Appendix

This is a technical appendix for the paper "Term Limits and Pork." If the paper is accepted, this appendix will be posted on the *LSQ* website, and the authors will also send it directly to any reader who requests a copy.

We first state two lemmas that specify first period legislator preferences over different types of proposals, given that voters use pork-rewarding strategies, as defined in Definition 1. We then state equilibrium strategies and finally demonstrate the optimality of legislators' and voters' strategies in Proposition 1.

Before presenting proofs we introduce some notation to specify whether the benefits of pork barrel projects in a given proposal go to district welfare maximizing (henceforth DWM) or social welfare maximizing (henceforth SWM) legislators. In the equilibria that we characterize, a legislator whose district does not receive any pork is never re-elected. Hence it will be sufficient to specify the types of legislators who receive pork under any proposal, and we do not need to specify whether non-pork-receiving legislators are DWMs or SWMs. We use a D or S in place of a 1 for any district that receives pork. For example, $(0, D, S)$ means that district 1 receives no pork, whereas district 2 (represented by a DWM) and district 3 (represented by a SWM) receive pork.

From the perspective of a SWM legislator, who cares equally about all districts, it does not matter which districts receive pork. Therefore, when evaluating a first period proposal, he only cares about three things: the number of districts that are represented by SWMs and receive pork, the number of districts that are represented by DWMs and receive pork, and the number of districts that receive no pork.

Lemma 1 (SWM legislator) *If voters use pork-rewarding strategies, a SWM legislator* *strictly prefers* $(0,0,0)$ *over any other first period policy.*

Proof. We first find a SWM's expected utility from $x^1 = (0,0,0)$ and then show that it is higher than his utility for any other possible first period policy. The other policies that we must consider are, w.l.o.g., $(0,0,D)$, $(0,0,S)$, $(0,D,S)$, $(0,D,D)$, $(0,S,S)$, (S,S,S), (S,S,D), and (S,D,D).

A SWM's first period utility from the policy $x^1 = (0,0,0)$ is 0. Given that voters use pork-rewarding strategies, all legislators are replaced for the second period. With probability $(1-\pi)^3$ three SWMs are elected in the second period and no pork is enacted. With probability $3(1-\pi)^2\pi$ two SWMs are elected along with a DWM, and no pork is enacted. With probability $3(1-\pi)\pi^2$ two DWMs and one SWM are elected. In this type of legislature, there is a $\frac{2}{3}$ chance that a DWM is chosen as the agenda setter, in which case two pork barrel projects are enacted, with a benefit of b and a cost of c for each project. With probability π^3 three DWMs are elected, in which case two pork barrel projects are enacted for sure. Adding up these terms, a SWM's expected utility from the first period policy $(0,0,0)$ is

$$\begin{aligned} E\left[U^S|x^1=(0,0,0)\right] &= 0+0\cdot(1-\pi)^3+0\cdot 3(1-\pi)^2\pi+\frac{2}{3}(2b-2c)\cdot 3(1-\pi)\pi^2 \\ &\quad +(2b-2c)\pi^3 \\ &= (b-c)\left[4\pi^2-2\pi^3\right] \end{aligned}$$

A SWM's first period utility from the policy $x^1 = (0,0,S)$ is $b-c$. Since voters use pork-rewarding strategies, a SWM is re-elected in the district that received pork, so the second period policy outcome depends on the electoral outcomes in the other two districts. With probability $(1-\pi)^2$ two SWMs win, in which case no pork is enacted.

With probability $2(1-\pi)\pi$ one new legislator is a SWM and the other is a DWM; in this case, since SWMs are a majority no pork is enacted. With probability π^2 two DWMs are elected. In this type of legislature, there is a $\frac{2}{3}$ chance that a DWM is chosen as the agenda setter, in which case two pork barrel projects are enacted, with benefit b and cost c for each project. Thus the SWM's expected utility is

$$\begin{aligned} E\left[U^S|x^1=(0,0,S)\right] &= b-c+0\cdot(1-\pi)^2+0\cdot 2(1-\pi)\pi+\frac{2}{3}(2b-2c)\pi^2 \\ &= (b-c)\left[1+\frac{4}{3}\pi^2\right] \end{aligned}$$

Since $c>b$ this is less than $E\left[U^S|x^1=(0,0,0)\right]$ whenever $1+\frac{4}{3}\pi^2>4\pi^2-2\pi^3$, which holds for all values of $\pi\in(0,1)$.

