

APPENDIX

This appendix formulates the mathematical model described in the main text. The Discrete Choice Submodel for aliens is given in §1, the Apprehension Submodel appears in §2, the Removal Submodel is depicted in §3, and the Illegal Wage Submodel is derived in §4. The submodels in these four sections correspond to relations (1)-(4) in the main text, respectively. Parameter estimation is carried out in §5 (parameters are listed in Tables 1-5), and supporting computational results are in Figures 1-9.

1 Discrete Choice Submodel

For Mexicans ($i = 1$) and OTMs ($i = 2$), we assume there are n_i aliens that are considering entering the U.S. illegally. In this section, we introduce two discrete choice models, one for Mexicans and one for OTMs, that specify the fraction of these potential border crossers that choose to illegally enter the U.S. ($j = 1$) and the fraction that choose to stay at home ($j = 2$). We need two models because apprehended Mexicans are typically offered voluntary departure, i.e., they are allowed to withdraw to Mexico without penalty rather than being detained, whereas OTMs cannot be returned to Mexico. We begin with the simpler model, which is for OTMs.

1.1 OTM Decision Model

We use the multinomial-logit model, which is the most widely used random utility model, particularly in the area of consumer choice (e.g., [1]). This model implicitly captures the heterogeneity in preferences, resources (e.g., money to buy fraudulent documents or hire a coyote, i.e., a human smuggler) and perceptions (e.g., about job opportunities or risk of being apprehended while crossing the border) among the population of potential border crossers. The model assumes that the utility for an OTM from choosing option j has a deterministic component u_{2j} plus a random component that is an independent and identically distributed (iid) logistic random variable with mean zero. According to the model, the probability P_{2j} that an OTM will choose option j , in terms of the expected utility u_{2j} received from this option and the scale parameter θ , is

$$P_{2j} = \frac{e^{\theta u_{2j}}}{e^{\theta u_{21}} + e^{\theta u_{22}}} \quad \text{for } j = 1, 2. \quad (1)$$

This model becomes deterministic as $\theta \rightarrow \infty$ and becomes a pure random choice model as $\theta \rightarrow 0$. Equation (1) is closely related to the logistic regression models used for analyzing the apprehension probability at the border, which replace u_{ij} by a linear function of independent variables.

Because the great majority of illegal immigrants are looking for a job in the U.S., the expected utilities are taken to be the expected undiscounted earnings over τ years minus the cost of migration. Let w_u be the mean annual wage of an illegal immigrant in the U.S., which is computed in §4, and let w_o be the mean annual wage in the home country; in our

model, these two quantities do not depend on whether the alien is Mexican or an OTM. If the potential crosser decides to stay at home, his utility is

$$u_{22} = w_o\tau. \quad (2)$$

Let c_2 denote the one-way cost of migration for an OTM. In the case where a crosser is apprehended, detained and removed, he does not pay his way home but incurs a cost d_2 due to lost wages in the home country and the physical and psychological toll of detention. We let P_{ai} be the probability that an alien of type i is apprehended while crossing the border; this quantity is computed in §2. Because OTMs are not offered voluntary departure to Mexico, apprehended OTMs are supposed to be detained and then removed to their home country. However, due to lack of beds in the detention and removal operations (DRO), an apprehended OTM will only be detained and removed with probability P_r , which is computed in §3. Otherwise, the apprehended OTM is released into the interior of the U.S., where he is free to (illegally) seek work; however, before being released, he incurs a cost of fd_2 , where f is the fraction of the total detention cost d_2 incurred before removal. Because f varies with the the number of DRO beds, which is a decision variable in our model, in a complex way, in §5.1 we approximate f by means of a quadratic function of the removal probability P_r . Hence, the expected utility for an OTM if he decides to attempt an illegal crossing is

$$u_{21} = (1 - P_{a2})(w_u\tau - c_2) + P_{a2}(1 - P_r)(w_u\tau - c_2 - fd_2) + P_{a2}P_r(w_o\tau - c_2 - d_2). \quad (3)$$

1.2 Mexican Decision Model

We assume that an unauthorized Mexican is offered voluntary departure to Mexico the first $a - 1$ times that he is apprehended, where a is a decision variable for the U.S. Government. Upon being apprehended for the a^{th} time, a Mexican alien is detained if there are DRO beds available and is offered voluntary departure to Mexico if there are no DRO beds available. Moreover, after the Mexican alien is detained, there is still the possibility for him to be forced out of DRO before his removal due to lack of bedspace, and in this case we assume that he is offered voluntary departure when he is forced out of DRO. We allow Mexicans who accept voluntary departure after apprehension (and perhaps some detention) to make the choice of whether to cross again or to stay in Mexico.

As explained in §5.1, we set $a = \infty$ in the base case, so that border patrol agents turn apprehended migrants back into Mexico. In this case, we assume that migrants keep crossing until they succeed, as in [2]. Because this case is much simpler than when a is finite, we treat it separately. When $a = \infty$, the probability that a Mexican will choose option j is

$$P_{1j} = \frac{e^{\theta u_{1j}}}{e^{\theta u_{11}} + e^{\theta u_{12}}} \quad \text{for } j = 1, 2, \quad (4)$$

where the utility from choosing option 2 is

$$u_{12} = w_o\tau. \quad (5)$$

If we let \tilde{d}_1 be the cost incurred each time an alien is apprehended at the border and returned to Mexico, then the expected utility from choosing option 1 is

$$u_{11} = w_u\tau - c_1 - \frac{P_{a1}}{1 - P_{a1}}\tilde{d}_1. \quad (6)$$

Now we turn to the case where a is finite. For stage $k = 1, 2, \dots$, let $u_{1j}^{(k)}$ denote the expected utility from stage k onward for a Mexican immigrant if he chooses option j at stage k , i.e., after being apprehended $k - 1$ times, and let $P_{1j}^{(k)}$ be the probability of making this choice. As in equation (3), we use the multinomial-logit model and get

$$P_{1j}^{(k)} = \frac{e^{\theta u_{1j}^{(k)}}}{e^{\theta u_{11}^{(k)}} + e^{\theta u_{12}^{(k)}}} \quad \text{for } j = 1, 2 \quad \text{and } k = 1, \dots, a. \quad (7)$$

If we view c_1 as the one-way cost between a Mexican's hometown and the U.S.-Mexico border, then $u_{12}^{(1)} = w_o\tau$ because the alien never travels to the border in this case, and $u_{12}^{(k)} = w_o\tau - c_1$ for $k \geq 2$ because the cost to get to the border is already sunk by the time he has been apprehended $k - 1$ times, but the cost to get home from the border still needs to be incurred for choice $j = 2$.

In the remainder of this subsection, we formulate and solve a recurrent relation between the $P_{11}^{(k)}$'s and the $u_{11}^{(k)}$'s. For stage $k = 1, \dots, a$, if the alien is apprehended then he is returned to Mexico and incurs the cost \tilde{d}_1 , and his expected utility at stage $k + 1$ will be $P_{11}^{(k+1)}u_{11}^{(k+1)} + P_{12}^{(k+1)}u_{12}^{(k+1)}$. Because the one-way travel cost is sunk after the alien arrives at the border, if we define $I_{\{x\}}$ to be the indicator function of the event x , then we have the relation

$$u_{11}^{(k)} = (1 - P_{a1})w_u\tau + P_{a1}(P_{11}^{(k+1)}u_{11}^{(k+1)} + P_{12}^{(k+1)}u_{12}^{(k+1)} - \tilde{d}_1) - c_1I_{\{k=1\}} \quad \text{for } k = 1, \dots, a-1. \quad (8)$$

Beginning at stage a , if the alien still chooses to cross, he risks being detained and removed. With probability $P_{a1}P_r$, he will be detained and removed and have utility $w_o\tau - c_1I_{\{k=1\}} - d_1$, where d_1 is the detection cost for Mexicans. With probability $P_{a1}(1 - P_r)$, he will not be detained in DRO until removal and will be offered voluntary departure after incurring a loss of fd_1 ; he can then make a decision at stage $a + 1$. Hence, at stage a the relation is

$$u_{11}^{(a)} = (1 - P_{a1})w_u\tau + P_{a1}(1 - P_r)(P_{11}^{(a+1)}u_{11}^{(a+1)} + P_{12}^{(a+1)}u_{12}^{(a+1)} - fd_1) + P_{a1}P_r(w_o\tau - d_1) - c_1I_{\{k=1\}}. \quad (9)$$

Moreover, because aliens face the same situation at each stage $k \geq a$, we have $u_{11}^{(k)} = u_{11}^{(a)}$ and $P_{11}^{(k)} = P_{11}^{(a)}$ for all $k \geq a$, where

$$u_{11}^{(a)} = (1 - P_{a1})w_u\tau + P_{a1}(1 - P_r)(P_{11}^{(a)}u_{11}^{(a)} + P_{12}^{(a)}u_{12}^{(a)} - fd_1) + P_{a1}P_r(w_o\tau - d_1) - c_1I_{\{k=1\}}. \quad (10)$$

Because $P_{12}^{(k)} = 1 - P_{11}^{(k)}$, equations (7), (8) and (10) are a system of $2a$ equations in terms of the $2a$ unknowns, $u_{11}^{(k)}, P_{11}^{(k)}$ for $k = 1, \dots, a$. We solve these equations using backwards recursion in a fashion reminiscent of optimal stopping problems [3]; indeed, the decision problem faced by an individual Mexican is an optimal stopping problem, but we are solving this problem over the aggregate Mexican alien population using the multinomial-logit model. First, we jointly solve equation (7) with $k = a$ and equation (10) for $P_{11}^{(a)}$ and $u_{11}^{(a)}$; the existence of a unique solution to these two equations, and an approach to numerically solve them, is shown in [4]. With $P_{11}^{(a)}$ and $u_{11}^{(a)}$ in hand, we use equation (8) for $k = a - 1$ to get $u_{11}^{(a-1)}$, and then use equation (7) to get $P_{11}^{(a-1)}$, and then recursively solve for $u_{11}^{(a-2)}, P_{11}^{(a-2)}, \dots$ in a similar manner until we obtain $u_{11}^{(1)}$ and $P_{11}^{(1)}$.

2 The Apprehension Submodel

The goal of this section is to compute the apprehension probability P_{ai} at the border for type i aliens. Our macro approach to the apprehension probability captures the impact of the alien flow and enforcement effort (both labor and technology), and ignores the micro-structural variables that provide insight into the personal and community characteristics of the types of people who are apt to be apprehended at the border; see [5] for an example of the latter approach. Espenshade and Acevedo [6] obtain a R^2 of 0.768 in a regression model that includes alien flow and enforcement effort. Our model also incorporates the fact that aliens adapt to the presence of enforcement effort by (often with the help of coyotes) choosing more remote routes [7].

Although the U.S. Government deploys a variety of technologies, including several helicopters and drones, for detecting border crossers, we focus on the Integrated Surveillance Intelligence Systems (ISIS), which are remote video surveillance systems consisting of a central command center and two infrared and two daytime cameras spanning a 3-5 mile radius [8]. As of 2005, these surveillance systems monitored approximately 15% of the U.S.-Mexico border [9], and hence are more prevalent than helicopters and drones. Although the Government has over 10,000 seismic, magnetic and thermal sensors on the U.S.-Mexico border [8], in areas without surveillance technology, border patrol agents need to travel – sometimes considerable distances – to investigate all sensor alarms, which are often nuisance alarms (e.g., animals); consequently, these sensor alarms are frequently ignored in these areas [8]. In areas with surveillance technology, the operator can use the video cameras to investigate the sensor alarms, which greatly enhances the effectiveness of these sensors [8].

We model the border as a straight line segment of length L . For $x \in [0, L]$, let the indicator function $I_{\{x\}}$ equal 1 if location x is monitored by surveillance technology, and equal 0 otherwise. Let $n_b(x)$ be the density of border patrol agents at location $x \in [0, L]$ in units of miles⁻¹, where $n_b = \int_0^L n_b(x) dx$ is a decision variable that represents the total number of border patrol agents (working at one time). Similarly, for $x \in [0, L]$, let $\lambda_{bi}(x)$ be the arrival rate of type i aliens ($i = 1$ is Mexican, $i = 2$ is OTM) at location x , which has units of miles⁻¹ × time⁻¹, where $\lambda_{bi} = \int_0^L \lambda_{bi}(x) dx$ is the total number of type i crossings per year. Due to the complexity of the model in this section, we do not capture the fact that arrivals to the border are seasonal; §3 incorporates seasonality of arrivals to DRO because it plays a bigger factor there than at the border (at least in the base case) due to the extreme lack of bedspace.

