Online Companion

The pdfs for the intraperson and interperson similarity scores for all three biometric systems appear in §1. In §2, we derive the six Texture lognormal parameters from §2.4 of the main text. Numerical precision issues are discussed in §3. In §4, we derive properties of the lognormal distribution that are used in §5 to derive the detection probability given in equation (40) of the main text. Also, Tables 1-3 provide the details of the optimal versions of the Minutiae-Face strategy, the Minutiae-Texture strategy, and the lower bound version of the Minutiae-Face strategy, respectively.

1 Probability Density Functions for the Similarity Scores

For the Minutiae system, the intraperson similarity score pdfs are

$$g_i^{(1)}(x) = \frac{1}{\sigma_i^{(1)}\sqrt{2\pi}x} e^{-(\ln x - \mu_i^{(1)})^2/2(\sigma_i^{(1)})^2}, \tag{1}$$

and the interperson similarity score pdfs are

$$h_i^{(1)}(y) = \frac{1}{\tilde{\sigma}_i^{(1)}\sqrt{2\pi}y} e^{-(\ln y - \tilde{\mu}_i^{(1)})^2/2(\tilde{\sigma}_i^{(1)})^2}. \tag{2}$$

For the Face system, the intraperson similarity score pdf is

$$g^{(\mathrm{f})}(x) = \frac{1}{(\theta^{(\mathrm{f})})^{\lambda^{(\mathrm{f})}}\Gamma(\lambda^{(\mathrm{f})})} x^{\lambda^{(\mathrm{f})}-1} e^{-x/\theta^{(\mathrm{f})}}, \tag{3}$$

and the interperson similarity score pdf is

$$h^{(\mathrm{f})}(y) = \frac{1}{\sigma^{(\mathrm{f})}\sqrt{2\pi}y} e^{-(\ln y - \mu^{(\mathrm{f})})^2/2(\sigma^{(\mathrm{f})})^2}. \tag{4}$$

The quality-8 intraperson similarity scores for Galaxy and Texture have the joint pdf

$$g_8(x_1, x_2) = \frac{1}{2\pi x_1 x_2 \sigma_8^{(1)}\sigma_8^{(2)}\sqrt{1-\rho_8^2}} e^{-q/2}, \tag{5}$$

where

$$q = \frac{1}{1-\rho_8^2}\left[\left(\frac{\ln x_1 - \mu_8^{(1)}}{\sigma_8^{(1)}}\right)^2 - 2\rho_8\left(\frac{\ln x_1 - \mu_8^{(1)}}{\sigma_8^{(1)}}\right)\left(\frac{\ln x_2 - \mu_8^{(2)}}{\sigma_8^{(2)}}\right) + \left(\frac{\ln x_2 - \mu_8^{(2)}}{\sigma_8^{(2)}}\right)^2\right], \tag{6}$$

and ρ_8 is the correlation coefficient of $\ln x_1$ and $\ln x_2$ (e.g., Yue 2000). Similarly, the quality-8 interperson similarity scores for Galaxy and Texture have the joint pdf

$$h_8(y_1, y_2) = \frac{1}{2\pi y_1 y_2 \tilde{\sigma}_8^{(1)} \tilde{\sigma}_8^{(2)} \sqrt{1-\tilde{\rho}_8^2}} e^{-\tilde{q}/2}, \tag{7}$$

where

$$\tilde{q} = \frac{1}{1-\tilde{\rho}_8^2} \left[\left(\frac{\ln y_1 - \tilde{\mu}_8^{(1)}}{\tilde{\sigma}_8^{(1)}} \right)^2 - 2\tilde{\rho}_8 \left(\frac{\ln y_1 - \tilde{\mu}_8^{(1)}}{\tilde{\sigma}_8^{(1)}} \right) \left(\frac{\ln y_2 - \tilde{\mu}_8^{(2)}}{\tilde{\sigma}_8^{(2)}} \right) + \left(\frac{\ln y_2 - \tilde{\mu}_8^{(2)}}{\tilde{\sigma}_8^{(2)}} \right)^2 \right]. \tag{8}$$

