_w}{N_w - r_wn_w}, 0) \tag{45}\n\]
otherwise. Then the probability that an illegal worker will stay at his reduced-wage job is the weighted average of the multinomial logit models,
\[
P_t \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} + (1 - P_t) \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}}. \tag{46}\n\]

We now describe the matching process for those who leave their job. Our matching process uses a Cobb-Douglas form with constant returns to scale, which is consistent with the empirical literature [24]. If $U$ is the number of unemployed workers looking for a job, $V$ is the number of matched jobs, and $M$ is the number of matched jobs, then
\[
M = \min\{U, V, A_m U^{\alpha_m} V^{1-\alpha_m}\}, \tag{47}\n\]
where $A_m$ and $\alpha_m$ are constants. In analyzing this matching process, note that we can interpret $\frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}}$ as a cdf that represents the fraction of illegal workers that prefer to work at hourly wage $x$ rather than return to their home country. Hence, $U$ is given by the number of illegal workers times the probability that a worker is originally in a targeted firm, quits his job because he is unwilling to work at wage $w_t$, but is willing to work at wage $w_r$,
\[
U = N_i P_t \left( \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} - \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} \right). \tag{48}\n\]
Similarly, $V$ is given by the number of illegal workers times the probability that a worker is originally in an untargeted firm and quits his job because he is unwilling to work at wage $w_r$,
\[
V = N_i (1 - P_t) \left( \frac{e^{\theta (w_0 \tau - c_1)}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} \right). \tag{49}\n\]
By equations (46)-(49), the annual illegal labor supply (for ease of presentation, in the remainder of this section we assume that the Cobb-Douglas term achieves the minimum in (47)) is
\[
N_i h \left[ \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} + (1 - P_t) \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} \right] + A_m \left[ \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} - \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} \right]^{\alpha_m} \left[ \frac{(1 - P_t) e^{\theta (w_0 \tau - c_1)}}{e^{\theta h w r \tau} + e^{\theta (w_0 \tau - c_1)}} \right]^{1-\alpha_m}. \tag{50}\n\]
The newly legal workers $\Delta_l$ also have the option to go home, but are getting paid exactly $w$, and hence the amount of newly legal labor supplied annually is
\[
\Delta_l h \frac{e^{\theta h w t}}{e^{\theta h w t} + e^{\theta (w_0 \tau - c_1)}}. \tag{51}\n\]
We assume that the new guest workers all stay, providing
\[
N_g h \tag{52}\n\]
hours of work annually.

The equilibrium condition is found by equating the labor demanded, which is the right side of (37), to the labor supplied, which is the sum of (41), (50), (51) and (52):

\[
A_w \left( \frac{1}{w} \right)^{1-\alpha_w} = N_l (\kappa_1 - \frac{\kappa_2}{w}) + \Delta_i h \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} + N_g h \\
+ N_i h \left[ P_t \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} + (1 - P_t) \frac{e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right] \\
+ A_m \left( \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} - \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right)^{\alpha_m} \left( \frac{(1-P_t)e^{\theta (w_o \tau - c_1)}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right)^{-\alpha_m}.
\]

We solve (53) for the equilibrium legal wage \( w \). By equation (50), we set the annual illegal wage \( w_u \) in §1 equal to

\[
w_u = \frac{\frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} + (1-P_t) e^{\theta h w r \tau} + A_m w_r \left( \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} - \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right)^{\alpha_m} \left( \frac{(1-P_t)e^{\theta (w_o \tau - c_1)}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right)^{-\alpha_m}}{\frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} + (1-P_t) e^{\theta h w r \tau} + A_m \left( \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} - \frac{P_t e^{\theta h w r \tau}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right)^{\alpha_m} \left( \frac{(1-P_t)e^{\theta (w_o \tau - c_1)}}{e^{\theta h w r \tau} + e^{\theta (w_o \tau - c_1)}} \right)^{-\alpha_m}}.
\]

Equation (54) corresponds to relation (4) in the main text.

## 5 Parameter Estimation

In the next four subsections, we estimate the parameters from the four submodels in §1-4. The parameter values for these four submodels are listed in Tables 1-4 and the base-case values of the decision variables appear in Table 5; although some of these parameters appear in several submodels, they appear in the table for the submodel in which they play the most central role.