We can determine λ_{bi} from our analysis in §1. Recall that n_i is the total annual number of potential border crossers of type i . Although the number of annual OTM border crossers is a binomial random variable with parameters n_2 and P_{21} , we simply assume that

$$\lambda_{b2} = n_2 P_{21} \tag{11}$$

because the variability in the binomial random variable can be safely ignored: $n_i \approx 10^6$, $P_{i1} \approx 0.9$, and hence three standard deviations divided by the mean $\approx \frac{3\sqrt{10^5}}{10^6} \approx 10^{-3}$. Similarly, if $a = \infty$, then we assume that

$$\lambda_{b1} = \frac{n_1 P_{11}}{1 - P_{a1}}. \tag{12}$$

If a is finite, the expected number of k^{th} crossings by Mexicans is $n_1 P_a^{k-1} \prod_{i=1}^k P_{11}^{(i)}$ for $k = 1, \dots, a$. For $k > a$, there is the possibility of detention and removal, and the expected number of k^{th} crossings is $n_1 P_{a1}^{a-1} \prod_{i=1}^a P_{11}^{(i)} (P_{a1}(1 - P_d) P_{11}^{(a)})^{k-a}$. Therefore, the arrival rate of illegal Mexican aliens to the border is (again, ignoring the variability in the binomial random variable)

$$\lambda_{b1} = n_1 \left[\sum_{k=1}^a P_{a1}^{k-1} \prod_{i=1}^k P_{11}^{(i)} + \sum_{k=a+1}^{\infty} (P_{a1}^{a-1} \prod_{i=1}^a P_{11}^{(i)}) (P_{a1}(1 - P_d) P_{11}^{(a)})^{k-a} \right], \quad (13)$$

$$= n_1 \left[\sum_{k=1}^{a-1} P_{a1}^{k-1} \prod_{i=1}^k P_{11}^{(i)} + \sum_{k=a}^{\infty} (P_{a1}^{a-1} \prod_{i=1}^a P_{11}^{(i)}) (P_{a1}(1 - P_d) P_{11}^{(a)})^{k-a} \right], \quad (14)$$

$$= n_1 \left[\sum_{k=1}^{a-1} P_{a1}^{k-1} \prod_{i=1}^k P_{11}^{(i)} + \frac{P_{a1}^{a-1} \prod_{i=1}^a P_{11}^{(i)}}{1 - P_{a1}(1 - P_d) P_{11}^{(a)}} \right]. \quad (15)$$

An alien arriving at location x will not necessarily cross there. We let $\lambda_{ci}(x)$ be the total crossing rate at location x (in units of miles $^{-1}$ \times time $^{-1}$), where $\int_0^L \lambda_{ci}(x) dx = \lambda_{bi}$. We are essentially modeling this process as a Stackelberg game with the U.S. Government as leader (choosing $n_b(x)$) and the aliens as followers (choosing where to cross, i.e., $\lambda_{ci}(x)$); as in §1.2, the aliens are not behaving in a coordinated manner, but rather their aggregate behavior is modeled using a multinomial logit model (see (24) below). However, we can think of this game as being part of an ongoing sequence of Stackelberg games. If we think in terms of discrete time periods, in each time period there are three decisions: first, the government chooses the spatial allocation of agents ($n_b(x)$), then the aliens choose where to arrive ($\lambda_{bi}(x)$), and finally the aliens choose where to cross ($\lambda_{ci}(x)$). It is natural to assume that the government adapts in period t by choosing $n_b(x)$ to be proportional to the $\lambda_{c1}(x) + \lambda_{c2}(x)$ from period $t - 1$ (i.e., $\frac{n_b(x)}{\lambda_{c1}(x) + \lambda_{c2}(x)}$ is a constant for all x). Similarly, it is not unreasonable to assume that the aliens choose $\lambda_{bi}(x)$ in period t to be proportional to the $\lambda_{ci}(x)$ from period $t - 1$. That is, our model assumes that the U.S. Government places its border patrol agents in regions that have experienced the most crossings recently, and that crossers (via word-of-mouth or coyotes) arrive at the border at locations where the recent crossings have occurred.

There are very busy areas and very quiet areas along the border. To capture the spatial heterogeneity in the simplest possible way, we assume that both $\lambda_{bi}(x)$ and $n_b(x)$ are sinusoidal functions with the same frequency ω_b (where $\omega_b L$ is an integer) and relative amplitudes $\alpha_b \in [0, 1]$ and $\tilde{\alpha}_b \in [0, 1]$:

$$\lambda_{bi}(x) = \frac{\lambda_{bi}}{L} + \frac{\lambda_{bi}}{L} \alpha_b \sin(2\pi\omega_b x) \quad \text{for } x \in [0, L], \quad (16)$$

$$n_b(x) = \frac{n_b}{L} + \frac{n_b}{L} \tilde{\alpha}_b \sin(2\pi\omega_b x) \quad \text{for } x \in [0, L]. \quad (17)$$

For simplicity, we assume that $\lambda_{bi}(x)$ and $n_b(x)$ are proportional to each other, which would be the case if $\lambda_{c1}(x)$ and $\lambda_{c2}(x)$ were proportional to each other (although they are not). These assumptions enable us to optimize $\lambda_{bi}(x)$ and $n_b(x)$ in a computationally tractable manner, via the parameters α_b and $\tilde{\alpha}_b$.

We introduce the decision variable s_b , which is the number of miles along the border that are monitored by surveillance technology. We assume that surveillance technology is implemented at the busiest parts of the border, so that

$$I_{\{x\}} = \begin{cases} 1 & \text{if } x \in \left[\left(\frac{i}{\omega_b} + \frac{1}{4\omega_b} \right) - \frac{1}{2\omega_b} \left(\frac{s_b}{L} \right), \left(\frac{i}{\omega_b} + \frac{1}{4\omega_b} \right) + \frac{1}{2\omega_b} \left(\frac{s_b}{L} \right) \right] \text{ for } i = 0, \dots, \omega_b L - 1; \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

The main modeling challenge is to determine $\lambda_{ci}(x)$ given both $\lambda_{bi}(x)$ and $n_b(x)$. Let $u_i(x, y)$ be the utility of a type i alien who arrives at location x (we assume the cost to get to location x on the border is sunk) and crosses at location y . Let $P_a(y)$ be the apprehension probability at location $y \in [0, L]$ and let k be the cost of traveling (in units of miles⁻¹) for aliens of type $i = 1, 2$. Then the utility for OTMs is given by

$$u_2(x, y) = (1 - P_a(y))w_u\tau + P_a(y)((1 - P_r)(w_u\tau - fd_2) + P_r(w_o\tau - d_2)) - k|x - y|. \quad (19)$$

For simplicity, we assume that Mexicans are not detained and will keep attempting to cross (at their chosen location) until they succeed, and that each time they are apprehended they are immediately returned to Mexico and suffer a cost \tilde{d}_1 . In essence, in this submodel we assume that if the U.S. Government became more aggressive about detaining Mexicans (e.g., reduced a to 1 or 2), then this would deter Mexicans at the initial decision stage of whether or not to travel to the border, and those who traveled to the border would be undeterred. Hence, we have

$$u_1(x, y) = w_u\tau - \frac{P_a(y)}{1 - P_a(y)}\tilde{d}_1 - k|x - y|. \quad (20)$$

At location y , there is a border patrol agent every $\frac{1}{n_b(y)}$ miles. Each agent can be viewed as the server in a single-server loss queueing system with (at this point unknown) arrival rate $\frac{P_a(y)[\lambda_{c1}(y) + \lambda_{c2}(y)]}{n_b(y)}$ and mean apprehension time m_b , which is only incurred when aliens are actually apprehended (i.e., aliens that go undetected, or are detected but not apprehended, experience no service time, because most of the apprehension time is devoted to transporting – or waiting for transportation for – the apprehended alien). It follows that the probability that the agent is busy with an apprehension is $\frac{\rho_b}{1 + \rho_b}$, where

$$\rho_b(y) = \frac{P_a(y)[\lambda_{c1}(y) + \lambda_{c2}(y)]m_b}{n_b(y)}. \quad (21)$$

We now derive an expression for the apprehension probability at location y , $P_a(y)$. If a crossing occurs while the agent at this location is busy with another apprehension, then the agent will not apprehend the crosser. We assume that the arrivals near location y follow a spatially homogeneous Poisson process and the location of the agent is random (i.e., he spends his time driving back and forth along the $\frac{1}{n_b(y)}$ miles), so that the distance between the agent and the alien at the time of crossing is the random distance between any two points that are uniformly distributed on $[0, \frac{1}{n_b(y)}]$, which has pdf

$$f(x) = \frac{2\left(\frac{1}{n_b(y)} - x\right)}{\left(\frac{1}{n_b(y)}\right)^2}. \quad (22)$$

If there is no technology deployed at location y (i.e., $I_{\{y\}} = 0$), then we assume that an idle agent will detect and apprehend the crosser with probability $e^{-\alpha_1 x}$ if the agent and alien are at a distance x apart at the time of crossing (i.e., apprehension occurs if the distance between them is less than an exponential random variable with parameter α_1). Similarly, if there is technology deployed at location y (i.e., $I_{\{y\}} = 1$), then we assume that an idle agent will detect and apprehend the crosser with probability $e^{-\alpha_2 x}$ if the agent and alien are at a distance x apart at the time of crossing, where it is natural to expect that $\alpha_2 < \alpha_1$ (i.e., technology aids in detection and apprehension). Taken together, the probability of apprehension at location y is

$$P_a(y) = \frac{1}{1 + \rho_b(y)} \left[I_{\{y\}} \int_0^{\frac{1}{n_b(y)}} f(x) e^{-\alpha_2 x} dx + (1 - I_{\{y\}}) \int_0^{\frac{1}{n_b(y)}} f(x) e^{-\alpha_1 x} dx \right]. \quad (23)$$

We now use a multinomial logit model with a continuum of choices to compute $P_{ci}(x, y)$, which is the probability that an alien arriving at location x will cross at location y . This model is given by

$$P_{ci}(x, y) = \frac{e^{\theta u_i(x, y)}}{\int_0^L e^{\theta u_i(x, y)} dy} \quad \text{for } i = 1, 2. \quad (24)$$

Hence, the crossing rate of a type i alien at location y is given by

$$\lambda_{ci}(y) = \int_0^L \lambda_{bi}(x) P_{ci}(x, y) dx. \quad (25)$$

Equation (25) is a fixed-point equation for $\lambda_{ci}(y)$ because $P_{ci}(x, y)$ on the right side of (25) is a function of $u_i(x, y)$ in equation (24), which is a function of $P_a(y)$ in equations (19)-(20), which is a function of $\rho_b(y)$ in (23), which is a function of $\lambda_{ci}(y)$ in (21).

Finally, we obtain the apprehension probability P_{ai} for type i aliens, which is

$$P_{ai} = \frac{\int_0^L \lambda_{ci}(y) P_a(y) dy}{\lambda_{bi}}. \quad (26)$$

Equation (26) corresponds to equation (2) in the main text.

3 The Removal Submodel

The goal of this section is to compute P_r , the probability that an illegal alien apprehended at the border is detained and removed, given that the U.S. Government would like to detain and remove him (i.e., given that he is either Mexican on at least his a^{th} apprehension or an OTM). The DRO facilities, which consist of eight large facilities in the U.S. plus contracted beds at a variety of other facilities, house aliens while they are waiting for removal proceedings to be completed. Most detained OTMs are ready to be voluntarily removed, but need to be detained until the home country verifies their identification (most come with no identification). There are mandatory detainees and non-mandatory detainees. Nearly all criminals are mandatory detainees, and some noncriminals are mandatory [10]. We model this facility as the following 2-class, infinite-server, partial-loss queueing system that has been analyzed

elsewhere [11]. Because nearby DRO facilities are used if the closest facility is full, we model this system by a single pooled queue, which should be an accurate approximation [12, 13]. We let aliens of class i ($i = m$ is mandatory, $i = n$ is nonmandatory) arrive according to a sinusoidal Poisson arrival process with average rate $\bar{\lambda}_i$, frequency ω_d and relative amplitude α_d ; i.e., the arrival rate for class i aliens at time t is $\bar{\lambda}_i + \bar{\lambda}_i \alpha_d \sin(2\pi\omega_d t)$.