2 Derivation of the Texture Lognormal Parameters

If we define

$$\kappa(c,t) = \mathbf{P}(cX^{(1)} + (1-c)X^{(2)} \le t), \tag{9}$$

$$= \int_0^{t/c} \int_0^{(t-cx_1)/(1-c)} g_8(x_1, x_2)\, dx_2\, dx_1, \tag{10}$$

$$\xi(c,t) = \mathbf{P}(cY^{(1)} + (1-c)Y^{(2)} \le t), \tag{11}$$

$$= \int_0^{t/c} \int_0^{(t-cx_1)/(1-c)} h_8(x_1, x_2)\, dx_2\, dx_1, \tag{12}$$

and their partial derivatives

$$\frac{\partial \kappa}{\partial c} = \int_0^{t/c} \frac{(t-x_1)}{(1-c)^2}\, g_8\!\left(x_1, \frac{(t-cx_1)}{(1-c)}\right) dx_1, \tag{13}$$

$$\frac{\partial \kappa}{\partial t} = \int_0^{t/c} \frac{1}{(1-c)}\, g_8\!\left(x_1, \frac{(t-cx_1)}{(1-c)}\right) dx_1, \tag{14}$$

$$\frac{\partial \xi}{\partial c} = \int_0^{t/c} \frac{(t-x_1)}{(1-c)^2}\, h_8\!\left(x_1, \frac{(t-cx_1)}{(1-c)}\right) dx_1, \tag{15}$$

$$\frac{\partial \xi}{\partial t} = \int_0^{t/c} \frac{1}{(1-c)}\, h_8\!\left(x_1, \frac{(t-cx_1)}{(1-c)}\right) dx_1, \tag{16}$$

then the data described in §2.4 of the main text can be expressed as 12 equations in terms of the 10 lognormal parameters in (5)-(8) and two Lagrange multipliers, λ_1 and λ_2. Equations (17)-(24) below stem from the two points on the Galaxy ROC curve and the two points on the 80-20 ROC curve, and equations (25)-(28) below are the first-order conditions (Ariel 1976) arising from the fact that the 80-20 weights maximize the detection probability subject to the false positive probability being less than or equal to 0.001 and 0.0001. If we let the gallery size $n = 600$ then the 12 equations are

$$1 - \kappa(1, 980) = 0.5631, \tag{17}$$

$$1 - \kappa(1, 1150) = 0.5022, \tag{18}$$

$$1 - \xi^n(1, 980) = 0.0010, \tag{19}$$

$$1 - \xi^n(1, 1150) = 0.0001, \tag{20}$$

$$1 - \kappa(0.2, 730) = 0.8188, \tag{21}$$

$$1 - \kappa(0.2, 900) = 0.7573, \tag{22}$$

$$1 - \xi^n(0.2, 730) = 0.0010, \tag{23}$$

$$1 - \xi^n(0.2, 900) = 0.0001, \tag{24}$$

$$\left.\frac{\partial \kappa}{\partial c}\right|_{\substack{c=0.2\\t=730}} + \lambda_1 n \xi^{n-1} \left.\frac{\partial \xi}{\partial c}\right|_{\substack{c=0.2\\t=730}} = 0, \tag{25}$$

$$\left.\frac{\partial \kappa}{\partial t}\right|_{\substack{c=0.2\\t=730}} + \lambda_1 n \xi^{n-1} \left.\frac{\partial \xi}{\partial t}\right|_{\substack{c=0.2\\t=730}} = 0, \tag{26}$$

$$\left.\frac{\partial \kappa}{\partial c}\right|_{\substack{c=0.2\\t=900}} + \lambda_2 n \xi^{n-1} \left.\frac{\partial \xi}{\partial c}\right|_{\substack{c=0.2\\t=900}} = 0, \tag{27}$$

$$\left.\frac{\partial \kappa}{\partial t}\right|_{\substack{c=0.2\\t=900}} + \lambda_2 n \xi^{n-1} \left.\frac{\partial \xi}{\partial t}\right|_{\substack{c=0.2\\t=900}} = 0. \tag{28}$$