### 5.1 Discrete Choice Parameters

We consider a time horizon of \( \tau = 2 \) yr. For lack of resources, the U.S. Attorneys Office does not prosecute an illegal immigrant until he has been apprehended 13 times [8]. Because \( P_{a}^{13} \approx 10^{-9} \), it appears unlikely that many of those detained were held because they were previously apprehended a specific number of times. Consequently, as noted in §1.2, we set the decision variable \( a = \infty \) in the base case.

We begin with the wages and costs, and then estimate some of the apprehension and detention parameters. The more difficult parameters to estimate are discussed last.

The hourly wage in 2004 for production workers in manufacturing in Mexico was $2.50 (Table 2 in [25]). Most OTM aliens are from Caribbean countries (e.g., Guatemala, Honduras, El Salvador) that have similar economic conditions. At 2000 hr/yr, we get \( w_o = 5000/yr \) as the annual wage in the home country. The average wage of Mexican immigrants in the U.S. is $22,300/yr [26]. Because there is currently a trivial amount of worksite enforcement, we set the annual illegal wage in the U.S. to be \( w_u = 22,300/yr \). Including the travel cost, the cost of a coyote (77% of aliens use coyotes [27]), the lost time,
and the danger, we roughly estimate the one-way cost of migration to be $c_1 = $1500 and 
$c_2 = $2500.

We assume the detention cost $d_i$ includes the cost of lost wages in the home country and 
an additional cost due to the physical and psychological toll of being detained. We assume 
$d_i$ is proportional to the square root of the number of days in detention. The concavity of 
the square root function attempts to crudely capture the phenomenon that the very act of 
being detained carries a significant psychological toll, regardless of the length of detention. 
Although a fixed plus variable cost of detention may be more appropriate, this would require 
an additional parameter to estimate. More specifically, we let 
$d_i = \psi \sqrt{\tilde{m}_i}$, where $\psi$ is the toll 
factor of apprehension (which is estimated later) and $\tilde{m}_i$ is the number of days that a type $i$ 
alien is detained until removal. Similarly, we let 
$\tilde{d}_2 = \psi \sqrt{2}$, where it is assumed that two days 
of U.S. salary are lost during the apprehension process (and the toll includes the possibility 
that the U.S. Government takes their fingerprints upon apprehension). In [11], we calculated 
that the mean uncensored residence time for nonmandatory aliens is 48.0 days, whereas the 
observed, right-censored residence times for Mexican noncriminals, OTM noncriminals, and 
all noncriminals is 10.5, 42.4, and 28.7 days, respectively, where the weighted average for 
all noncriminals uses the unblocked arrival rates (45,925 Mexican noncriminals and 61,312 
OTM noncriminals) to determine the weights. If we assume that apprehended aliens in our 
model are noncriminal, and apply the censoring factor $48.0 \div 28.7$ to the censored residence times 
for Mexican noncriminals and OTM noncriminals, we get $\tilde{m}_1 = 48.0 \div 28.7 (10.5) = 17.6$ days and 
$\tilde{m}_2 = 48.0 \div 28.7 (42.4) = 70.9$ days.

The parameter $f$ is the fraction of the total detention cost incurred before release from 
DRO. The parameter $g$ is the expected number of days detained before release from DRO 
(and voluntary departure to Mexico) conditioned on not being detained until removal, di-
vided by the expected number of days detained until removal. Since the cost $d_i$ is proportional 
to the square root of the detention duration, we approximate $f$ by $\sqrt{g}$ (although the expected 
value of the ratio does not equal the ratio of the expected values in the computation of $g$, the 
two should be similar because the denominator is bigger than the numerator). In 2003, there 
were 28,000 blocked arrivals (i.e., people who were not detained at all) and 43,000 preempted 
arrivals (i.e., aliens who were detained but were released before removal) out of 93,976 total 
arriages, so that probability of removal in 2003 was $P_r = 1 - \frac{71,000}{93,976} = 0.245$. Recalling that the 
mean censored residence time is 28.7 days and the mean uncensored residence time is 48.0 
days, we get that in 2003 $g = \frac{43,000}{25,782} \frac{28.7}{48.0} = 0.36$. As mentioned in §1.1, the parameter 
g varies in a complicated way with the number of DRO beds. To enable the optimization of 
the entire system, we assume that $g$ can be expressed as a quadratic function of the removal 
probability $P_r$. Fitting the three parameters of the quadratic function to the two $(g,P_r)$ 
极端点, (0,0) and (1,1), and the 2003 point (0.36,0.245) yields the relationship

$$f = \sqrt{g} = \sqrt{1.637P_r - 0.637P_r^2}. \tag{55}$$

The current removal probability is estimated to be $P_r = 0.137$ in §5.3, and so $f = 0.46$ in 
the base case. We assume that the base-case apprehension probability is $P_{a1} = P_{a2} = 0.2$ 
[7], which is lower than it was in the 1970s and 1980s [6].