An SWM's first period utility from the policy $x^1=(0,0,D)$ is $b-c$. Since voters use pork-rewarding strategies, a DWM is re-elected in the district that received pork, so the second period policy outcome depends on the electoral outcomes in the other two districts. With probability $(1-\pi)^2$ two SWMs win, in which case no pork is enacted. With probability $2(1-\pi)\pi$ one new legislator is a SWM and the other is a DWM. There is a $\frac{2}{3}$ chance that a DWM is chosen as the agenda setter, in which case two projects are enacted, with a benefit of b and a cost of c for each project. With probability π^2 two new DWMs win and thus two projects are enacted. A SWM's expected utility from $(0,0,D)$ is thus

$$\begin{aligned} E\left[U^S|x^1=(0,0,D)\right] &= b-c+0\cdot(1-\pi)^2+\frac{2}{3}(2b-2c)\cdot 2(1-\pi)\pi+(2b-2c)\pi^2 \\ &= (b-c)\left[1+\frac{8}{3}(1-\pi)\pi+2\pi^2\right] \end{aligned}$$

Since $c>b$ we know that $E\left[U^S|x^1=(0,0,S)\right]>E\left[U^S|x^1=(0,0,D)\right]$. Thus we can also conclude that $E\left[U^S|x^1=(0,0,0)\right]>E\left[U^S|x^1=(0,0,D)\right]$.

The other possible proposals we need to consider are $(0, D, S)$, $(0, D, D)$, $(0, S, S)$, (S, S, S), (S, S, D), and (S, D, D). For each of these proposals, at least two pork projects are enacted in the first period. In contrast, under $(0, 0, 0)$, the maximum possible number of pork projects enacted in both periods combined is two. And, since $\pi < 1$, there is a strictly positive probability that fewer than two total projects will be enacted if $x^1 = (0, 0, 0)$. Thus, since a SWM only cares about minimizing the total amount of pork enacted, he strictly prefers $(0, 0, 0)$ over any of these proposals.■

For a DWM, things are somewhat more complicated. Unlike a SWM, a DWM cares about which districts receive pork. Thus, for example, his expected utility from $(D, 0, S)$ is different from his expected utility from $(0, D, S)$. We characterize the preferences for a DWM in district 1. This characterization is w.l.o.g., when the districts are relabeled appropriately, so we do not explicitly state preferences for DWMs in districts 2 and 3. We use the notation $E\left[U^{D_1}|x^1 = (\cdot, \cdot, \cdot)\right]$ to refer to the expected utility for a DWM in district 1.

Lemma 2 (DWM legislator) *If voters use pork-rewarding strategies, a DWM legislator in district 1 has the following preferences:*

1. $E\left[U^{D_1}|x^1 = (0, 0, 0)\right] > E\left[U^{D_1}|x^1 = (0, 0, S)\right]$
2. $E\left[U^{D_1}|x^1 = (0, 0, 0)\right] > E\left[U^{D_1}|x^1 = (0, 0, D)\right]$
3. $E\left[U^{D_1}|x^1 = (D, 0, 0)\right] > E\left[U^{D_1}|x^1 = (0, 0, 0)\right]$
4. $E\left[U^{D_1}|x^1 = (0, 0, 0)\right] > E\left[U^{D_1}|x^1 = (0, D, D)\right]$
5. $E\left[U^{D_1}|x^1 = (0, 0, 0)\right] > E\left[U^{D_1}|x^1 = (0, S, D)\right]$