Equations (17)-(20) can be solved independently of equations (21)-(28) for the four marginal Galaxy parameters. We first solve (17)-(20) and then substitute these four values into (21)-(28) to solve for the remaining six lognormal parameters and the two Lagrange multipliers. The six lognormal parameters that are used in our analysis are displayed in Table 2 of the main text.

3 Numerical Precision Issues

In this section, we address the question of numerical precision, which forced us to take a different approach for deriving the false positive probability than was used in Wein and Baveja (2005) and §3.2 in the main text. By equation (24) in the main text, the false positive probability f_i, given that visitor image quality is i, can be expressed as

$$f_i = 1-(1-\epsilon)^n, \tag{29}$$

$$\approx n\epsilon, \tag{30}$$

where ϵ is the probability that the k^{th} record and the visitor's prints will erroneously match. For the setting considered in Wein and Baveja (2005) and in this paper, $f_i \approx 10^{-2}$ (for poor image qualities) and $n \approx 10^7$, implying that $\epsilon \approx 10^{-9}$. Our approach in the single-stage strategies in Wein and Baveja (2005) and in the two-stage strategy in §3.2 in the main text was to compute $1-\epsilon$ to an absolute error of 10^{-16} (machine precision) using MATLAB which resulted in ϵ having 7 digits of precision. However, for the two-stage strategy in this section, which requires the evaluation of bivariate lognormal cdfs and the computation of double integrals, it is not feasible to get even a few digits of precision in ϵ via the computation of $1-\epsilon$. Hence, in this subsection, we derive an expression for ϵ directly and then evaluate it with a relative error of 10^{-9} using CUBPACK in MATLAB (Cools and Haegemans 2003, Nuyens 2006), which together with the computation of (29) via MATLAB's MAPLE interface in 32-digit precision, achieves the requisite precision. Although this approach is not necessary to compute the detection probability for the Minutiae-Texture strategy, we nonetheless use this approach in §3.3 of the main text so as to streamline our presentation.

4 Properties of the Lognormal Distribution

Let Z_1 be a lognormal random variable with parameters (μ_1, σ_1) and cdf $L_1(z_1)$. Let $\Phi(a)$ denote the standard normal distribution with mean 0 and variance 1. The result $1-\Phi(a) = \Phi(-a)\ \ \forall a \in \mathbb{R}$ implies that

$$\mathbf{P}(Z_1 \geq z_1) = L_1\Big(\frac{e^{2\mu_1}}{z_1}\Big) \quad \forall z_1 \geq 0, \tag{31}$$

$$\mathbf{P}(z_1 \leq Z_1 \leq z_2) = L_1\Big(\frac{e^{2\mu_1}}{z_1}\Big) - L_1\Big(\frac{e^{2\mu_1}}{z_2}\Big) \quad \forall z_1, z_2 \geq 0. \tag{32}$$

Let S_1 and S_2 be two standard normal random variables with correlation ρ and joint cdf $\Phi_\rho(s_1, s_2)$. Define $S_3 = -S_1$. Then S_3 is also a standard normal random variable and the joint cdf of S_3 and S_2 is $\Phi_{-\rho}(s_1, s_2)$, and hence

$$\mathbf{P}(S_1 \geq s_1, S_2 \leq s_2) = \mathbf{P}(S_3 \leq -s_1, S_2 \leq s_2), \tag{33}$$