This leaves six unestimated parameters: $\theta$, $\psi$ (embedded within $d_i$), $n_1$, $n_2$, $\lambda_{b1}$, and 
$\lambda_{b2}$. The parameters $\theta$ and $\psi$ will be jointly estimated with two other parameters in §5.2.
\text{ Here we provide one of the four equations that will be needed for their derivation. This equation is based on an analysis in [28], which estimates that the natural logarithm of the monthly apprehension rate at the U.S.-Mexico border increases by 0.049 times the ratio, for Mexican aliens, of the U.S. wage and the Mexican wage. Let } \lambda_{b1} \text{ be the value of } \lambda_{b1} \text{ in (12) but with } w_u \text{ replaced by } w_u + w_0, \text{ which represents an increase of the wage ratio by 1.0. Noting that the monthly apprehension rate in [28] corresponds to } \frac{P_{a1}\lambda_{b1}}{12} \text{ in our model, our equation is given by}

\[
\ln \left( \frac{P_{a1}\lambda_{b1}}{12} \right) = \ln \left( \frac{P_{a1}\lambda_{b1}}{12} \right) - 0.049. 
\]

\text{We complete this subsection by deriving values for } n_1, n_2, \lambda_{b1}, \text{ and } \lambda_{b2}. \text{ After obtaining values of } \theta = 6.83 \times 10^{-5}/$ \text{ and } \psi = $1067/\text{day} \text{ in §5.2, we can compute } P_{1j} \text{ and } P_{2j} \text{ in the base case, which yields } P_{11} = 0.903 \text{ and } P_{21} = 0.887. \text{ In 2005, 155k OTMs were apprehended at the U.S.-Mexico border [29], and so } \lambda_{b2} = 155k/\text{yr}. \text{ By equation (27), } \lambda_{b2} = n_2P_{21} = \frac{\lambda_{b2}}{P_{21}} = 775k/\text{yr} \text{ and } n_2 = 874k. \text{ In the base case, we know } \lambda_{b1} = 0. \text{ In 2004, 1.16M Mexican aliens were apprehended by Border Patrol agents [30]. Assuming these apprehensions all took place at the U.S.-Mexico border, in the base case } (a = \infty) \text{ we have } \frac{n_1P_{11}P_{21}}{1-P_{a1}} = 1.16M, \text{ which gives } n_1 = 5.14M \text{ and, by equation (12), } \lambda_{b1} = 5.80M/\text{yr.}

\textbf{5.2 Apprehension Parameters}

\text{We need to estimate values for the parameters } L, \omega_b, \alpha_b, k, \alpha_1, \alpha_2, m_b, \text{ and } \theta, \text{ along with base-case values of the decision variables } n_b \text{ and } s_b, \text{ and the costs } c_b \text{ and } c_s \text{ for agents and surveillance technology. We start with the straightforward parameters, then turn to the decision variables and costs, and then finally to the more difficult parameters.}

\text{We let } L = 1933 \text{ miles [8] and assume } m_b = 1 \text{ hr based on conversations with government employees. The cost of a coyote increased } $800 \text{ when they needed to take more rural routes [7], and we assume } k = $4/\text{mile because each sector averages approximately 200 miles in length and it is clear from data that people cross in adjacent sectors in the presence of increased enforcement [7, 31]. We assume there are 10 peak crossing points that are evenly distributed across the U.S.-Mexico border (currently seven fences are in place with several more proposed [32], so that } \omega_b = \frac{10}{7} = 5.17 \times 10^{-3}/\text{mile. In the base case we assume the arrival rate } \lambda_{b1}(x) \text{ and border agent density } n_b(x) \text{ have the same amplitude, so that } \alpha_b = \bar{\alpha}_b. \text{ The largest apprehension rate per mile among the nine sectors divided by the smallest apprehension rate per mile among the nine sectors is approximately 30 [32], and so we set } \frac{\alpha_b+1}{\alpha_b-1} = 40 \text{, which gives } \alpha_b = \frac{39}{41} = 0.951.