6. $E\left[U^{D_1}|x^1=(0,0,0)\right] > E\left[U^{D_1}|x^1=(0,S,S)\right]$

7. $E\left[U^{D_1}|x^1=(0,0,0)\right] > E\left[U^{D_1}|x^1=(D,D,D)\right]$

8. $E\left[U^{D_1}|x^1=(0,0,0)\right] > E\left[U^{D_1}|x^1=(D,S,S)\right]$

9. $E\left[U^{D_1}|x^1=(D,D,0)\right] > E\left[U^{D_1}|x^1=(D,D,S)\right]$

10. $E\left[U^{D_1}|x^1=(D,S,0)\right] > E\left[U^{D_1}|x^1=(0,0,0)\right]$

11. $E\left[U^{D_1}|x^1=(D,D,0)\right] > (<)(=) E\left[U^{D_1}|x^1=(0,0,0)\right]$ *if* $b > (<)(=) \bar{b}$

12. $E\left[U^{D_1}|x^1=(D,D,S)\right] > (<)(=) E\left[U^{D_1}|x^1=(0,0,0)\right]$ *if* $b > (<)(=) c\left[\frac{13-12\pi^2+6\pi^3}{15-12\pi^2+6\pi^3}\right]$

To prove the lemma we first calculate a DWM's expected utility when no pork is enacted in the first period. A DWM's first period utility from the policy $x^1=(0,0,0)$ is 0. Since voters use pork-rewarding strategies, an entirely new slate of legislators takes office in the second period. With probability $(1-\pi)^3$ three SWMs win, in which case no pork is enacted. With probability $3(1-\pi)^2\pi$ two SWMs and one DWM are elected, in which case no pork is enacted. With probability $2\cdot\pi^2(1-\pi)$ a DWM is elected in district 1 along with one other DWM and one SWM. There is a $\frac{2}{3}$ chance that one of the DWMs is chosen as the agenda setter, in which case district 1 receives benefit b from pork and pays cost $\frac{2}{3}c$. With probability $(1-\pi)\pi^2$ a SWM is elected in district 1 along with two DWMs. There is a $\frac{2}{3}$ chance that one of the DWMs is chosen as the agenda setter, in which case district 1 receives no pork yet pays cost $\frac{2}{3}c$. With probability π^3, three DWMs are elected, in which case pork is enacted for sure; there is a $\frac{2}{3}$ chance that district 1 will be one of the districts that receives pork, and the district pays a cost of $\frac{2}{3}c$

regardless of which two districts receive pork. Thus, we have

$$\begin{aligned}
E\left[U^{D_1}|x^1=(0,0,0)\right] &= 0+0\cdot(1-\pi)^3+0\cdot 3(1-\pi)^2\pi+\frac{2}{3}\left(b-\frac{2}{3}c\right)\cdot 2\pi^2(1-\pi)+ \\
&\quad +\frac{2}{3}\left(-\frac{2}{3}c\right)(1-\pi)\pi^2+(\frac{2}{3}b-\frac{2}{3}c)\pi^3 \\
&= (b-c)\left[\frac{4}{3}\pi^2-\frac{2}{3}\pi^3\right]
\end{aligned}$$

For part 1 of the lemma, we calculate $E\left[U^{D_1}|x^1=(0,0,S)\right]$. District 1 pays a cost of $\frac{1}{3}c$ in the first period. In the second period, with probability π^2, DWMs are elected from districts 1 and 2, and there is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter, in which case district 1 receives benefit b and pays a cost of $\frac{2}{3}c$. Otherwise no pork is enacted in the second period. Thus, we have

$$\begin{aligned}
E\left[U^{D_1}|x^1=(0,0,S)\right] &= -\frac{1}{3}c+0\cdot(1-\pi)^2+0\cdot 2(1-\pi)\pi+\frac{2}{3}\left(b-\frac{2}{3}c\right)\pi^2 \\
&= \frac{2}{3}\pi^2 b-c\left[\frac{1}{3}+\frac{4}{9}\pi^2\right]
\end{aligned}$$

For $E\left[U^{D_1}|x^1=(0,0,0)\right]>E\left[U^{D_1}|x^1=(0,0,S)\right]$ we need

$$\begin{aligned}
(b-c)\left[\frac{4}{3}\pi^2-\frac{2}{3}\pi^3\right] &> \frac{2}{3}\pi^2 b-c\left[\frac{1}{3}+\frac{4}{9}\pi^2\right] \\
\frac{2}{3}b\pi^2(1-\pi) &> -\frac{c}{3}\left[3-8\pi^2+6\pi^3\right]
\end{aligned}$$

The inequality holds since for all $\pi\in(0,1)$ the left hand side is strictly greater than zero and the right hand side is strictly less than zero.