$$= \Phi_{-\rho}(-s_1, s_2) \quad \forall s_1, s_2 \geq 0. \tag{34}$$

Now consider Z_1 and Z_2 to be lognormally distributed random variables with parameters (μ_1, σ_1) and (μ_2, σ_2) respectively and joint correlation ρ. Let $L_1(z_1)$ and $L_2(z_2)$ be the respective marginal cdfs and $L(z_1, z_2)$ be the joint cdf. Further, let $\bar{L}(z_1, z_2)$ be the joint cdf of Z_1 and Z_2 if the correlation was $-\rho$ instead of ρ. Then equation (34) implies that

$$\mathbf{P}(Z_1 \geq z_1, Z_2 \leq z_2) = \bar{L}\Big(\frac{e^{2\mu_1}}{z_1}, z_2\Big) \quad \forall z_1, z_2 \geq 0, \tag{35}$$

$$\mathbf{P}(a_1 \leq Z_1 \leq a_2, Z_2 \leq b_1) = \bar{L}\Big(\frac{e^{2\mu_1}}{a_1}, b_1\Big) - \bar{L}\Big(\frac{e^{2\mu_1}}{a_2}, b_1\Big) \quad \forall a_1, a_2, b_1 \geq 0. \tag{36}$$

Finally, let $l_{z_2}(z_1)$ denote the conditional pdf of Z_1 given $Z_2 = z_2$. From Yue (2000), we have that $l_{z_2}(z_1)$ is also a lognormal pdf with mean μ and standard deviation σ of the underlying normal distribution given by

$$\mu = \mu_1 + \frac{\rho\sigma_1(\ln(z_2) - \mu_2)}{\sigma_2}, \tag{37}$$

$$\sigma = \sigma_1\sqrt{(1 - \rho^2)}. \tag{38}$$

We use this result to ease the computational burden by simplifying double integrals of the following type to single integrals,

$$\int_0^a \int_0^b m(z_2) l(z_1, z_2) \, dz_2 \, dz_1 = \int_0^b m(z_2) L_{z_2}(a) l(z_2) \, dz_2, \tag{39}$$

where $m(z_2)$ is any function of z_2.

5 Derivation of the Detection Probability for the Minutiae-Texture Strategy

In this section, we derive the detection probability d_i, which is given in equation (40) of the main text, in two steps: first we express d_i in terms of the probabilities of the various events in equations (34)-(39) of the main text, and then we evaluate these probabilities using the results in §4.

To perform the first step, we let $D_i^{(1)}$ be the event that prints of an illegal visitor of image quality $Q_v = i$ and his corresponding prints on the watchlist pass the Minutiae biometric rule, and let $D_i^{(2)}$ be the corresponding event for the Texture matching process. It follows by equations (34)-(39) of the main text that $D_i^{(1)} = \{(M_1 \cup M_2 \cup M_3)|Q_v = i\}$ and $D^{(2)} = \{(N_1 \cup N_2 \cup N_3)|Q_v = i\}$. The detection probability can be expressed as

$$d_i = \mathbf{P}(D_i^{(1)} \cap D_i^{(2)}), \tag{40}$$

$$= \mathbf{P}(D_i^{(1)}) + \mathbf{P}(D_i^{(2)}) - \mathbf{P}(D_i^{(1)} \cup D_i^{(2)}). \tag{41}$$

If we denote the complement of a generic event E by $\overline{E}$, then the three expressions on the right side of (41) can be rewritten as

$$\mathbf{P}(D_i^{(1)}) = \Big(\mathbf{P}(M_1) + \mathbf{P}(M_2 \cap \overline{M_1}) + \mathbf{P}(M_3 \cap \overline{M_2} \cap \overline{M_1})\Big), \tag{42}$$

$$\mathbf{P}(D_i^{(2)}) = \Big(\mathbf{P}(N_1) + \mathbf{P}(N_2 \cap \overline{N_1}) + \mathbf{P}(N_3 \cap \overline{N_2} \cap \overline{N_1})\Big), \tag{43}$$