\text{Now we turn to the decision variables and costs. Assuming around-the-clock surveillance and 11,380 border patrol agents [17] who work 2000 hr/yr but spend only 63\% of their time on the border [27], we have } n_b = \frac{0.63(11,380)(2000 \text{ hr/yr})}{8766 \text{ hr/yr}} = 1636 \text{ agents at a given time on the border. We assume that a border patrol agent costs $176k per year to work 40 hr/week [33], and that 63\% of agent hours are on the border. Hence, the annual cost per } n_b \text{ is } c_b = \frac{($176k/\text{yr})(168 \text{ hr/wk})}{0.63(40 \text{ hr/wk})} = 1.173M/\text{yr. There are several different kinds of technologies, and we focus on surveillance cameras, which are far more prevalent than helicopters and drones, and much more effective than thermal sensors [8]. Page 53 of [8] claims that 25\% of the Laredo, El Paso and McAllen sectors are covered by surveillance technology, and the}
GAO reports that approximately 300 miles of the Northern and Southern borders are covered [9]. We set \( s_b = 0.15L = 290 \) miles in the base case. These surveillance systems cost \$90k/mile ([32], which is consistent with the estimate that each camera system costs \$650k [8]. However, we need to convert this to an annual cost. Each operator is in charge of several dozen cameras [8], and there are four cameras to cover five miles, and so this extra cost is approximately \$3000/yr. As a ballpark figure, we use \( c_s = \$30k/mile \cdot yr \), assuming a lifetime of approximately five years and some maintenance.

We conclude this subsection by computing the four remaining parameters, \( \theta, \psi \) (embedded in \( d_i \) and \( \tilde{d}_1 \)), \( \alpha_1 \) and \( \alpha_2 \), from four equations. One of these equations is (56). Two more equations are constructed by setting the right side of equation (26) equal to 0.2 for \( i = 1, 2 \) (i.e., \( P_{a_1} = P_{a_2} = 0.2 \)), which is the base-case apprehension probability [7]. Page 41 of [8] states that technology (which covers approximately 15% of the U.S.-Mexico border) is responsible for up to 60% of apprehensions in some sectors on the U.S.-Mexico border. This 60% figure would appear to be an upper bound and we estimate the efficacy of surveillance technology by assuming that it plays a role in 45% of all apprehensions. Hence, by equations (23) and (26), the fourth equation is

\[
\int_0^L \frac{\lambda_c(y)I(y)}{\int_0^{\rho_b(y)} \frac{f(x)e^{-\alpha_2 x}}{1+\rho_b(y)} \, dx} \, dy = 0.09, \tag{57}
\]

where we set \( s_b = 0.15L \) in the definition of the indicator function in equation (18). The solution to these four equations is \( \theta = 6.83 \times 10^{-5} / $, \( \psi = 1067 / \sqrt{\text{day}} \) (and hence \( d_1 = 1509 \), \( d_1 = 4476 \), and \( d_2 = 8983 \)), \( \alpha_1 = 9.08 / \text{mile} \), and \( \alpha_2 = 0.29 / \text{mile} \).

### 5.3 DRO Parameters

We begin by estimating the constants \( a_n \) and \( b_n \) in equation (29). Because \( a = \infty \) in the base case, no Mexicans are sent to DRO because they have been apprehended a specified number of times. Hence, we take \( a_n \) to be the annual number of nonmandatory Mexicans that are currently sent to DRO. In 2003, 45,925 Mexican noncriminals arrived to DRO [11]. The estimated fraction of noncriminal detainees that are nonmandatory is 0.6949, and so we set \( a_n = 0.6949(45,925) = 31,913/yr \).

We estimate the multiplier factor \( b_n \) by the total potential (i.e., blocked and unblocked) nonmandatory OTM arrival rate to DRO divided by the total potential nonmandatory OTM apprehension rate at the U.S.-Mexico border; for lack of data, we assume that all detentions at the border are nonmandatory. The number of unblocked nonmandatory arrivals to DRO in 2003 was estimated to be 65,976 [11], and hence the number of nonblocked OTM nonmandatory arrivals at DRO was 65,976-31,913=34,063. In addition, 28,000 nonmandatory OTMs were apprehended at the border but were blocked from entering DRO [11]. We also need to estimate the number of nonmandatory OTMs apprehended away from the border but blocked from entering DRO. To obtain this estimate, first note that in 2003, there were 905,065 apprehensions by border patrol agents on the U.S.-Mexico border (Table 37 in [34]), and 882,012 Mexican aliens apprehended by border patrol agents (Table 38 in [34]). If we assume all of these Mexican aliens were apprehended on the U.S.-Mexico border, then the number of
OTMs detained at the border was 905,065 – 882,012 = 23,053. There were 854,976 voluntary departures of Mexicans on the U.S.-Mexico border in 2003 (Table 41 in [34]), and hence the number of Mexicans detained at the border was 882,012 – 854,976 = 27,036 and the total number of aliens detained at the border was 23,053+27,036=50,093. Recalling that there were 65,976 nonmandatory arrivals to DRO in 2003, the remaining 65,976=50,093=15,883 nonmandatory aliens were apprehended elsewhere (e.g., in the interior). If we assume that the likelihood of a potential nonmandatory detainee being blocked from entering DRO is independent of where he was apprehended, then the number of blocked nonmandatory apprehensions occuring away from the border was 28,000, so the estimate $b_n = \frac{70,941}{51,053} = 1.39$.