For part 2 of the lemma, we calculate $E\left[U^{D_1}|x^1=(0,0,D)\right]$. District 1 pays a cost of $\frac{1}{3}c$ in the first period. In the second period, with probability $\pi(1-\pi)$, a DWM is elected from district 1 along with a SWM from district 2, in which case there is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter, and district 1 receives benefit b and pays cost $\frac{2}{3}c$. With probability $(1-\pi)\pi$ a SWM is elected from district 1 along with a DWM from district 2, in which case there is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter and district 1 receives no project yet pays cost $\frac{2}{3}c$. With probability π^2 two new DWMs are elected. In this case, there is a $\frac{2}{3}$ chance that district 1 will be one of the districts that receives pork, and the district always pays a cost of $\frac{2}{3}c$ regardless of which two districts receive pork. Thus we have

$$\begin{aligned} E\left[U^{D_1}|x^1=(0,0,D)\right] &= -\frac{1}{3}c+\frac{2}{3}\left(b-\frac{2}{3}c\right)\cdot\pi(1-\pi)+\frac{2}{3}\left(-\frac{2}{3}c\right)(1-\pi)\pi \\ &\quad +\left(\frac{2}{3}b-\frac{2}{3}c\right)\pi^2 \\ &= \frac{2}{3}\pi b-c\left[\frac{1}{3}+\frac{8}{9}\pi-\frac{2}{9}\pi^2\right] \end{aligned}$$

For $E\left[U^{D_1}|x^1=(0,0,0)\right] > E\left[U^{D_1}|x^1=(0,0,D)\right]$ we need

$$\begin{aligned} (b-c)\left[\frac{4}{3}\pi^2-\frac{2}{3}\pi^3\right] &> \frac{2}{3}\pi b-c\left[\frac{1}{3}+\frac{8}{9}\pi-\frac{2}{9}\pi^2\right] \\ (c-b)\frac{2}{3}\pi\left[1-2\pi+\pi^2\right] &> -\frac{c}{3}\left[1+\frac{2}{3}\pi-\frac{2}{3}\pi^2\right] \end{aligned}$$

The inequality holds since for all $\pi\in(0,1)$ the left hand side is strictly greater than zero and the right hand side is strictly less than zero.

For part 3 of the lemma, we calculate $E\left[U^{D_1}|x^1=(D,0,0)\right]$. In the first period, district 1 receives benefit b and pays cost $\frac{1}{3}c$. In the second period, with probability $(1-\pi)^2$ two new SWMs are elected and no pork is enacted. With probability $2\pi(1-\pi)$ a SWM and a DWM are elected, in which case there is a $\frac{2}{3}$ chance that a DWM is the agenda setter and district 1 receives benefit b and pays cost $\frac{2}{3}c$. With probability π^2 two DWMs are elected. In this case, there is a $\frac{2}{3}$ chance that district 1 will be one of the districts that receives pork, and district 1 always pays a cost of $\frac{2}{3}c$ regardless of which two districts receive pork. Thus, we have

$$E\left[U^{D_1}|x^1=(D,0,0)\right] = b-\frac{c}{3}+0\cdot(1-\pi)^2+\frac{2}{3}\left(b-\frac{2}{3}c\right)2\pi(1-\pi) + \left(\frac{2}{3}b-\frac{2}{3}c\right)\pi^2$$

For $E\left[U^{D_1}|x^1=(D,0,0)\right] > E\left[U^{D_1}|x^1=(0,0,0)\right]$ we need

$$b-\frac{c}{3}+\frac{2}{3}\left(b-\frac{2}{3}c\right)2\pi(1-\pi)+\left(\frac{2}{3}b-\frac{2}{3}c\right)\pi^2 > (b-c)\left[\frac{4}{3}\pi^2-\frac{2}{3}\pi^3\right]$$
$$b-\frac{c}{3}+\frac{2}{3}\left(b-\frac{2}{3}c\right)2\pi(1-\pi) > (b-c)\left[\frac{2}{3}\pi^2-\frac{2}{3}\pi^3\right]$$

The inequality holds for all $\pi \in (0,1)$ since the left hand side is strictly greater than zero and the right hand side is strictly less than zero.