Let λ_{d1} be the rate at which Mexicans are apprehended at the border on at least their a^{th} apprehension and let λ_{d2} be the rate at which OTMs are apprehended at the border. We know from §1.1 that

$$\lambda_{d2} = n_2 P_{21} P_{a2}. \quad (27)$$

Similarly, λ_2 equals the expected number of a^{th} and later crossings multiplied by apprehension probability, which by (15) is

$$\lambda_{d1} = n_1 \frac{P_{a1}^{a-1} \prod_{i=1}^a P_{11}^{(i)}}{1 - P_{a1}(1 - P_d) P_{11}^{(a)}} P_{a1} \quad (28)$$

if a is finite, and $\lambda_{d1} = 0$ if $a = \infty$.

Aliens arriving to DRO are apprehended not only at the U.S.-Mexico border, but also at the other borders, at the ports-of-entry, in the interior, and from jails (i.e., they have finished serving jail time but are yet to be removed). Hence, we need to determine a relationship between the mean arrival rates of mandatory and nonmandatory aliens to DRO, $\bar{\lambda}_m$ and $\bar{\lambda}_n$, and the U.S.-Mexico border rates in equations (27) and (28). Because most of the criminals entering DRO come directly from jails, not from the border, we assume that λ_m is an exogenous parameter that is independent of λ_{di} . We assume that

$$\bar{\lambda}_n = a_n + b_n[\lambda_{d1} + \lambda_{d2}], \quad (29)$$

where the constants a_n and b_n are computed in §5.3 using DRO data. Equation (29) implicitly assumes that the nonmandatory apprehensions away from the U.S.-Mexico border are roughly proportional to the nonmandatory apprehensions at the U.S.-Mexico border, as they were before and after the Hold The Line operation [6]. The rationale is that as the number of aliens apprehended at the border increases, the number that successfully cross the border also increases, which should increase the number of apprehensions that occur in the interior.

The residence time for class i aliens is an exponential random variable with mean m_i for $i = \{m, n\}$. When a mandatory detainee arrives to the system he is detained. If there is not sufficient bed space, then one is rented and he is moved into a DRO bed later if room becomes available [8]. When a nonmandatory detainee arrives to the system, he is blocked from entering the system if no beds are available. As explained in §1, a blocked Mexican is returned to Mexico and a blocked OTM is released into the interior of the U.S. If all DRO beds are filled but at least one nonmandatory detainee is being detained, then a nonmandatory detainee is released into the interior of the U.S. and the bed is given to the arriving mandatory detainee. In this case, we say that the mandatory detainee is preempted.

An equation for the mean total number of nonmandatory aliens released (i.e, blocked plus preempted) per year is derived in [11], which allows us to compute the detection probability P_r in terms of the seven DRO parameters: the number of DRO beds (s), the mean arrival rate of mandatory ($\bar{\lambda}_m$) and nonmandatory ($\bar{\lambda}_n$) detainees, the mean residence time

of mandatory (m_m) and nonmandatory (m_n) detainees, and the frequency (ω_d) and relative amplitude (α_d) of the sinusoidal arrival rates. To this end, for $i = \{m, n\}$, let f_{it} and F_{it} be the probability density function (pdf) and cumulative distribution function (cdf) of a normal random variable with mean and variance

$$\bar{\lambda}_i m_i \left(1 + \frac{\alpha_d}{1 + (2\pi m_i \omega_d)^2} [\sin(2\pi \omega_d t) - 2\pi m_i \omega_d \cos(2\pi \omega_d t)] \right). \quad (30)$$

Then the mean number of detainees released per year (the frequency is $\omega_d = 1/\text{yr}$) is

$$R = \int_0^{\omega_d^{-1}} R(t) dt, \quad (31)$$

where

$$R(t) = \lambda_n(t) \left(\int_0^s \frac{f_{mt}(x) f_{nt}(s-x)}{F_{nt}(s-x)} dx + 1 - F_{mt}(s+1) \right), \quad (32)$$

and the probability of removal is

$$P_r = 1 - \frac{R}{\lambda_n}, \quad (33)$$

which is the mathematical version of relation (3) in the main text.

Our main performance measure, denoted by P_T , is the probability that an OTM terrorist successfully enters the country, which is given by

$$P_T = 1 - P_{a2} + P_{a2}(1 - P_r). \quad (34)$$

Preempted nonmandatory aliens are released (on bond) from the DRO facilities but are still processed, and approximately 1% of these overflow cases are released on order of supervision. However, more than 90% of nondetained aliens do not appear in court, and only 13% of nondetained aliens (and only 6% from countries believed to support terrorist activities) with final removal orders are actually removed [14]. Electronic home-monitoring of low-risk nondetained aliens has been tested and appears to be moderately successful [15]. From a homeland security viewpoint, we assume in equation (34) that a terrorist who is released on bond will not appear in court.

4 The Illegal Wage Submodel

The goal here is to construct a model that quantifies how changes in worksite enforcement, including fines for hiring illegal aliens and the number of worksite inspectors, as well a guest-worker program that brings new aliens into the country as legal workers and an amnesty program that would legalize illegal workers currently in the U.S., would impact the wages of illegal aliens that sneak across the border.

It appears that the 1986 Immigration Reform and Control ACT (IRCA) sanctions did not work because of the widespread use of illegal documents and contracting firms, the easy way out for employers (they were protected as long as they filled out I-9 forms and looked at workers' identification cards), and the low level of sanctions enforcement [16]. We assume an idealized system that works according to plan (e.g., perfect document security for

employment verification, employers cannot pass the risk along to subcontractors), with five controllable variables: the number of currently illegal workers that are legalized, the number of newly legal workers who participate in a guest-worker program, the number of worksite inspectors, the extent to which the inspections are targeted (characterized by the parameter r_w in equation (35)), and the size of the fine (we focus on a hiring fine, even though the IRCA enforcements relied mostly on paperwork fines).

Our model is described in three subsections: how the Government deploys its worksite enforcement resources and what the employers do in the face of this enforcement, the labor demand, and the labor supply.

4.1 Work Site Enforcement

Suppose m_w worksite enforcement agents are hired, each of which performs inspections at μ_w firms per year. Let $n_w = m_w \mu_w$ be the number of firms inspected per year, N_w be the total number of firms in the U.S. that hire illegal aliens, and N_i be the total number of illegal workers in the U.S. For mathematical simplicity, we assume that the number of illegal workers in a firm is an exponential random variable with mean $\frac{N_i}{N_w}$, which succinctly captures the phenomenon that many illegal workers are concentrated in a handful of industries [17].

The only study to explicitly address how the Government should allocate enforcement resources across firms is [18], which solves a constrained optimization problem and finds that only firms in industries with the largest number of illegal employees should be inspected, and that among inspected industries, the fraction of firms in an industry that should be inspected is linearly increasing with the number of illegal workers per firm in that industry. However, in contrast to this policy, approximately 60% of the IRCA investigations were aimed at suspected companies/industries, and the remaining investigations were essentially random [19].

We follow what was used during IRCA and assume a hybrid targeted-random strategy, where a fraction r_w of inspections are targeted and the remaining inspections are random. We assume that the Government knows which firms have the most illegal workers, and the targeted inspections are aimed at these firms. That is, the firms in the highest $\frac{r_w n_w}{N_w}$ fractile of illegal workers are targeted, which corresponds to the firms with more than $\frac{N_i}{N_w} \ln\left(\frac{N_w}{r_w n_w}\right)$ illegal workers. The remaining fraction $1 - r_w$ of inspections are randomly sampled from the untargeted industries, so that the annual probability of inspecting a firm that hires x illegal aliens is

$$p_w(x) = \begin{cases} \frac{(1-r_w)n_w}{N_w - r_w n_w} & \text{if } x < \frac{N_i}{N_w} \ln\left(\frac{N_w}{r_w n_w}\right); \\ 1 & \text{if } x \geq \frac{N_i}{N_w} \ln\left(\frac{N_w}{r_w n_w}\right). \end{cases} \quad (35)$$

In the face of sanctions, we assume that employers pass the expected penalties on to the illegal workers in the form of lower wages [18, 20]. Due to our assumption about perfect document security, employers can fully discern the legal status of workers. Therefore, illegal laborers at a firm with x illegal laborers are paid, assuming that employment decisions are made on an annual basis so that the expected fine is allocated over one year,

$$w_i(x) = \max\{w - p_w(x)f_w, 0\}, \quad (36)$$

where w is the legal wage that is determined in §4.3 and f_w is the fine per illegal worker per hour of work.

4.2 Labor Demand

We take the view that immigrants tend to fill jobs that native workers do not want, and that in the absence of these workers, many of these jobs would be replaced by capital or move offshore [21]. Hence, we ignore high-skilled labor in our model, and assume that each of the N_w firms has a Cobb-Douglas production function with two factors, unskilled workers and capital [20], i.e., the output equals $Y = L^{\alpha_w} K^{1-\alpha_w}$, where r is the cost of capital and w is the cost of labor, which includes wages and expected sanctions; because employers pass the expected penalties on to the workers, this w is the same as in equation (36). The cost-minimizing demand for labor is $\left(\frac{\alpha_w r}{(1-\alpha_w)w}\right)^{1-\alpha_w} Y$ [22]. If we assume that the parameters α_w , r and w and the output Y are common across all N_w firms (the assumption on Y is only for notational simplicity), then the annual labor hours demanded is $\left(\frac{\alpha_w r}{(1-\alpha_w)w}\right)^{1-\alpha_w} N_w Y$. To ease the parameter estimation task, we express this as

$$L_d = A_w \left(\frac{1}{w}\right)^{1-\alpha_w}, \quad (37)$$

where $A_w = \left(\frac{\alpha_w r}{1-\alpha_w}\right)^{1-\alpha_w} N_w Y$.

4.3 Labor Supply

The unskilled labor is supplied by both legal and illegal workers currently in the U.S., which are assumed to have equal skill levels. Let the total legal labor supply be initially fixed at N_l and recall that N_i be the total number of illegal workers currently in the U.S. We model legalization by the decision variable $\Delta_l \in [0, N_i]$, which is the number of illegal workers that are legalized, and we model a guest-worker program by the decision variable N_g , which is the number of new aliens that are legally brought into the U.S. to work. We assume that the guest-worker slots are allocated to Mexicans and OTMs in proportion to n_i , so that the number of potential border crossers that are Mexican and OTM is reduced to

$$n_1 - \frac{n_1}{n_1 + n_2} N_g, \quad (38)$$

$$n_2 - \frac{n_2}{n_1 + n_2} N_g. \quad (39)$$

Hence, we have four pools of labor supply: N_l originally legal, $N_i - \Delta_l$ illegal, Δ_l newly legal and N_g new guest workers. The legal laborers (i.e., all but the $N_i - \Delta_l$ illegal workers) are paid the equilibrium legal wage w , while an illegal laborer at a firm with x illegal laborers is paid $w_i(x)$ in equation (36).

As in [20], we use the neo-classical labor supply function to compute the number of legal labor hours the N_l originally legal people are willing to work at wage w . This model (§4.1 of [23]) assumes each person has the utility $\alpha_0 \log(q_0 - \gamma_0) + (1 - \alpha_0) \log(q - \gamma)$, where q_0 is the

number of waking hours minus the number of labor hours, q is the quantity of goods bought at price p , α_0 is a weighting factor, and γ_0 and γ are committed leisure and consumption, respectively. The utility-maximizing amount of legal labor supplied per person, subject to the budget constraint $wq_0 + pq = wT + \mu$, where T is 24 hr, μ is nonlabor income, and p is an index of consumption good prices, is (§4.1 of [23])

$$(1 - \alpha_0)(T - \gamma_0) + \frac{\alpha_0 \gamma p}{w} - \frac{\alpha_0 \mu}{w}. \quad (40)$$

To reduce the number of parameters to estimate, we multiply (40) by $365N_l$ and express the annual legal labor hours supplied by

$$N_l \left(\kappa_1 - \frac{\kappa_2}{w} \right), \quad (41)$$

where $\kappa_1 = 365(1 - \alpha_0)(T - \gamma_0)$ and $\kappa_2 = 365\alpha_0(\mu - \gamma p)$.