We assume there are $s = 22,580$ DRO beds, which was allotted in the 2006 budget. The values of the mean mandatory arrival rate $\lambda_m$, the mean residence times $m_m$ and $m_n$, and the relative amplitude $\alpha_d$ were estimated in [11] using 2003 data and appear in Table 1. The only major change at the border since 2003 is a large increase in OTM arrivals, i.e., $\lambda_d = 155k/yr$. Hence, we set $\lambda_d = 0$ in the base case and $\lambda_d = 31,913 + 1.39(155k) = 247,363/yr$, and use equation (33) to solve for the removal probability, which is $P_r = 0.137$.

Finally, U.S. Immigration and Customs Enforcement requested $31M for 950 additional detention beds (and additional support personnel) for fiscal year 2008 [35], and so we set $c_d = \frac{31M}{950} = $32.6k/yr.

### 5.4 Wage Parameters

We assume there are $N_w = 217k$ firms in the U.S. that hire illegal aliens [36]. Estimates of the service rate of a worksite inspector vary from approximately 7 firms per year [37] to nearly 50 per yr [38], and we assume $\mu_w = 20/yr$. The fraction of inspected firms that are inspected in a random manner is taken to be the same as during IRCA, $r_w = 0.4$ [37]. The current penalty is $10,000 per illegal employee, and so in the base case, $f_w = \frac{10,000}{h} = $5/worker-hr, where $h = 2000$ hours worked per worker per year. Also, in the base case, we assume that $m_w = 65$ (its value in 2004 [39]), which leads to $n_w = 1300$ firms investigated per year. The fiscal year 2007 budget requested $41.7M for 171 additional worksite inspectors [40], and so we set $c_w = \frac{41.7M}{171} = $243.9k/yr.

We set the annual illegal wage $w_u$ to be the wage of U.S. immigrants, which is $22,300/yr [26]. In the Cobb-Douglas matching function in equation (47), we set the elasticity with respect to unemployment $\alpha_m = 0.7$ [41]. The labor demand function in equation (37) can be rewritten as $\ln L_d = \ln A_w + (\alpha_w - 1)\ln w$, and we set $\alpha_w - 1$ equal to the wage elasticity of demand, -0.63 [42], to get $\alpha_w = 0.37$.

Although there are 140.8M legal laborers in the U.S. [17], the subset of these workers that are in direct competition for jobs with illegal immigrants is geographically diverse [43] and difficult to estimate directly. In 2005, 29.5% of (i.e., 41.5M) workers aged 18 and older had income < $20k and 39.0% (i.e., 54.9M) had income < $25k [44]. Only a fraction of these
workers are in direct job competition with illegal immigrants, and we use the rough estimate \( N_l = 30M \). Hence, we still have six unknowns: the Cobb-Douglas matching parameter \( A_m \), the illegal labor supply \( N_i \), the equilibrium legal wage \( w \), the Cobb-Douglas labor demand parameter \( A_w \), and the labor supply parameters \( \kappa_1 \) and \( \kappa_2 \). We now state the six equations that can be solved simultaneously for these six unknowns.