For part 4 of the lemma, we calculate $E\left[U^{D_1}|x^1=(0,D,D)\right]$. District 1 pays a cost of $\frac{2}{3}c$ in the first period. In the second period, with probability π a DWM is elected from district 1. In this case, there is a $\frac{2}{3}$ chance that district 1 will be one of the districts that receives pork, and the district always pays a cost of $\frac{2}{3}c$ regardless of which two districts receive pork. With probability $(1-\pi)$ a SWM is elected from district 1, in which case there is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter and district 1 receives no project yet pays cost $\frac{2}{3}c$. Thus, we have

$$\begin{aligned} E\left[U^{D_1}|x^1=(0,D,D)\right] &= -\frac{2}{3}c+\left(\frac{2}{3}b-\frac{2}{3}c\right)\pi+\frac{2}{3}\left(-\frac{2}{3}c\right)\cdot(1-\pi) \\ &= \frac{2}{3}\pi b-c\left[\frac{10}{9}-\frac{2}{9}\pi\right] \end{aligned}$$

For $E\left[U^{D_1}|x^1=(0,0,0)\right] > E\left[U^{D_1}|x^1=(0,D,D)\right]$ we need

$$(b-c)\left[\frac{4}{3}\pi^2-\frac{2}{3}\pi^3\right] > \frac{2}{3}\pi b-c\left[\frac{10}{9}-\frac{2}{9}\pi\right]$$
$$(c-b)\frac{2}{3}\pi\left[1-2\pi+\pi^2\right] > -c\left[\frac{10}{9}+\frac{4}{9}\pi\right]$$

The inequality holds since for all $\pi \in (0,1)$ the left hand side is strictly greater than zero and the right hand side is strictly less than zero.

For part 5 of the lemma, we calculate $E\left[U^{D_1}|x^1 = (0,S,D)\right]$. District 1 pays a cost of $\frac{2}{3}c$ in the first period. In the second period, with probability π a DWM is elected from district 1, in which case there is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter and district 1 receives benefit b and pays a cost of $\frac{2}{3}c$. With probability $(1-\pi)$ a SWM is elected so no pork is enacted in the second period. Thus, we have

$$\begin{aligned} E\left[U^{D_1}|x^1 = (0,S,D)\right] &= -\frac{2}{3}c + \frac{2}{3}\left(b - \frac{2}{3}c\right)\pi \\ &= \frac{2}{3}\pi b - c\left[\frac{2}{3} + \frac{4}{9}\pi\right] \end{aligned}$$

For $E\left[U^{D_1}|x^1 = (0,0,0)\right] > E\left[U^{D_1}|x^1 = (0,S,D)\right]$ we need

$$\begin{aligned} (b-c)\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right] &> \frac{2}{3}\pi b - c\left[\frac{2}{3} + \frac{4}{9}\pi\right] \\ (c-b)\frac{2}{3}\pi\left[1 - 2\pi + \pi^2\right] &> -c\left[\frac{2}{3} - \frac{2}{9}\pi\right] \end{aligned}$$

The inequality holds since for all $\pi \in (0,1)$ the left hand side is strictly greater than zero and the right hand side is strictly less than zero.

For part 6 of the lemma, we calculate $E\left[U^{D_1}|x^1 = (0,S,S)\right]$. District 1 receives no benefit and pays cost $\frac{2}{3}c$ in the first period. In the second period both SWMs are re-elected so no pork is enacted. Thus, we have

$$E\left[U^{D_1}|x^1 = (0,S,S)\right] = -\frac{2}{3}c$$

For $E\left[U^{D_1}|x^1 = (0,0,0)\right] > E\left[U^{D_1}|x^1 = (0,S,S)\right]$ we need

$$\begin{aligned} (b-c)\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right] &> -\frac{2}{3}c \\ b\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right] + c\left[\frac{2}{3} - \frac{4}{3}\pi^2 + \frac{2}{3}\pi^3\right] &> 0 \end{aligned}$$

This inequality holds for all $\pi \in (0,1)$.