Because the base-case amount of worksite enforcement is negligible, and because we are interested in the impact of greater levels of enforcement, we implicitly assume that most illegal workers are already employed before enforcement. In our model, illegal aliens who have their wages lowered due to worksite enforcement have the option of staying in their present job, quitting and looking for a job with a U.S. firm that has a lower probability of being inspected, or returning home. Because there are only two different illegal wages offered (see (35)-(36)), we assume that illegal workers who quit a job at an untargeted firm will return to their home country because the other untargeted firms pay the same wage and the targeted firms pay a lower wage. In contrast, we consider the possibility that illegal workers who quit a job at a targeted firm may prefer to fill a vacated job at an untargeted firm rather than go home. Hence, we consider a matching process between the illegal workers who quit a job at a targeted firm, and the jobs at untargeted firms that were vacated by illegal aliens; illegal workers who are not matched return to their home country. This matching process takes place before the firms decide to raise the legal wage and/or increase the amount of capital in response to the vacated jobs.

We again use a multinomial logit model similar to equation (1) to compute how many people stay in their reduced-wage job. The expected utility if an illegal alien returns home is $w_0\tau - c_1$ where w_0 is the annual home wage and c_1 is the cost to return home (we use c_1 because most illegal workers are Mexican). The expected utility if he stays in the U.S. is $hx\tau$ if he is offered the hourly wage x , where h is the number of hours worked per worker per year.

By equation (35), the probability that an illegal worker works in a targeted firm is

$$P_t = \frac{N_w \int_{\frac{N_i}{N_w} \ln(\frac{N_w}{r_w n_w})}^{\infty} x \frac{N_w}{N_i} e^{-N_w x / N_i} dx}{N_i}, \quad (42)$$

$$= \frac{r_w n_w}{N_w} \left(1 + \ln\left(\frac{N_w}{r_w n_w}\right) \right). \quad (43)$$

Equation (36) implies that an illegal worker will be offered the hourly wage

$$w_t = \max(w - f_w, 0) \quad (44)$$

with probability P_t (i.e., if he works at a targeted firm), and the hourly wage

$$w_r = \max\left(w - \frac{f_w(1 - r_w)n_w}{N_w - r_w n_w}, 0\right) \quad (45)$$

otherwise. Then the probability that an illegal worker will stay at his reduced-wage job is the weighted average of the multinomial logit models,

$$P_t \frac{e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_0 \tau - c_1)}} + (1 - P_t) \frac{e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_0 \tau - c_1)}}. \quad (46)$$

We now describe the matching process for those who leave their job. Our matching process uses a Cobb-Douglas form with constant returns to scale, which is consistent with the empirical literature [24]. If U is the number of unemployed workers looking for a job, V is the number of vacated jobs, and M is the number of matches, then

$$M = \min\{U, V, A_m U^{\alpha_m} V^{1-\alpha_m}\}, \quad (47)$$

where A_m and α_m are constants. In analyzing this matching process, note that we can interpret $\frac{e^{\theta h x \tau}}{e^{\theta h x \tau} + e^{\theta(w_0 \tau - c_1)}}$ as a cdf that represents the fraction of illegal workers that prefer to work at hourly wage x rather than return to their home country. Hence, U is given by the number of illegal workers times the probability that a worker is originally in a targeted firm, quits his job because he is unwilling to work at wage w_t , but is willing to work at wage w_r ,

$$U = N_i P_t \left(\frac{e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_0 \tau - c_1)}} - \frac{e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_0 \tau - c_1)}} \right). \quad (48)$$

Similarly, V is given by the number of illegal workers times the probability that a worker is originally in an untargeted firm and quits his job because he is unwilling to work at wage w_r ,

$$V = N_i (1 - P_t) \left(\frac{e^{\theta(w_0 \tau - c_1)}}{e^{\theta h w_r \tau} + e^{\theta(w_0 \tau - c_1)}} \right). \quad (49)$$

By equations (46)-(49), the annual illegal labor supply (for ease of presentation, in the remainder of this section we assume that the Cobb-Douglas term achieves the minimum in (47)) is

$$N_i h \left[\frac{P_t e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_0 \tau - c_1)}} + \frac{(1 - P_t) e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_0 \tau - c_1)}} \right. \\ \left. + A_m \left(\frac{P_t e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_0 \tau - c_1)}} - \frac{P_t e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_0 \tau - c_1)}} \right)^{\alpha_m} \left(\frac{(1 - P_t) e^{\theta(w_0 \tau - c_1)}}{e^{\theta h w_r \tau} + e^{\theta(w_0 \tau - c_1)}} \right)^{1-\alpha_m} \right]. \quad (50)$$

The newly legal workers Δ_l also have the option to go home, but are getting paid exactly w , and hence the amount of newly legal labor supplied annually is

$$\Delta_l h \frac{e^{\theta h w \tau}}{e^{\theta h w \tau} + e^{\theta(w_0 \tau - c_1)}}. \quad (51)$$

We assume that the new guest workers all stay, providing

$$N_g h \quad (52)$$

hours of work annually.

The equilibrium condition is found by equating the labor demanded, which is the right side of (37), to the labor supplied, which is the sum of (41), (50), (51) and (52):

$$\begin{aligned}
A_w \left(\frac{1}{w}\right)^{1-\alpha_w} &= N_l \left(\kappa_1 - \frac{\kappa_2}{w}\right) + \Delta_i h \frac{e^{\theta h w \tau}}{e^{\theta h w \tau} + e^{\theta(w_o \tau - c_1)}} + N_g h \\
&+ N_i h \left[P_t \frac{e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_o \tau - c_1)}} + (1 - P_t) \frac{e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} \right. \\
&\left. + A_m \left(\frac{P_t e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} - \frac{P_t e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_o \tau - c_1)}} \right)^{\alpha_m} \left(\frac{(1 - P_t) e^{\theta(w_o \tau - c_1)}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} \right)^{1-\alpha_m} \right].
\end{aligned} \tag{53}$$

We solve (53) for the equilibrium legal wage w . By equation (50), we set the annual illegal wage w_u in §1 equal to

$$\begin{aligned}
w_u &= \frac{\frac{P_t e^{\theta h w_t \tau} w_t}{e^{\theta h w_t \tau} + e^{\theta(w_o \tau - c_1)}} + \frac{(1 - P_t) e^{\theta h w_r \tau} w_r}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} + A_m w_r \left(\frac{P_t e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} - \frac{P_t e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_o \tau - c_1)}} \right)^{\alpha_m} \left(\frac{(1 - P_t) e^{\theta(w_o \tau - c_1)}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} \right)^{1-\alpha_m}}{\frac{P_t e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_o \tau - c_1)}} + \frac{(1 - P_t) e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} + A_m \left(\frac{P_t e^{\theta h w_r \tau}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} - \frac{P_t e^{\theta h w_t \tau}}{e^{\theta h w_t \tau} + e^{\theta(w_o \tau - c_1)}} \right)^{\alpha_m} \left(\frac{(1 - P_t) e^{\theta(w_o \tau - c_1)}}{e^{\theta h w_r \tau} + e^{\theta(w_o \tau - c_1)}} \right)^{1-\alpha_m}}.
\end{aligned} \tag{54}$$

Equation (54) corresponds to relation (4) in the main text.

5 Parameter Estimation

In the next four subsections, we estimate the parameters from the four submodels in §1-4. The parameter values for these four submodels are listed in Tables 1-4 and the base-case values of the decision variables appear in Table 5; although some of these parameters appear in several submodels, they appear in the table for the submodel in which they play the most central role.

5.1 Discrete Choice Parameters

We consider a time horizon of $\tau = 2$ yr. For lack of resources, the U.S. Attorneys Office does not prosecute an illegal immigrant until he has been apprehended 13 times [8]. Because $P_a^{13} \approx 10^{-9}$, it appears unlikely that many of those detained were held because they were previously apprehended a specific number of times. Consequently, as noted in §1.2, we set the decision variable $a = \infty$ in the base case.

We need values for the parameters w_o , w_u , c_i , d_i , \tilde{d}_i , f , P_{ai} , P_r , λ_{di} , n_i , and λ_{bi} . We begin with the wages and costs, and then estimate some of the apprehension and detention parameters. The more difficult parameters to estimate are discussed last.

The hourly wage in 2004 for production workers in manufacturing in Mexico was \$2.50 (Table 2 in [25]). Most OTM aliens are from Caribbean countries (e.g., Guatemala, Honduras, El Salvador) that have similar economic conditions. At 2000 hr/yr, we get $w_o = \$5000/\text{yr}$ as the annual wage in the home country. The average wage of Mexican immigrants in the U.S. is \$22,300/yr [26]. Because there is currently a trivial amount of worksite enforcement, we set the annual illegal wage in the U.S. to be $w_u = \$22,300/\text{yr}$. Including the travel cost, the cost of a coyote (77% of aliens use coyotes [27]), the lost time,

and the danger, we roughly estimate the one-way cost of migration to be $c_1 = \$1500$ and $c_2 = \$2500$.

We assume the detention cost d_i includes the cost of lost wages in the home country and an additional cost due to the physical and psychological toll of being detained. We assume d_i is proportional to the square root of the number of days in detention. The concavity of the square root function attempts to crudely capture the phenomenon that the very act of being detained carries a significant psychological toll, regardless of the length of detention. Although a fixed plus variable cost of detention may be more appropriate, this would require an additional parameter to estimate. More specifically, we let $d_i = \psi\sqrt{\tilde{m}_i}$, where ψ is the toll factor of apprehension (which is estimated later) and \tilde{m}_i is the number of days that a type i alien is detained until removal. Similarly, we let $\tilde{d}_1 = \psi\sqrt{2}$, where it is assumed that two days of U.S. salary are lost during the apprehension process (and the toll includes the possibility that the U.S. Government takes their fingerprints upon apprehension). In [11], we calculated that the mean uncensored residence time for nonmandatory aliens is 48.0 days, whereas the observed, right-censored residence times for Mexican noncriminals, OTM noncriminals, and all noncriminals is 10.5, 42.4, and 28.7 days, respectively, where the weighted average for all noncriminals uses the unblocked arrival rates (45,925 Mexican noncriminals and 61,312 OTM noncriminals) to determine the weights. If we assume that apprehended aliens in our model are noncriminal, and apply the censoring factor $\frac{48.0}{28.7}$ to the censored residence times for Mexican noncriminals and OTM noncriminals, we get $\tilde{m}_1 = \frac{48.0}{28.7}(10.5) = 17.6$ days and $\tilde{m}_2 = \frac{48.0}{28.7}(42.4) = 70.9$ days.

The parameter f is the fraction of the total detention cost incurred before release from DRO. The parameter g is the expected number of days detained before release from DRO (and voluntary departure to Mexico) conditioned on not being detained until removal, divided by the expected number of days detained until removal. Since the cost d_i is proportional to the square root of the detention duration, we approximate f by \sqrt{g} (although the expected value of the ratio does not equal the ratio of the expected values in the computation of g , the two should be similar because the denominator is bigger than the numerator). In 2003, there were 28,000 blocked arrivals (i.e., people who were not detained at all) and 43,000 preempted arrivals (i.e., aliens who were detained but were released before removal) out of 93,976 total arrivals, so that probability of removal in 2003 was $P_r = 1 - \frac{71,000}{93,976} = 0.245$. Recalling that the mean censored residence time is 28.7 days and the mean uncensored residence time is 48.0 days, we get that in 2003 $g = \frac{43,000}{28,000+43,000} \frac{28.7}{48.0} = 0.36$. As mentioned in §1.1, the parameter g varies in a complicated way with the number of DRO beds. To enable the optimization of the entire system, we assume that g can be expressed as a quadratic function of the removal probability P_r . Fitting the three parameters of the quadratic function to the two (g, P_r) extreme points, (0,0) and (1,1), and the 2003 point (0.36,0.245) yields the relationship

$$f = \sqrt{g} = \sqrt{1.637P_r - 0.637P_r^2}. \quad (55)$$

The current removal probability is estimated to be $P_r = 0.137$ in §5.3, and so $f = 0.46$ in the base case. We assume that the base-case apprehension probability is $P_{a1} = P_{a2} = 0.2$ [7], which is lower than it was in the 1970s and 1980s [6].

This leaves six unestimated parameters: θ , ψ (embedded within d_i), n_1 , n_2 , λ_{b1} , and λ_{b2} . The parameters θ and ψ will be jointly estimated with two other parameters in §5.2.