$$\begin{aligned}\mathbf{P}(D_i^{(1)} \cup D_i^{(2)}) = &\Big(\mathbf{P}(N_1) + \mathbf{P}(N_2 \cap \overline{N_1}) + \mathbf{P}(N_3 \cap \overline{N_2} \cap \overline{N_1})\Big) + \\ &\Big(\mathbf{P}(M_1 \cap (\bigcap_{j=1}^{3} \overline{N_j})) + \mathbf{P}(M_2 \cap \overline{M_1} \cap (\bigcap_{j=1}^{3} \overline{N_j})) + \mathbf{P}(M_3 \cap \overline{M_2} \cap \overline{M_1} \cap (\bigcap_{j=1}^{3} \overline{N_j}))\Big).\end{aligned} \tag{44}$$

Substituting (42)-(44) into (41) yields

$$\begin{aligned}d_i = &\Big(\mathbf{P}(M_1) + \mathbf{P}(M_2 \cap \overline{M_1}) + \mathbf{P}(M_3 \cap \overline{M_2} \cap \overline{M_1})\Big) - \\ &\Big(\mathbf{P}(M_1 \cap (\bigcap_{j=1}^{3} \overline{N_j})) + \mathbf{P}(M_2 \cap \overline{M_1} \cap (\bigcap_{j=1}^{3} \overline{N_j})) + \mathbf{P}(M_3 \cap \overline{M_2} \cap \overline{M_1} \cap (\bigcap_{j=1}^{3} \overline{N_j}))\Big).\end{aligned} \tag{45}$$

Turning to the second step of the derivation, we use the lognormal results in §4 and the definitions in (34)-(39) of the main text to write the six probabilities on the right side of equation (45) in terms of the relevant intraperson score distributions:

$$\mathbf{P}\left(M_1\right) = G_i^{(1)}\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}\Big), \tag{46}$$

$$\mathbf{P}\left(M_2 \cap \overline{M_1}\right) = G_i^{(1)}\left(t_{1i}\right) G_i^{(1)}\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}\Big), \tag{47}$$

$$\begin{aligned}
&\mathbf{P}\left(M_3 \cap \overline{M_2} \cap \overline{M_1}\right) \\
&= \int_0^{t_{1i}} \mathbf{P}\left(t_{2i} - x_1 \leq X_r^{(1)} \leq t_{1i}\right) g_i^{(1)}\left(x_1\right)\ dx_1, && (48)\\
&= \int_{t_{2i}-t_{1i}}^{t_{1i}} \mathbf{P}\left(t_{2i} - x_1 \leq X_r^{(1)} \leq t_{1i}\right) g_i^{(1)}\left(x_1\right)\ dx_1, && (49)\\
&= \int_{t_{2i}-t_{1i}}^{t_{1i}} \Big(G_i^{(1)}\Big(\frac{e^{2\mu_i^{(1)}}}{(t_{2i}-x_1)}\Big) - G_i^{(1)}\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}\Big)\Big) g_i^{(1)}\left(x_1\right)\ dx_1, && (50)\\
&= \int_{t_{2i}-t_{1i}}^{t_{1i}} G_i^{(1)}\Big(\frac{e^{2\mu_i^{(1)}}}{(t_{2i}-x_1)}\Big) g_i^{(1)}\left(x_1\right)\ dx_1 + G_i^{(1)}\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}\Big)\Big[G_i^{(1)}\left(t_{2i}-t_{1i}\right) - G_i^{(1)}\left(t_{1i}\right)\Big], && (51)
\end{aligned}$$