By equation (47), the parameter \( A_m \) represents the employment rate among illegals when there are an equal number of unhired workers and vacant jobs. There are 7.20M illegal immigrants working in the U.S. [17] and the first equation sets \( N_i = \frac{7.2M}{A_m} \). The second and third equations are (53)-(54) using the base-case decisions in Table 5. The next two equations compute the labor supply parameters in terms of the legal wage rate. The annual wages of U.S. high school dropouts ($24,800) and Mexican immigrants ($22,300) are comparable [26], and the former have an unemployment rate of 14.3% [17]. If we assume that high school dropouts are competing with illegal immigrants for jobs, then our fourth equation states that the current legal wage generates a 14.3% unemployment rate among the legal labor population:

\[
\kappa_1 - \frac{\kappa_2}{w} = \left(2000 \frac{\text{hr}}{\text{yr}}\right)(1 - 0.143) = 1714 \frac{\text{hr}}{\text{yr}}. \tag{58}
\]

The aggregate labor supply elasticity is believed to be approximately three [45], and our fifth equation states that a 1% increase in the equilibrium wage leads to a 3% increase in the amount of legal labor supplied:

\[
\kappa_1 - \frac{\kappa_2}{1.01w} = 1.03 \left(2000 \frac{\text{hr}}{\text{yr}}\right)(1 - 0.143) = 1765.4 \frac{\text{hr}}{\text{yr}}. \tag{59}
\]

Both sides of equation (53) are equal to 2000 hr/yr times the total number of workers, which is 7.2M illegal workers plus \((1-0.143)N_l\) legal workers. Hence, our final equation is

\[
A_w \left(\frac{1}{w}\right)^{1-\alpha_w} = 2000 \frac{\text{hr}}{\text{yr}} [7.2M + (1 - 0.143)N_l]. \tag{60}
\]

Solving these six equations jointly for the six unknown parameters yields an employment rate of \( A_m = 0.927 \), an illegal labor supply of \( N_i = 7.76M \), an equilibrium legal wage of \( w = 11.24/\text{hr} \) (and hence, by equation (54), an equilibrium illegal wage in the base case of \( w_u = $22.3k/\text{yr} \), or 11.15$/hr), the Cobb-Douglas labor demand parameter \( A_w = $3.9 \times 10^{12}/\text{yr} \), and the labor supply parameters \( \kappa_1 = 6905 \text{ hr/yr} \) and \( \kappa_2 = $58.3k/\text{yr} \).
References


<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>Time horizon</td>
<td>2 yr</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Discrete choice parameter</td>
<td>$6.83 \times 10^{-5}/$</td>
</tr>
<tr>
<td>$w_o$</td>
<td>Wage in home country</td>
<td>$5000/yr$</td>
</tr>
<tr>
<td>$w_u$</td>
<td>Illegal wage in U.S. in base case</td>
<td>$22.3k/yr$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>One-way cost of Mexican migration</td>
<td>$1500$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>One-way cost of OTM</td>
<td>$2500$</td>
</tr>
<tr>
<td>$d_1$</td>
<td>Cost of Mexican detention and removal</td>
<td>$4476$</td>
</tr>
<tr>
<td>$d_1$</td>
<td>Cost of Mexican detention</td>
<td>$1509$</td>
</tr>
<tr>
<td>$d_2$</td>
<td>Cost of OTM detention and removal</td>
<td>$8983$</td>
</tr>
<tr>
<td>$f$</td>
<td>Fraction of detention cost incurred</td>
<td>0.46</td>
</tr>
<tr>
<td>$n_1$</td>
<td>Potential Mexican border crossers per year</td>
<td>5.14M/yr</td>
</tr>
<tr>
<td>$n_2$</td>
<td>Potential OTM border crossers per year</td>
<td>874k/yr</td>
</tr>
<tr>
<td>$\lambda_b$</td>
<td>Attempted annual border crossings by Mexicans</td>
<td>5.80M/yr</td>
</tr>
<tr>
<td>$\lambda_{b2}$</td>
<td>Attempted annual border crossings by OTMs</td>
<td>775k/yr</td>
</tr>
</tbody>
</table>

Table 1: Values for parameters in the Discrete Choice Submodel.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{a1}, P_{a2}$</td>
<td>Apprehension probability in base case</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda_{d1}$</td>
<td>Mexican apprehension rate ($\geq a^{th}$ time)</td>
<td>0/yr</td>
</tr>
<tr>
<td>$\lambda_{d2}$</td>
<td>OTM apprehension rate</td>
<td>155k/yr</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of border</td>
<td>1933 miles</td>
</tr>
<tr>
<td>$m_b$</td>
<td>Mean time to apprehend an alien</td>
<td>1 hr</td>
</tr>
<tr>
<td>$k$</td>
<td>Cost to travel along the border</td>
<td>$4/mile$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Frequency of sinusoidal arrival rates</td>
<td>$5.17 \times 10^{-3}$/mile</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>Relative amplitude of sinusoidal arrival rates</td>
<td>0.951</td>
</tr>
<tr>
<td>$\hat{\alpha}_b$</td>
<td>Relative amplitude of sinusoidal border patrol agent density</td>
<td>0.951</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Cost per border patrol agent</td>
<td>$1.173M/yr$</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Cost for surveillance technology</td>
<td>$30k/mile\cdot yr$</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Exponential apprehension parameter without technology</td>
<td>9.08/mile</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Exponential apprehension parameter with technology</td>
<td>0.29/mile</td>
</tr>
</tbody>
</table>