Here we provide one of the four equations that will be needed for their derivation. This equation is based on an analysis in [28], which estimates that the natural logarithm of the monthly apprehension rate at the U.S.-Mexico border increases by 0.049 times the ratio, for Mexican aliens, of the U.S. wage and the Mexican wage. Let $\tilde{\lambda}_{b1}$ be the value of λ_{b1} in (12) but with w_u replaced by $w_u + w_0$, which represents an increase of the wage ratio by 1.0. Noting that the monthly apprehension rate in [28] corresponds to $\frac{P_{a1}\lambda_{b1}}{12}$ in our model, our equation is given by

$$\ln\left(\frac{P_{a1}\lambda_{b1}}{12}\right) = \ln\left(\frac{P_{a1}\tilde{\lambda}_{b1}}{12}\right) - 0.049. \quad (56)$$

We complete this subsection by deriving values for n_1 , n_2 , λ_{b1} , and λ_{b2} . After obtaining values of $\theta = 6.83 \times 10^{-5}/\$$ and $\psi = \$1067/\sqrt{\text{day}}$ in §5.2, we can compute P_{1j} and P_{2j} in the base case, which yields $P_{11} = 0.903$ and $P_{21} = 0.887$. In 2005, 155k OTMs were apprehended at the U.S.-Mexico border [29], and so $\lambda_{d2} = 155\text{k}/\text{yr}$. By equation (27), $\lambda_{b2} = n_2 P_{21} = \frac{\lambda_{d2}}{P_{a2}} = 775\text{k}/\text{yr}$ and $n_2 = 874\text{k}$. In the base case, we know $\lambda_{d1} = 0$. In 2004, 1.16M Mexican aliens were apprehended by Border Patrol agents [30]. Assuming these apprehensions all took place at the U.S.-Mexico border, in the base case ($a = \infty$) we have $\frac{n_1 P_{11} P_{a1}}{1 - P_{a1}} = 1.16\text{M}$, which gives $n_1 = 5.14\text{M}$ and, by equation (12), $\lambda_{b1} = 5.80\text{M}/\text{yr}$.

5.2 Apprehension Parameters

We need to estimate values for the parameters L , ω_b , α_b , k , α_1 , α_2 , m_b , and θ , along with base-case values of the decision variables n_b and s_b , and the costs c_b and c_s for agents and surveillance technology. We start with the straightforward parameters, then turn to the decision variables and costs, and then finally to the more difficult parameters.

We let $L = 1933$ miles [8] and assume $m_b = 1$ hr based on conversations with government employees. The cost of a coyote increased \$800 when they needed to take more rural routes [7], and we assume $k = \$4/\text{mile}$ because each sector averages approximately 200 miles in length and it is clear from data that people cross in adjacent sectors in the presence of increased enforcement [7, 31]. We assume there are 10 peak crossing points that are evenly distributed across the U.S.-Mexico border (currently seven fences are in place with several more proposed [32], so that $\omega_b = \frac{10}{L} = 5.17 \times 10^{-3}/\text{mile}$. In the base case we assume the arrival rate $\lambda_{bi}(x)$ and border agent density $n_b(x)$ have the same amplitude, so that $\alpha_b = \tilde{\alpha}_b$. The largest apprehension rate per mile among the nine sectors divided by the smallest apprehension rate per mile among the nine sectors is approximately 40 [32], and so we set $\frac{\alpha_b + 1}{\alpha_b - 1} = 40$, which gives $\alpha_b = \frac{39}{41} = 0.951$.

Now we turn to the decision variables and costs. Assuming around-the-clock surveillance and 11,380 border patrol agents [17] who work 2000 hr/yr but spend only 63% of their time on the border [27], we have $n_b = \frac{0.63(11,380)(2000 \text{ hr/yr})}{8766 \text{ hr/yr}} = 1636$ agents at a given time on the border. We assume that a border patrol agent costs \$176k per year to work 40 hr/week [33], and that 63% of agent hours are on the border. Hence, the annual cost per n_b is $c_b = \frac{(\$176\text{k}/\text{yr})(168 \text{ hr}/\text{wk})}{0.63(40 \text{ hr}/\text{wk})} = \$1.173\text{M}/\text{yr}$. There are several different kinds of technologies, and we focus on surveillance cameras, which are far more prevalent than helicopters and drones, and much more effective than thermal sensors [8]. Page 53 of [8] claims that 25% of the Laredo, El Paso and McAllen sectors are covered by surveillance technology, and the

GAO reports that approximately 300 miles of the Northern and Southern borders are covered [9]. We set $s_b = 0.15L = 290$ miles in the base case. These surveillance systems cost \$90k/mile ([32], which is consistent with the estimate that each camera system costs \$650k [8]. However, we need to convert this to an annual cost. Each operator is in charge of several dozen cameras [8], and there are four cameras to cover five miles, and so this extra cost is approximately \$3000/yr. As a ballpark figure, we use $c_s = \$30\text{k}/\text{mile}\cdot\text{yr}$, assuming a lifetime of approximately five years and some maintenance.

We conclude this subsection by computing the four remaining parameters, θ , ψ (embedded in d_i and \tilde{d}_1), α_1 and α_2 , from four equations. One of these equations is (56). Two more equations are constructed by setting the right side of equation (26) equal to 0.2 for $i = 1, 2$ (i.e., $P_{a1} = P_{a2} = 0.2$), which is the base-case apprehension probability [7]. Page 41 of [8] states that technology (which covers approximately 15% of the U.S.-Mexico border) is responsible for up to 60% of apprehensions in some sectors on the U.S.-Mexico border. This 60% figure would appear to be an upper bound and we estimate the efficacy of surveillance technology by assuming that it plays a role in 45% of all apprehensions. Hence, by equations (23) and (26), the fourth equation is

$$\frac{\int_0^L \frac{\lambda_{ci}(y)I_{\{y\}} \int_0^{n_b(y)} f(x)e^{-\alpha_2 x} dx}{1+\rho_b(y)} dy}{\lambda_{bi}} = 0.09, \quad (57)$$

where we set $s_b = 0.15L$ in the definition of the indicator function in equation (18). The solution to these four equations is $\theta = 6.83 \times 10^{-5}/\text{\$}$, $\psi = \$1067/\sqrt{\text{day}}$ (and hence $\tilde{d}_1 = \$1509$, $d_1 = \$4476$, and $d_2 = \$8983$), $\alpha_1 = 9.08/\text{mile}$, and $\alpha_2 = 0.29/\text{mile}$.

5.3 DRO Parameters

We begin by estimating the constants a_n and b_n in equation (29). Because $a = \infty$ in the base case, no Mexicans are sent to DRO because they have been apprehended a specified number of times. Hence, we take a_n to be the annual number of nonmandatory Mexicans that are currently sent to DRO. In 2003, 45,925 Mexican noncriminals arrived to DRO [11]. The estimated fraction of noncriminal detainees that are nonmandatory is 0.6949, and so we set $a_n = 0.6949(45,925) = 31,913/\text{yr}$.

We estimate the multiplier factor b_n by the total potential (i.e., blocked and unblocked) nonmandatory OTM arrival rate to DRO divided by the total potential nonmandatory OTM apprehension rate at the U.S.-Mexico border; for lack of data, we assume that all detentions at the border are nonmandatory. The number of unblocked nonmandatory arrivals to DRO in 2003 was estimated to be 65,976 [11], and hence the number of unblocked OTM nonmandatory arrivals at DRO was 65,976-31,913=34,063. In addition, 28,000 nonmandatory OTMs were apprehended at the border but were blocked from entering DRO [11]. We also need to estimate the number of nonmandatory OTMs apprehended away from the border but blocked from entering DRO. To obtain this estimate, first note that in 2003, there were 905,065 apprehensions by border patrol agents on the U.S.-Mexico border (Table 37 in [34]), and 882,012 Mexican aliens apprehended by border patrol agents (Table 38 in [34]). If we assume all of these Mexican aliens were apprehended on the U.S.-Mexico border, then the number of

OTMs detained at the border was $905,065 - 882,012 = 23,053$. There were 854,976 voluntary departures of Mexicans on the U.S.-Mexico border in 2003 (Table 41 in [34]), and hence the number of Mexicans detained at the border was $882,012 - 854,976 = 27,036$ and the total number of aliens detained at the border was $23,053 + 27,036 = 50,093$. Recalling that there were 65,976 nonmandatory arrivals to DRO in 2003, the remaining $65,976 - 50,093 = 15,883$ nonmandatory aliens were apprehended elsewhere (e.g., in the interior). If we assume that the likelihood of a potential nonmandatory detainee being blocked from entering DRO is independent of where he was apprehended, then the number of blocked nonmandatory apprehensions occurring away from the border was $\frac{28,000}{50,093} 15,883 = 8878$. Taken together, the total potential nonmandatory arrival rate to DRO, which is the numerator for calculating b_n , is 34,063 (actual arrivals) plus 28,000 (blocked at the border) plus 8878 (blocked away from the border), or 70,941. The denominator for b_n is the actual number of OTMs detained at the border (23,053) plus the number blocked (28,000), or 51,053. Hence, we obtain the estimate $b_n = \frac{70,941}{51,053} = 1.39$.

We assume there are $s = 22,580$ DRO beds, which was allotted in the 2006 budget. The values of the mean mandatory arrival rate $\bar{\lambda}_m$, the mean residence times m_m and m_n , and the relative amplitude α_d were estimated in [11] using 2003 data and appear in Table 1. The only major change at the border since 2003 is a large increase in OTM arrivals, i.e., $\lambda_{d2} = 155\text{k}/\text{yr}$. Hence, we set $\lambda_{d1} = 0$ in the base case and set $\bar{\lambda}_n = 31,913 + 1.39(155\text{k}) = 247,363/\text{yr}$, and use equation (33) to solve for the removal probability, which is $P_r = 0.137$.

Finally, U.S. Immigration and Customs Enforcement requested \$31M for 950 additional detention beds (and additional support personnel) for fiscal year 2008 [35], and so we set $c_d = \frac{\$31\text{M}}{950} = \$32.6\text{k}/\text{yr}$.

5.4 Wage Parameters

We assume there are $N_w = 217\text{k}$ firms in the U.S. that hire illegal aliens [36]. Estimates of the service rate of a worksite inspector vary from approximately 7 firms per year [37] to nearly 50 per yr [38], and we assume $\mu_w = 20/\text{yr}$. The fraction of inspected firms that are inspected in a random manner is taken to be the same as during IRCA, $r_w = 0.4$ [37]. The current penalty is \$10,000 per illegal employee, and so in the base case, $f_w = \frac{\$10,000}{h} = \$5/\text{worker-hr}$, where $h = 2000$ hours worked per worker per year. Also, in the base case, we assume that $m_w = 65$ (its value in 2004 [39]), which leads to $n_w = 1300$ firms investigated per year. The fiscal year 2007 budget requested \$41.7M for 171 additional worksite inspectors [40], and so we set $c_w = \frac{\$41.7\text{M}}{171} = \$243.9\text{k}/\text{yr}$.

We set the annual illegal wage w_u to be the wage of U.S. immigrants, which is \$22,300/yr [26]. In the Cobb-Douglas matching function in equation (47), we set the elasticity with respect to unemployment $\alpha_m = 0.7$ [41]. The labor demand function in equation (37) can be rewritten as $\ln L_d = \ln A_w + (\alpha_w - 1) \ln w$, and we set $\alpha_w - 1$ equal to the wage elasticity of demand, -0.63 [42], to get $\alpha_w = 0.37$.

Although there are 140.8M legal laborers in the U.S. [17], the subset of these workers that are in direct competition for jobs with illegal immigrants is geographically diverse [43] and difficult to estimate directly. In 2005, 29.5% of (i.e., 41.5M) workers aged 18 and older had income $< \$20\text{k}$ and 39.0% (i.e., 54.9M) had income $< \$25\text{k}$ [44]. Only a fraction of these

workers are in direct job competition with illegal immigrants, and we use the rough estimate $N_l = 30\text{M}$. Hence, we still have six unknowns: the Cobb-Douglas matching parameter A_m , the illegal labor supply N_i , the equilibrium legal wage w , the Cobb-Douglas labor demand parameter A_w , and the labor supply parameters κ_1 and κ_2 . We now state the six equations that can be solved simultaneously for these six unknowns.