$$\begin{aligned}
&\mathbf{P}\Big(M_1 \cap \big(\bigcap_{j=1}^{3} \overline{N_j}\big)\Big) \\
&= \int_0^{u_{1i}} \int_{t_{1i}}^{\infty} \mathbf{P}\left(X_r^{(2)} \leq \min\left(u_{2i} - x_2, u_{1i}\right)\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, && (52)\\
&= \int_0^{u_{2i}-u_{1i}} \int_{t_{1i}}^{\infty} \mathbf{P}\left(X_r^{(2)} \leq u_{1i}\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2 + \int_{u_{2i}-u_{1i}}^{u_{1i}} \int_{t_{1i}}^{\infty} \mathbf{P}\left(X_r^{(2)} \leq u_{2i} - x_2\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, && (53)\\
&= G_i^{(2)}\left(u_{1i}\right) \int_0^{u_{2i}-u_{1i}} \int_{t_{1i}}^{\infty} g_i\left(x_1, x_2\right)\ dx_1\ dx_2 + \int_{u_{2i}-u_{1i}}^{u_{1i}} \int_{t_{1i}}^{\infty} G_i^{(2)}\left(u_{2i} - x_2\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, && (54)\\
&= G_i^{(2)}\left(u_{1i}\right) \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{2i} - u_{1i}\Big) + \int_{u_{2i}-u_{1i}}^{u_{1i}} G_i^{(2)}\left(u_{2i} - x_2\right) \widetilde{G}_{i_{x_2}}^{(1)}\Big(\frac{e^{2\mu_i^{(1)}(x_2)}}{t_{1i}}\Big) g_i^{(2)}\left(x_2\right)\ dx_2, && (55)
\end{aligned}$$

$$\mathbf{P}\Big(M_2 \cap \overline{M_1} \cap (\bigcap_{j=1}^{3} \overline{N_j})\Big)$$

$$= \int_0^{u_{1i}} \int_0^{t_{1i}} \mathbf{P}\left(X_r^{(1)} \geq t_{1i}, X_r^{(2)} \leq \min\left(u_{2i} - x_2, u_{1i}\right)\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, \tag{56}$$

$$\begin{aligned} &= \int_0^{u_{2i}-u_{1i}} \int_0^{t_{1i}} \mathbf{P}\left(X_r^{(1)} \geq t_{1i}, X_r^{(2)} \leq u_{1i}\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2 \\ &+ \int_{u_{2i}-u_{1i}}^{u_{1i}} \int_0^{t_{1i}} \mathbf{P}\left(X_r^{(1)} \geq t_{1i}, X_r^{(2)} \leq u_{2i} - x_2\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, \end{aligned} \tag{57}$$

$$= \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{1i}\Big) G_i\left(t_{1i}, u_{2i} - u_{1i}\right) + \int_{u_{2i}-u_{1i}}^{u_{1i}} \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{2i} - x_2\Big) \widetilde{G}_{i_{x_2}}^{(1)}\left(t_{1i}\right) g_i^{(2)}\left(x_2\right)\ dx_2, \tag{58}$$

$$\mathbf{P}\Big(M_3 \cap \overline{M_2} \cap \overline{M_1} \cap (\bigcap_{j=1}^{3} \overline{N_j})\Big)$$

$$= \int_0^{u_{1i}} \int_0^{t_{1i}} \mathbf{P}\left(t_{2i} - x_1 \leq X_r^{(1)} \leq t_{1i}, X_r^{(2)} \leq \min\left(u_{2i} - x_2, u_{1i}\right)\right) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, \tag{59}$$

$$\begin{aligned} &= \int_0^{u_{2i}-u_{1i}} \int_{t_{2i}-t_{1i}}^{t_{1i}} \Bigg(\bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{(t_{2i} - x_1)}, u_{1i}\Big) - \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{1i}\Big)\Bigg) g_i\left(x_1, x_2\right)\ dx_1\ dx_2 \\ &+ \int_{u_{2i}-u_{1i}}^{u_{1i}} \int_{t_{2i}-t_{1i}}^{t_{1i}} \Bigg(\bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{(t_{2i} - x_1)}, u_{2i} - x_2\Big) - \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{2i} - x_2\Big)\Bigg) g_i\left(x_1, x_2\right)\ dx_1\ dx_2, \end{aligned} \tag{60}$$