Table 2: Values for parameters in the Apprehension Submodel.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n$</td>
<td>Constant factor for nonmandatory detainees</td>
<td>31,913/yr</td>
</tr>
<tr>
<td>$b_n$</td>
<td>Multiplicative factor for nonmandatory detainees</td>
<td>1.39</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>Mean arrival rate of nonmandatory detainees</td>
<td>144,323/yr</td>
</tr>
<tr>
<td>$\bar{\lambda}_m$</td>
<td>Mean arrival rate of mandatory detainees</td>
<td>247,363/yr</td>
</tr>
<tr>
<td>$m_m$</td>
<td>Mean residence time of mandatory detainees</td>
<td>45.8 days</td>
</tr>
<tr>
<td>$m_n$</td>
<td>Mean residence time of nonmandatory detainees</td>
<td>48.0 days</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>Frequency of sinusoidal arrival rates</td>
<td>1/yr</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>Relative amplitude of sinusoidal arrival rates</td>
<td>0.1474</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Removal probability in base case</td>
<td>0.137</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Cost per DRO bed</td>
<td>$32.6k/yr</td>
</tr>
</tbody>
</table>

Table 3: Values for parameters in the Removal Submodel. Under Expedited Removal, $m_n$ is reduced to 19.0 days [46].

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_w$</td>
<td>Service rate of worksite inspectors</td>
<td>20/yr</td>
</tr>
<tr>
<td>$N_w$</td>
<td>Total number of firms hiring illegals</td>
<td>217k</td>
</tr>
<tr>
<td>$n_w$</td>
<td>Total number of firms inspected per year</td>
<td>1300/yr</td>
</tr>
<tr>
<td>$h$</td>
<td>Hours worked per worker per year</td>
<td>2000 hr/yr</td>
</tr>
<tr>
<td>$A_w$</td>
<td>Linear parameter for Cobb-Douglas demand model</td>
<td>$3.9 \times 10^{12}$/yr</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>Exponential parameter for Cobb-Douglas demand model</td>
<td>0.37</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>Labor supply parameter</td>
<td>6905 hr/yr</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>Labor supply parameter</td>
<td>$58.3k/yr$</td>
</tr>
<tr>
<td>$A_m$</td>
<td>Linear parameter for Cobb-Douglas matching model</td>
<td>0.927</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>Exponential parameter for Cobb-Douglas matching model</td>
<td>0.7</td>
</tr>
<tr>
<td>$w$</td>
<td>Legal wage rate in base case</td>
<td>$11.24/hr</td>
</tr>
<tr>
<td>$N_l$</td>
<td>Legal labor supply</td>
<td>30M</td>
</tr>
<tr>
<td>$N_i$</td>
<td>Illegal labor supply</td>
<td>7.76M</td>
</tr>
<tr>
<td>$c_w$</td>
<td>Cost per worksite inspector</td>
<td>$243.9k/yr$</td>
</tr>
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Table 4: Values for parameters in the Illegal Wage Submodel.
<table>
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<th>Notation</th>
<th>Description</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Number of apprehensions until detention of Mexican</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$n_b$</td>
<td>Number of border patrol agents at border</td>
<td>1636</td>
</tr>
<tr>
<td>$s_b$</td>
<td>Length of border monitored by surveillance technology</td>
<td>290 miles</td>
</tr>
<tr>
<td>$s$</td>
<td>Number of DRO beds</td>
<td>22,580</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Fraction of worksite inspectors that are random</td>
<td>0.4</td>
</tr>
<tr>
<td>$m_w$</td>
<td>Number of worksite inspectors</td>
<td>65</td>
</tr>
<tr>
<td>$f_w$</td>
<td>Fine per illegal worker-hour</td>
<td>$5$/worker-hr</td>
</tr>
<tr>
<td>$\Delta_l$</td>
<td>Number of illegal workers legalized</td>
<td>0</td>
</tr>
<tr>
<td>$N_g$</td>
<td>Number of new guest workers</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: Decision variables and their base-case values.
Figure Legends

Fig. 1. Border-crossing behavior in the base case. The OTMs’ arrival rate to each location on the border ($\lambda_b(x)$ in equation (16)), which is proportional to the density of border patrol agents at each location on the border ($n_b(x)$ in equation (17)), and the OTMs’ crossing rate at each location on the border ($\lambda_c(x)$ in equation (25)). Also depicted at the top of the plot is the spatial location of surveillance technology ($I_x$ in equation (18)).