For part 7 of the lemma, we calculate $E\left[U^{D_1}|x^1 = (D,D,D)\right]$. District 1 receives benefit b and pays cost c in the first period. All three legislators are re-elected. In the second period there is a $\frac{2}{3}$ chance that district 1 will be one of the districts that receives pork, and the district always pays a cost of $\frac{2}{3}c$ regardless of which two districts receive pork. Thus we have

$$\begin{aligned} E\left[U^{D_1}|x^1 = (D,D,D)\right] &= b - c + \left(\frac{2}{3}b - \frac{2}{3}c\right) \\ &= \frac{5}{3}(b-c) \end{aligned}$$

Since $\frac{5}{3} > \left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right]$ for all $\pi \in (0,1)$ we know that $E\left[U^{D_1}|x^1 = (D,D,D)\right] < E\left[U^{D_1}|x^1 = (0,0,0)\right] = (b-c)\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right]$.

For part 8 of the lemma, we calculate $E\left[U^{D_1}|x^1 = (D,S,S)\right]$. In the first period, district 1 receives benefit b and pays cost c. All three legislators are re-elected and in the second period no pork is enacted. Thus $E\left[U^{D_1}|x^1 = (D,S,S)\right] = b - c$, which is strictly less than $E\left[U^{D_1}|x^1 = (0,0,0)\right] = (b-c)\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right]$ for all $\pi \in (0,1)$.

For part 9 of the lemma, we calculate $E\left[U^{D_1}|x^1 = (D,D,0)\right]$ as well as $E\left[U^{D_1}|x^1 = (D,D,S)\right]$. When $x^1 = (D,D,0)$, district 1 receives benefit b and pays cost $\frac{2}{3}c$ in the first period. In the second period, with probability $(1-\pi)$ the new legislator in district 3 is a SWM. There is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter, in which case district 1 receives benefit b and pays cost $\frac{2}{3}c$. With probability π the new legislator is a DWM and in this case there is a $\frac{2}{3}$ chance that district 1 will be one of the districts that receives pork, whereas the district always pays a cost of $\frac{2}{3}c$ regardless of which two districts receive pork. Thus, we have

$$\begin{aligned} E\left[U^{D_1}|x^1=(D,D,0)\right] &= b-\frac{2}{3}c+\frac{2}{3}\left(b-\frac{2}{3}c\right)(1-\pi)+\left(\frac{2}{3}b-\frac{2}{3}c\right)\pi \\ &= (b-c)\frac{2}{3}\pi+\left(b-\frac{2}{3}c\right)\left[\frac{5}{3}-\frac{2}{3}\pi\right] \end{aligned}$$

When $x^1 = (D,D,S)$, district 1 receives benefit b and pays cost c in the first period. All legislators are re-elected. There is a $\frac{2}{3}$ chance that one of the DWMs is the agenda setter, in which case district 1 receives benefit b and pays cost $\frac{2}{3}c$. Thus we have

$$E\left[U^{D_1}|x^1=(D,D,S)\right]=b-c+\frac{2}{3}\left(b-\frac{2}{3}c\right)$$

For $E\left[U^{D_1}|x^1=(D,D,0)\right] > E\left[U^{D_1}|x^1=(D,D,S)\right]$ we need

$$\begin{aligned} (b-c)\frac{2}{3}\pi+\left(b-\frac{2}{3}c\right)\left[\frac{5}{3}-\frac{2}{3}\pi\right] &> b-c+\frac{2}{3}\left(b-\frac{2}{3}c\right) \\ (c-b)\left(1-\frac{2}{3}\pi\right) &> \left(b-\frac{2}{3}c\right)\cdot\left(\frac{2}{3}\pi-1\right) \end{aligned}$$

Since $b\in\left(\frac{2}{3}c,c\right)$ the left hand side is strictly greater than zero and the right hand side is strictly less than zero for all $\pi\in(0,1)$.