By equation (47), the parameter A_m represents the employment rate among illegals when there are an equal number of unhired workers and vacant jobs. There are 7.20M illegal immigrants working in the U.S. [17] and the first equation sets $N_i = \frac{7.2\text{M}}{A_m}$. The second and third equations are (53)-(54) using the base-case decisions in Table 5. The next two equations compute the labor supply parameters in terms of the legal wage rate. The annual wages of U.S. high school dropouts (\$24,800) and Mexican immigrants (\$22,300) are comparable [26], and the former have an unemployment rate of 14.3% [17]. If we assume that high school dropouts are competing with illegal immigrants for jobs, then our fourth equation states that the current legal wage generates a 14.3% unemployment rate among the legal labor population:

$$\kappa_1 - \frac{\kappa_2}{w} = \left(2000 \frac{\text{hr}}{\text{yr}}\right)(1 - 0.143) = 1714 \frac{\text{hr}}{\text{yr}}. \quad (58)$$

The aggregate labor supply elasticity is believed to be approximately three [45], and our fifth equation states that a 1% increase in the equilibrium wage leads to a 3% increase in the amount of legal labor supplied:

$$\kappa_1 - \frac{\kappa_2}{1.01w} = 1.03 \left(2000 \frac{\text{hr}}{\text{yr}}\right)(1 - 0.143) = 1765.4 \frac{\text{hr}}{\text{yr}}. \quad (59)$$

Both sides of equation (53) are equal to 2000 hr/yr times the total number of workers, which is 7.2M illegal workers plus $(1-0.143)N_l$ legal workers. Hence, our final equation is

$$A_w \left(\frac{1}{w}\right)^{1-\alpha_w} = 2000 \frac{\text{hr}}{\text{yr}} [7.2\text{M} + (1 - 0.143)N_l]. \quad (60)$$

Solving these six equations jointly for the six unknown parameters yields an employment rate of $A_m = 0.927$, an illegal labor supply of $N_i = 7.76\text{M}$, an equilibrium legal wage of $w = 11.24/\text{hr}$ (and hence, by equation (54), an equilibrium illegal wage in the base case of $w_u = \$22.3\text{k}/\text{yr}$, or 11.15\$/hr), the Cobb-Douglas labor demand parameter $A_w = \$3.9 \times 10^{12}/\text{yr}$, and the labor supply parameters $\kappa_1 = 6905 \text{ hr}/\text{yr}$ and $\kappa_2 = \$58.3\text{k}/\text{yr}$.

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Notation	Description	Value
τ	Time horizon	2 yr
θ	Discrete choice parameter	$6.83 \times 10^{-5}/\$$
w_o	Wage in home country	\$5000/yr
w_u	Illegal wage in U.S. in base case	\$22.3k/yr
c_1	One-way cost of Mexican migration	\$1500
c_2	One-way cost of OTM	\$2500
d_1	Cost of Mexican detention and removal	\$4476
\tilde{d}_1	Cost of Mexican detention	\$1509
d_2	Cost of OTM detention and removal	\$8983
f	Fraction of detention cost incurred	0.46
n_1	Potential Mexican border crossers per year	5.14M/yr
n_2	Potential OTM border crossers per year	874k/yr
λ_{b1}	Attempted annual border crossings by Mexicans	5.80M/yr
λ_{b2}	Attempted annual border crossings by OTMs	775k/yr

Table 1: Values for parameters in the Discrete Choice Submodel.

Notation	Description	Value
P_{a1}, P_{a2}	Apprehension probability in base case	0.2
λ_{d1}	Mexican apprehension rate ($\geq a^{\text{th}}$ time)	0/yr
λ_{d2}	OTM apprehension rate	155k/yr
L	Length of border	1933 miles
m_b	Mean time to apprehend an alien	1 hr
k	Cost to travel along the border	\$4/mile
ω_b	Frequency of sinusoidal arrival rates	$5.17 \times 10^{-3}/\text{mile}$
α_b	Relative amplitude of sinusoidal arrival rates	0.951
$\tilde{\alpha}_b$	Relative amplitude of sinusoidal border patrol agent density	0.951
c_b	Cost per border patrol agent	\$1.173M/yr
c_s	Cost for surveillance technology	\$30k/mile·yr
α_1	Exponential apprehension parameter without technology	9.08/mile
α_2	Exponential apprehension parameter with technology	0.29/mile

Table 2: Values for parameters in the Apprehension Submodel.

Notation	Description	Value
a_n	Constant factor for nonmandatory detainees	31,913/yr
b_n	Multiplicative factor for nonmandatory detainees	1.39
λ_m	Mean arrival rate of mandatory detainees	144,323/yr
λ_n	Mean arrival rate of nonmandatory detainees	247,363/yr
m_m	Mean residence time of mandatory detainees	45.8 days
m_n	Mean residence time of nonmandatory detainees	48.0 days
ω_d	Frequency of sinusoidal arrival rates	1/yr
α_d	Relative amplitude of sinusoidal arrival rates	0.1474
P_r	Removal probability in base case	0.137
c_d	Cost per DRO bed	\$32.6k/yr

Table 3: Values for parameters in the Removal Submodel. Under Expedited Removal, m_n is reduced to 19.0 days [46].

Notation	Description	Value
μ_w	Service rate of worksite inspectors	20/yr
N_w	Total number of firms hiring illegals	217k
n_w	Total number of firms inspected per year	1300/yr
h	Hours worked per worker per year	2000 hr/yr
A_w	Linear parameter for Cobb-Douglas demand model	$\$3.9 \times 10^{12}/\text{yr}$
α_w	Exponential parameter for Cobb-Douglas demand model	0.37
κ_1	Labor supply parameter	6905 hr/yr
κ_2	Labor supply parameter	\$58.3k/yr
A_m	Linear parameter for Cobb-Douglas matching model	0.927
α_m	Exponential parameter for Cobb-Douglas matching model	0.7
w	Legal wage rate in base case	\$11.24/hr
N_l	Legal labor supply	30M
N_i	Illegal labor supply	7.76M
c_w	Cost per worksite inspector	\$243.9k/yr

Table 4: Values for parameters in the Illegal Wage Submodel.

Notation	Description	Value
a	Number of apprehensions until detention of Mexican	∞
n_b	Number of border patrol agents at border	1636
s_b	Length of border monitored by surveillance technology	290 miles
s	Number of DRO beds	22,580
r_w	Fraction of worksite inspectors that are random	0.4
m_w	Number of worksite inspectors	65
f_w	Fine per illegal worker-hour	\$5/worker-hr
Δ_l	Number of illegal workers legalized	0
N_g	Number of new guest workers	0

Table 5: Decision variables and their base-case values.

Figure Legends

Fig. 1. Border-crossing behavior in the base case. The OTMs' arrival rate to each location on the border ($\lambda_{b2}(x)$ in equation (16)), which is proportional to the density of border patrol agents at each location on the border ($n_b(x)$ in equation (17)), and the OTMs' crossing rate at each location on the border ($\lambda_{c2}(x)$ in equation (25)). Also depicted at the top of the plot is the spatial location of surveillance technology ($I_{\{x\}}$ in equation (18)).

Fig. 2. Impact of the Mexican detention policy. The probability that a terrorist successfully enters the U.S. vs. the number of apprehensions until a Mexican border crosser is detained (i.e., is placed in a DRO facility if there is available space). Other decision variables are set at their base-case values.

Fig. 3. Impact of isolated changes in non-worksite decision variables. The probability that a terrorist successfully enters the U.S. vs. **(a)** the number of border patrol agents on the border, **(b)** the number of miles of border monitored by surveillance technology, and **(c)** the number of DRO beds. Other decision variables are set at their base-case values.

Fig. 4. The optimal P_T vs. cost curve in the base case (i.e., as in Fig. 1 in the main text, where $\tilde{\alpha}_b = 0.951$) and when border patrol agents are distributed evenly across the border (i.e., when the relative amplitude $\tilde{\alpha}_b = 0$).

Fig. 5. Annual illegal wage vs. number of worksite inspectors for various fractions of inspections that are not targeted (r_w). The fine f_w is **(a)** \$5/worker-hr and **(b)** \$25/worker-hr.

Fig. 6. Annual illegal wage vs. number of worksite inspectors for various numbers of legalized workers (Δ_l , which is expressed as a fraction of the illegal labor supply N_i) and new guest workers (N_g). The fine f_w is **(a)** \$5/worker-hr and **(b)** \$25/worker-hr.

Fig. 7. The OTM crossing probability vs. the annual illegal wage for various values of the multinomial logit parameter, θ .

Fig. 8. These curves compare the impact of marginal investments in border patrol agents vs. worksite inspectors on **(a)** the apprehension probability P_{a2} , **(b)** the OTM crossing probability P_{21} , and **(c)** the alternative objective function $P_{21}[1 - P_{a2} + P_{a2}(1 - P_r)]$, which is proportional to the number of OTMs who successfully sneak into the U.S. In each case, the initial annual budget of \$3.6B uses technology along the entire border, has evenly spaced agents (i.e., $\tilde{\alpha}_b = 0$, and has sufficient DRO beds so that $P_r = 1$ (and the alternate objective function equals $P_{21}(1 - P_{a2})$), but has base-case values of 1636 border patrol agents and 65 worksite inspectors. Each curve is generated by adding either border patrol agents or worksite inspectors.

Fig. 9. The optimal budget allocation across border patrol agents, border technology, and DRO beds in the case where 50% of the apprehended aliens are identifiable (see §4.8 in the main text). The numbers appearing along the curves are the optimal number of agents (on the border at any given time), miles of technology, and DRO beds at various budgets. The * denotes the base-case allocation with Expedited Removal, which represents current practice.