Fig. 2. Impact of the Mexican detention policy. The probability that a terrorist successfully enters the U.S. vs. the number of apprehensions until a Mexican border crosser is detained (i.e., is placed in a DRO facility if there is available space). Other decision variables are set at their base-case values.

Fig. 3. Impact of isolated changes in non-worksite decision variables. The probability that a terrorist successfully enters the U.S. vs. (a) the number of border patrol agents on the border, (b) the number of miles of border monitored by surveillance technology, and (c) the number of DRO beds. Other decision variables are set at their base-case values.

Fig. 4. The optimal $P_T$ vs. cost curve in the base case (i.e., as in Fig. 1 in the main text, where $\alpha_b = 0.951$) and when border patrol agents are distributed evenly across the border (i.e., when the relative amplitude $\alpha_b = 0$).

Fig. 5. Annual illegal wage vs. number of worksite inspectors for various fractions of inspections that are not targeted ($r_w$). The fine $f_w$ is (a) $5$/worker-hr and (b) $25$/worker-hr.

Fig. 6. Annual illegal wage vs. number of worksite inspectors for various numbers of legalized workers ($\Delta_l$, which is expressed as a fraction of the illegal labor supply $N_i$) and new guest workers ($N_g$). The fine $f_w$ is (a) $5$/worker-hr and (b) $25$/worker-hr.

Fig. 7. The OTM crossing probability vs. the annual illegal wage for various values of the multinomial logit parameter, $\theta$.

Fig. 8. These curves compare the impact of marginal investments in border patrol agents vs. worksite inspectors on (a) the apprehension probability $P_{a2}$, (b) the OTM crossing probability $P_{21}$, and (c) the alternative objective function $P_{21}[1 - P_{a2} + P_{a2}(1 - P_T)]$, which is proportional to the number of OTMs who successfully sneak into the U.S. In each case, the initial annual budget of $3.6B uses technology along the entire border, has evenly spaced agents (i.e., $\alpha_b = 0$, and has sufficient DRO beds so that $P_r = 1$ (and the alternate objective function equals $P_{21}(1 - P_{a2})$), but has base-case values of 1636 border patrol agents and 65 worksite inspectors. Each curve is generated by adding either border patrol agents or worksite inspectors.

Fig. 9. The optimal budget allocation across border patrol agents, border technology, and DRO beds in the case where 50% of the apprehended aliens are identifiable (see §4.8 in the main text). The numbers appearing along the curves are the optimal number of agents (on the border at any given time), miles of technology, and DRO beds at various budgets. The * denotes the base-case allocation with Expedited Removal, which represents current practice.
FIGURE 1
FIGURE 2

The graph shows the probability that a terrorist successfully enters the U.S., denoted by $P_T$, as a function of the number of apprehensions until the detention of a Mexican, denoted by $a$. The probability decreases sharply as the number of apprehensions increases, indicating a higher likelihood of interception with more apprehensions.
FIGURE 3
FIGURE 4

annual budget ($ billions)

probability that terrorist successfully enters the U.S., $P_T$

$\tilde{\alpha}_b = 0.951$

$\tilde{\alpha}_b = 0$
FIGURE 5
\[ \Delta_l = 0, \quad N_g = 0 \]
\[ \Delta_l = 0, \quad N_g = 4M \]
\[ \Delta_l = 0, \quad N_g = 8M \]
\[ \Delta_l = 0.5N, \quad N_g = 0 \]
\[ \Delta_l = 0.5N, \quad N_g = 4M \]
\[ \Delta_l = 0.5N, \quad N_g = 8M \]
\[ \Delta_l = N, \quad N_g = 0 \]
\[ \Delta_l = N, \quad N_g = 4M \]
\[ \Delta_l = N, \quad N_g = 8M \]

(a)

(b)

FIGURE 6
FIGURE 7
FIGURE 8
probability that terrorist successfully enters the U.S., $P_T$

fraction allocated to DRO beds
fraction allocated to border patrol agents
fraction allocated to technology

base case: $P_T = .887$, budget = $2.7$ billion

FIGURE 9