For part 10 of the lemma, we calculate $E\left[U^{D_1}|x^1=(D,S,0)\right]$. District 1 receives benefit b and pays cost $\frac{2}{3}c$ in the first period. In the second period with probability $(1-\pi)$ the newly elected legislator is a SWM, in which case no pork is enacted. With probability π, the new legislator is a DWM. There $\frac{2}{3}$ chance that one of the DWMs is the agenda setter, in which case district 1 receives benefit b and pays a cost of $\frac{2}{3}c$. Thus, we have

$$E\left[U^{D_1}|x^1=(D,S,0)\right]=b-\frac{2}{3}c+\frac{2}{3}\left(b-\frac{2}{3}c\right)\pi=\left(b-\frac{2}{3}c\right)\left[1+\frac{2}{3}\pi\right]$$

For $E\left[U^{D_1}|x^1=(D,S,0)\right] > E\left[U^{D_1}|x^1=(0,0,0)\right]$ we need

$$\left(b-\frac{2}{3}c\right)\left[1+\frac{2}{3}\pi\right]>(b-c)\left[\frac{4}{3}\pi^2-\frac{2}{3}\pi^3\right]$$

Since $b \in \left(\frac{2}{3}c, c\right)$ this inequality holds for all $\pi \in (0,1)$.

For part 11 of the lemma, we solve for conditions under which $E\left[U^{D_1}|x^1 = (D,D,0)\right] > E\left[U^{D_1}|x^1 = (0,0,0)\right]$, i.e.,

$$\begin{aligned}
(b-c)\frac{2}{3}\pi + \left(b - \frac{2}{3}c\right)\left[\frac{5}{3} - \frac{2}{3}\pi\right] &> (b-c)\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right] \\
b\left[\frac{2}{3}\pi + \frac{5}{3} - \frac{2}{3}\pi - \frac{4}{3}\pi^2 + \frac{2}{3}\pi^3\right] &> c\left[\frac{2}{3}\pi + \frac{10}{9} - \frac{4}{9}\pi - \frac{4}{3}\pi^2 + \frac{2}{3}\pi^3\right] \\
b &> c\left[\frac{\frac{2}{3}\pi^3 - \frac{4}{3}\pi^2 + \frac{2}{9}\pi + \frac{10}{9}}{\frac{5}{3} - \frac{4}{3}\pi^2 + \frac{2}{3}\pi^3}\right] \\
b &> c\left[\frac{6\pi^3 - 12\pi^2 + 2\pi + 10}{15 - 12\pi^2 + 6\pi^3}\right] = \bar{b}
\end{aligned}$$

For part 12 of the lemma, we solve for conditions under which $E\left[U^{D_1}|x^1 = (D,D,S)\right] > E\left[U^{D_1}|x^1 = (0,0,0)\right]$, i.e.,

$$\begin{aligned}
(b-c) + \frac{2}{3}\left(b - \frac{2}{3}c\right) &> (b-c)\left[\frac{4}{3}\pi^2 - \frac{2}{3}\pi^3\right] \\
b\left[\frac{5}{3} - \frac{4}{3}\pi^2 + \frac{2}{3}\pi^3\right] &> c\left[\frac{13}{9} - \frac{4}{3}\pi^2 + \frac{2}{3}\pi^3\right] \\
b &> c\left[\frac{13 - 12\pi^2 + 6\pi^3}{15 - 12\pi^2 + 6\pi^3}\right]
\end{aligned}$$

■

We now prove the main proposition.

Lemma 3 (part 1 of main proposition) *For $b \in \left(\frac{2}{3}c, \bar{b}\right)$ there is an equilibrium in which voters use pork-rewarding strategies. Furthermore, in any equilibrium in which voters use pork-rewarding strategies no pork is enacted in the first period.*

We first analyze optimal behavior by legislators, then show that pork-rewarding strategies are optimal for voters given this legislator behavior.

The possible proposals that could be adopted to enact pork are, w.l.o.g., $(0, 0, D)$, $(0, 0, S)$, $(D, D, 0)$, $(0, S, D)$, $(0, S, S)$, (S, S, S), (S, S, D), (D, D, S), (D, D, D). If voters use pork-rewarding strategies, then we know by Lemma 1 that a SWM legislator will vote for $(0, 0, 0)$ over any other first period policy proposal. The proposals $(0, S, S)$, (S, S, S), and (S, S, D) thus cannot receive majority support since all SWM legislators will vote against them.