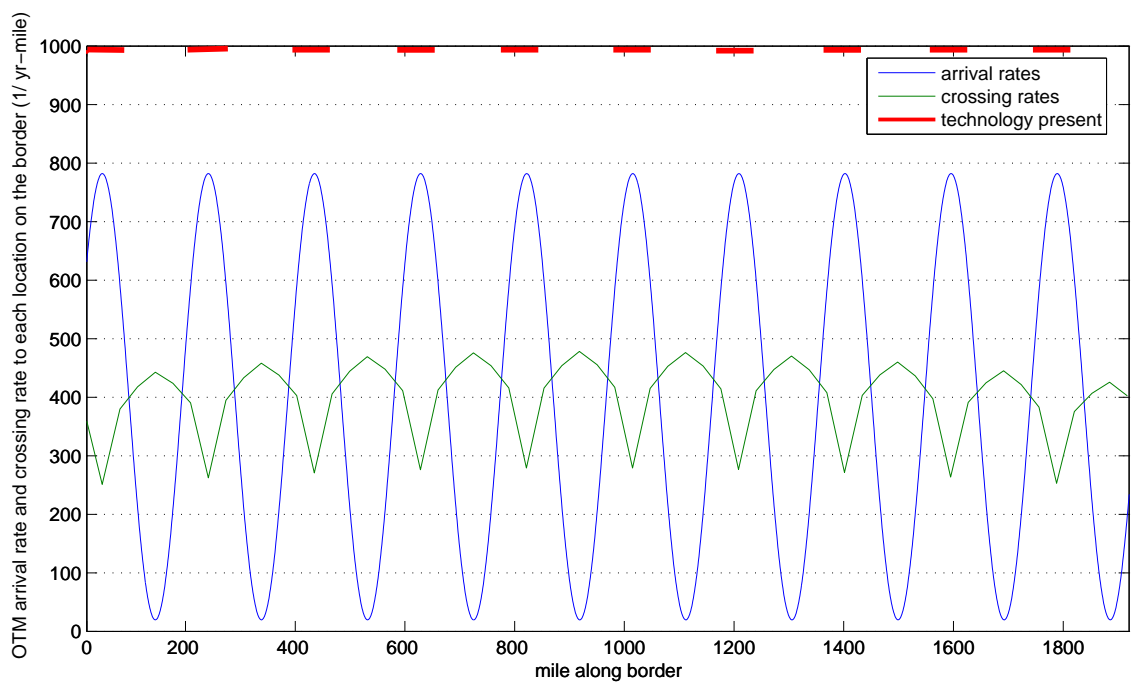


FIGURE 1

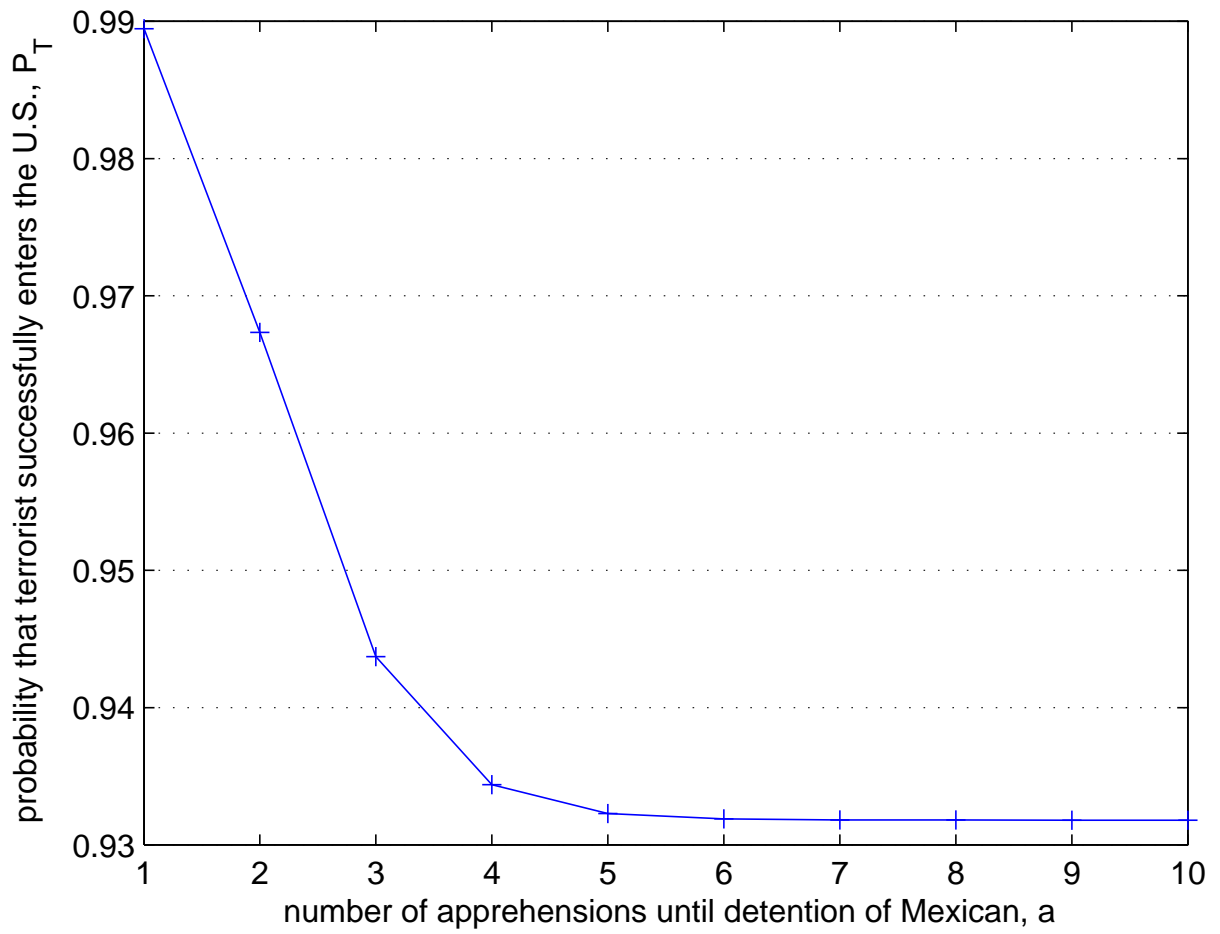
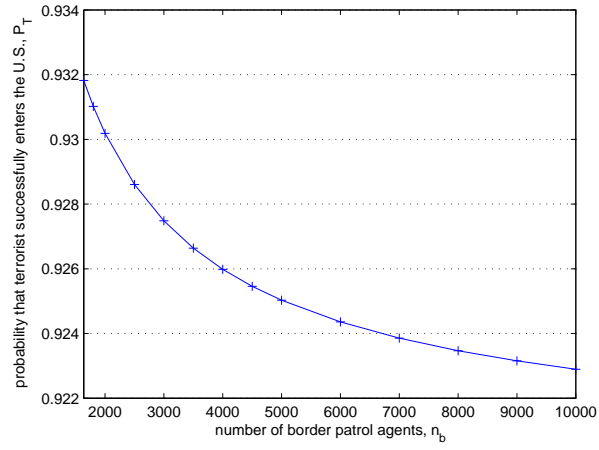
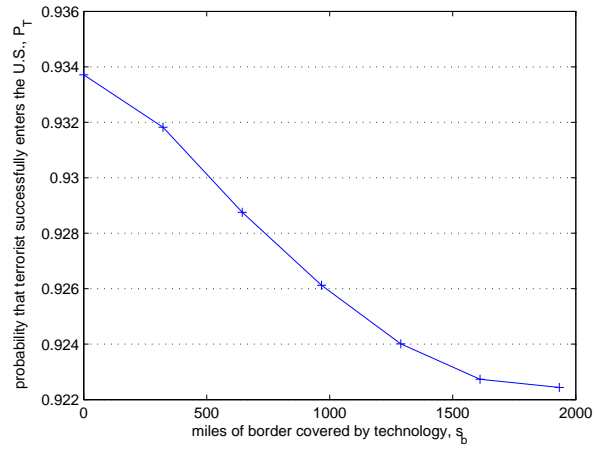


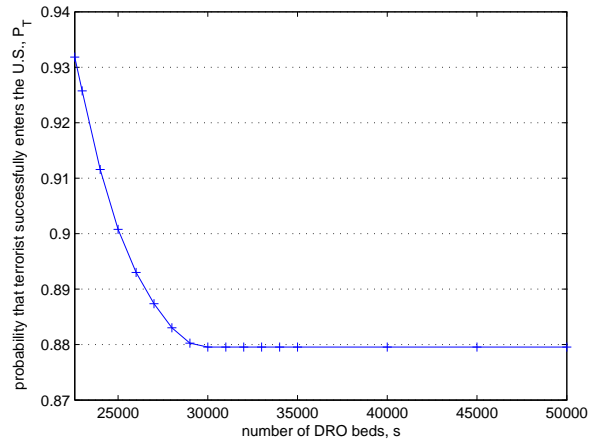
FIGURE 2



(a)



(b)



(c)

FIGURE 3

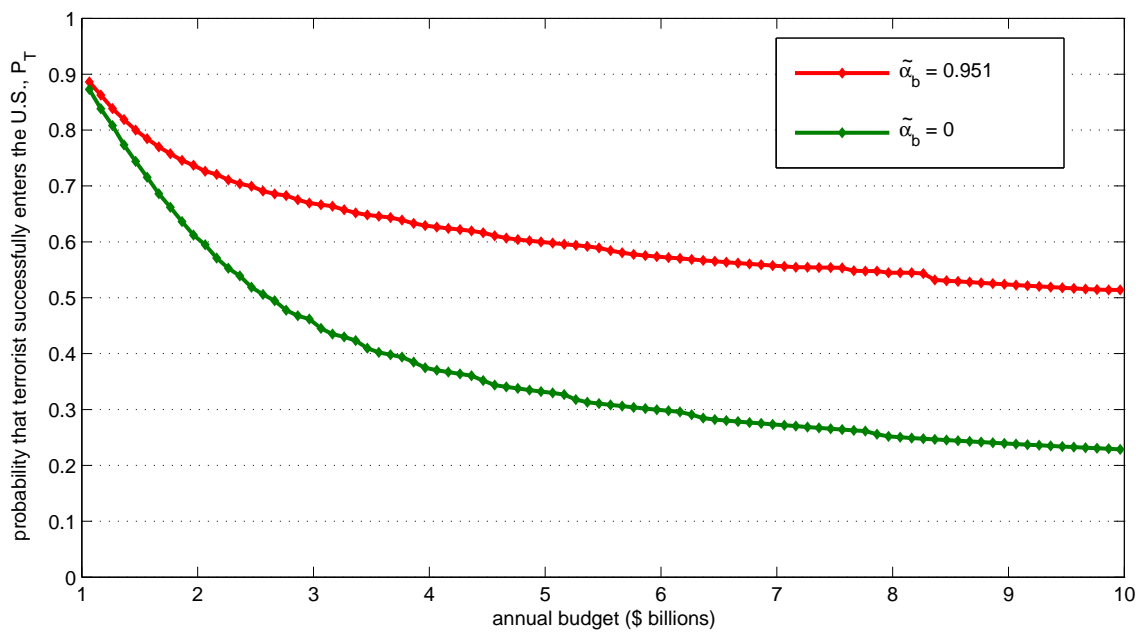
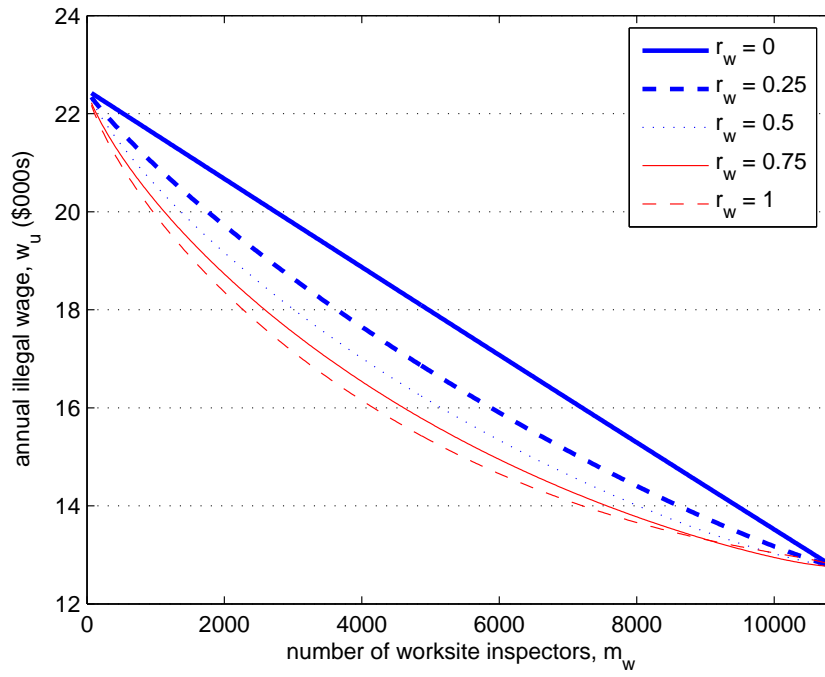
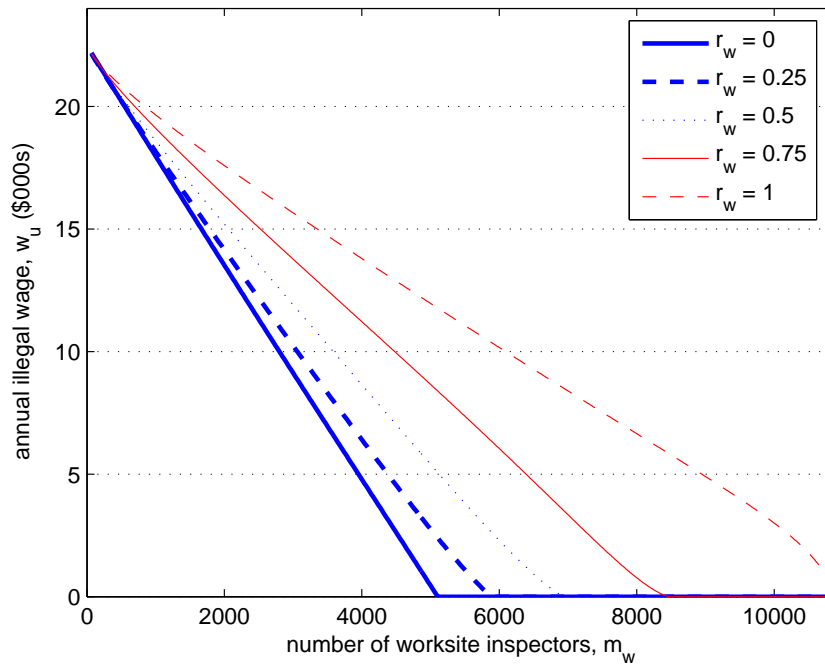