$$\begin{aligned} &= \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{1i}\Big)\Big(G_i\left(t_{2i} - t_{1i}, u_{2i} - u_{1i}\right) - G_i\left(t_{1i}, u_{2i} - u_{1i}\right)\Big) \\ &+ \int_{t_{2i}-t_{1i}}^{t_{1i}} \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{(t_{2i} - x_1)}, u_{1i}\Big) \widetilde{G}_{i_{x_1}}^{(2)}\left(u_{2i} - u_{1i}\right) g_i^{(1)}\left(x_1\right)\ dx_1 \\ &- \int_{u_{2i}-u_{1i}}^{u_{1i}} \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{t_{1i}}, u_{2i} - x_2\Big) \Big(\widetilde{G}_{i_{x_2}}^{(1)}\left(t_{1i}\right) - \widetilde{G}_{i_{x_2}}^{(1)}\left(t_{2i} - t_{1i}\right)\Big) g_i^{(2)}\left(x_2\right)\ dx_2 \\ &+ \int_{u_{2i}-u_{1i}}^{u_{1i}} \int_{t_{2i}-t_{1i}}^{t_{1i}} \bar{G}_i\Big(\frac{e^{2\mu_i^{(1)}}}{(t_{2i} - x_1)}, u_{2i} - x_2\Big) g_i\left(x_1, x_2\right)\ dx_1\ dx_2. \end{aligned} \tag{61}$$

Replacing the various expressions derived above into equation (45) for d_i and noting that (47) and (58) cancel out with terms in (51) and (61), respectively, we get equation (40) in the main text.

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Image Quality	Minutiae Thresholds	Face Threshold	d_i	f_i	$E[N_i]$
1	3597, 4911	443	0.841	0.00160	0.09
2	3278, 4451	388	0.841	0.00160	0.09
3	3035, 4111	328	0.841	0.00160	0.09
4	2660, 3757	394	0.841	0.00160	0.09
5	2202, 3128	376	0.841	0.00161	0.09
6	1523, 2111	309	0.841	0.00167	0.09
7	1499, 1649	416	0.841	0.00245	0.09
8	740, 741	783	0.841	0.03449	1.97

Table 1: Results for the Minutiae-Face strategy, including the optimal threshold values, and the detection probability, false positive probability and mean candidate list for each visitor image quality.

Image Quality	Minutiae Thresholds	Texture Thresholds	d_i	f_i	$E[N_i]$
1	2934, 4421	0, 0	0.894	0.00166	368.79
2	2695, 3928	0, 0	0.894	0.00166	368.79
3	2507, 3588	0, 0	0.894	0.00166	368.79
4	2232, 3253	0, 0	0.894	0.00166	368.79
5	1884, 2479	0, 0	0.894	0.00172	368.79
6	1413, 1764	0, 0	0.894	0.00354	368.80
7	1229, 1392	0, 0	0.894	0.06693	368.86
8	216, 427	1526, 1565	0.894	0.00309	8195.40

Table 2: Results for the Minutiae-Texture strategy, including the optimal threshold values, and the detection probability, false positive probability and mean candidate list for each visitor image quality.

Image Quality	Minutiae Thresholds	Texture Thresholds	d_i	f_i	$E[N_i]$
1	2571, 4350	12, 17	0.913	0.00166	1108
2	2512, 3651	12, 17	0.913	0.00166	1108
3	2229, 3427	12, 17	0.913	0.00166	1108
4	2076, 3004	12, 17	0.913	0.00166	1108
5	1683, 2258	12, 17	0.913	0.00183	1108
6	949, 978	1347, 1352	0.913	0.02085	12442
7	693, 717	1310, 1342	0.913	0.02283	12352
8	243, 258	1437, 1442	0.913	0.00922	9263

Table 3: Results for the lower bound version of the Minutiae-Texture strategy, including the optimal threshold values, and the detection probability, false positive probability and mean candidate list for each visitor image quality.