FIGURE 4

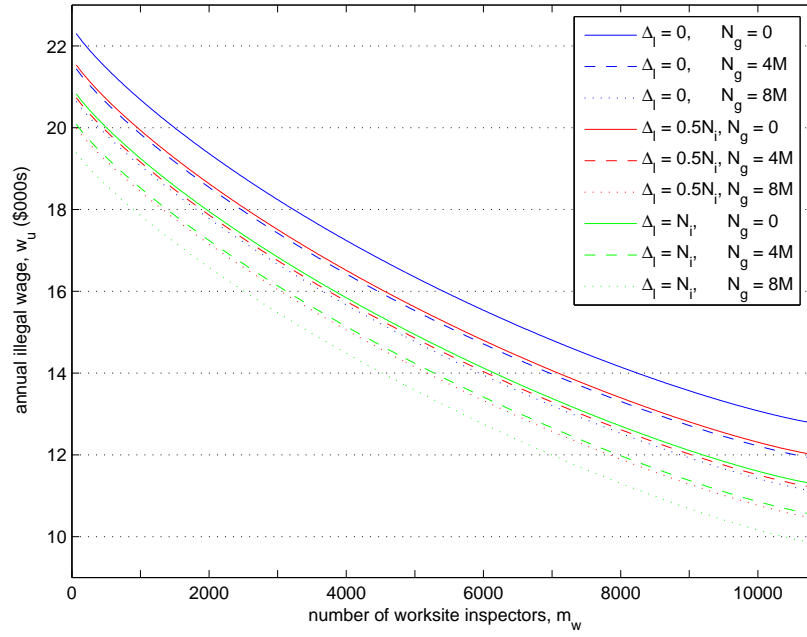


(a)

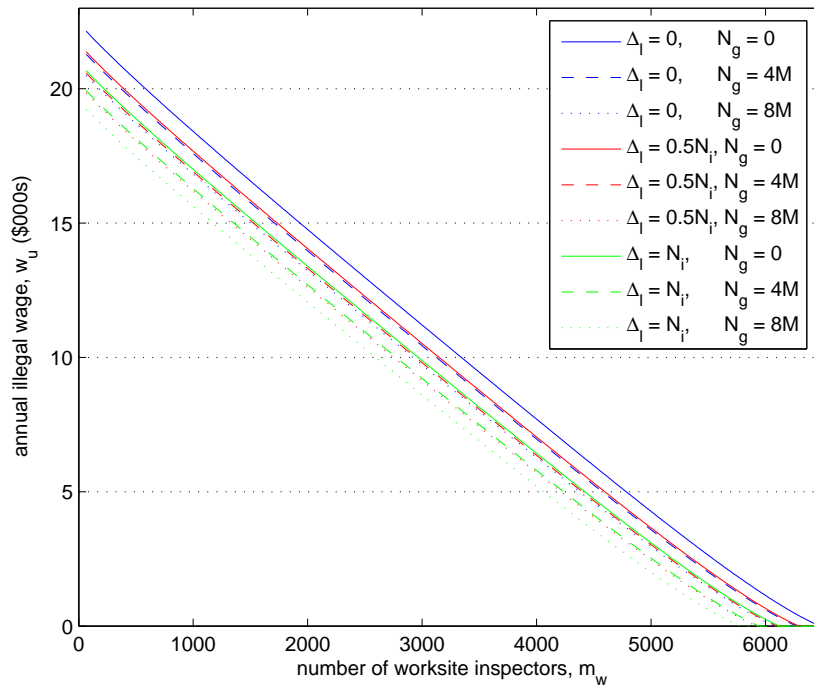


(b)

FIGURE 5



(a)



(b)

FIGURE 6

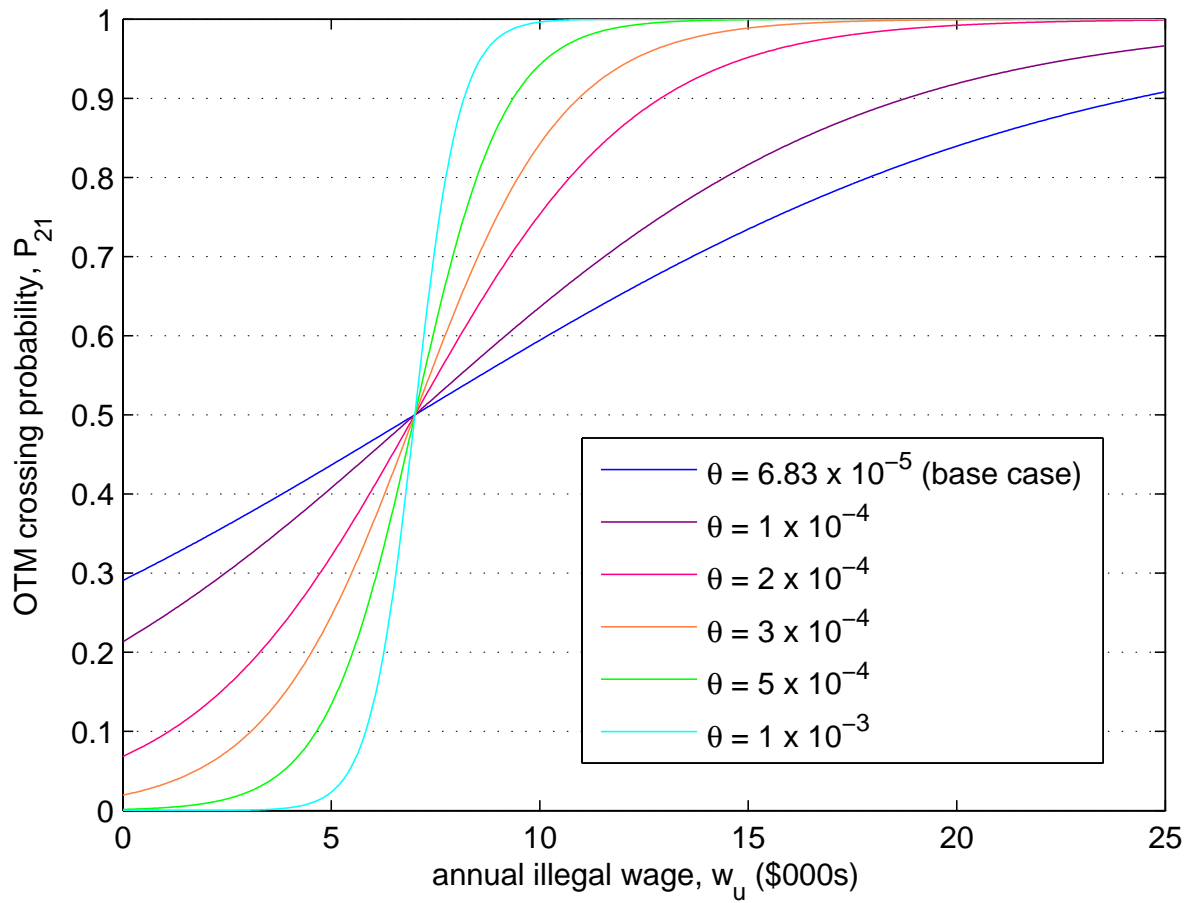
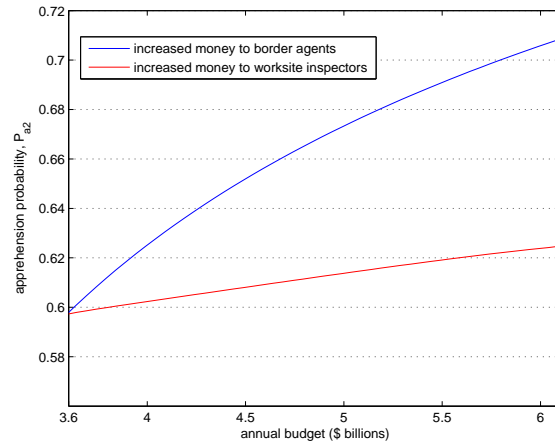
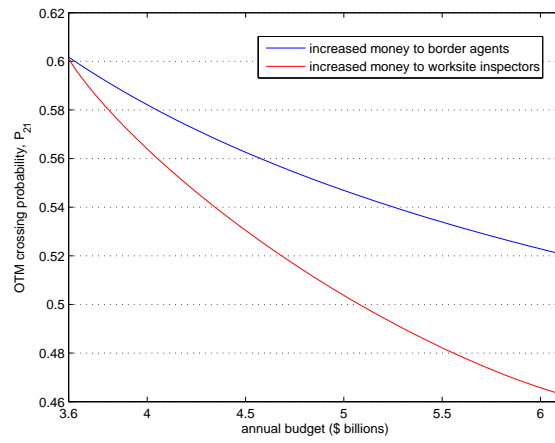


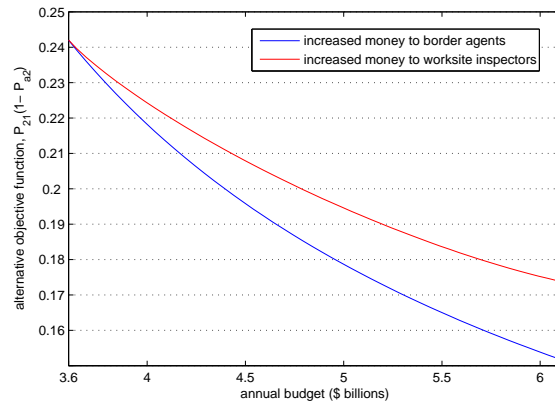
FIGURE 7



(a)



(b)



(c)

FIGURE 8

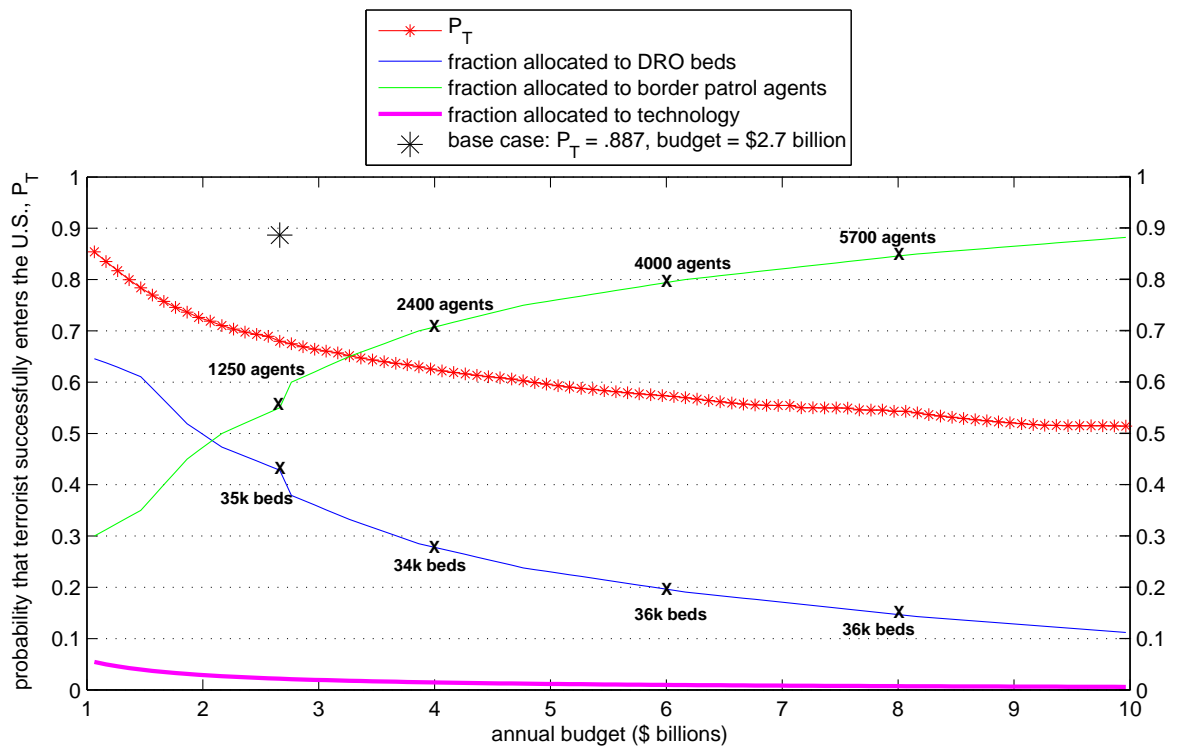


FIGURE 9