The proposals $(0, 0, D)$ and $(0, 0, S)$ cannot receive majority support since they would require at least one aye vote from a legislator whose district did not receive pork. By Lemma 1, no SWM would vote aye and by parts 1 and 2 of Lemma 2, a DWM who did not receive pork would never vote aye for these proposals.

Similarly, the proposal $(0, S, D)$ cannot pass. By Lemma 1, the SWM from district 2 would vote nay. By Lemma 1 and part 5 of Lemma 2, either type of legislator in district 1 would vote nay.

The proposal (D, D, S) cannot pass. By part 11 of Lemma 2, the legislators in districts 1 and 2 strictly prefer $(0, 0, 0)$ over $(D, D, 0)$ and by part 9 of Lemma 2 they both prefer $(D, D, 0)$ over (D, D, S). Thus they strictly prefer $(0, 0, 0)$ over (D, D, S).

The proposal $(D, D, 0)$ cannot pass because, by part 11 of Lemma 2, legislators 1 and 2 both strictly prefer $(0, 0, 0)$ when $b < \bar{b}$.

Thus no proposal can defeat $(0, 0, 0)$ so it is optimal for the first period agenda setter to propose $(0, 0, 0)$, as in the equilibrium we specify.

For voter strategies to be optimal, we must show that the voter's behavior is optimal in each of six information sets. The beliefs for these information sets are denoted $\mu_d(0, 0)$, $\mu_d(0, 1)$, $\mu_d(0, 2)$, $\mu_d(1, 1)$, $\mu_d(1, 2)$, and $\mu_d(1, 3)$ where the first argument represents the number of pork barrel projects that the voter's district receives and the second argument represents the total number of pork barrel projects enacted.

Given the legislator behavior specified in the equilibrium for $b < \bar{b}$, optimality of voter behavior is simple to establish since the only belief that is specified via Bayes' Rule is $\mu_d(0,0)$, which of course must be equal to the prior π. Given that $\mu_d(0,0) = \pi$ a voter can mix, using any strategy, including the one specified in our equilibrium where she never re-elects the legislator.[1] For the other five information sets, which are never reached in equilibrium, we specify the following beliefs: $\mu_d(0,1) = \mu_d(0,2) = 0$ and $\mu_d(1,1) = \mu_d(1,2) = \mu_d(1,3) = 1$. Thus if a district receives no pork, its legislator is assumed to be a SWM and it is optimal for the voters to replace him. If a district receives pork, its legislator is assumed to be a DWM and it is optimal for the voters to retain him. Thus it is optimal for voters to use pork-rewarding strategies.■

Lemma 4 (part 2 of main proposition) *For $b \in (\bar{b}, c)$ there is an equilibrium in which:*

1. *A social welfare maximizing legislator exhibits the following behavior:*

   (a) *If I'm chosen as agenda setter, propose zero pork projects.*

   (b) *If the first period policy proposal is $(0,0,0)$ vote aye. Otherwise vote nay.*

2. *A district welfare maximizing legislator exhibits the following behavior:*

   (a) *If I'm chosen as agenda setter and there is at least one other district welfare*

[1] This may seem odd, but it is important to note that if voters use pork-rewarding strategies then $x^1 = (0,0,0)$ is the only possible first period policy outcome when $b < \bar{b}$.

maximizer in the legislature, propose pork in my district and one other district. If there are no other district welfare maximizers, propose zero pork projects.

(b) *If the first period policy proposal gives pork only to my district, or to my district and exactly one other district, vote aye. If the proposal gives pork to all three districts and the other two districts are represented by a SWM and a DWM vote aye if and only if* $b > c\left[\frac{13-12\pi^2+6\pi^3}{15-12\pi^2+6\pi^3}\right]$. *In all other cases vote nay.*

3. *Voters use pork-rewarding strategies.*

Proof. Optimality of legislators' voting strategies follows directly from Lemmas 1 and 2. Here we demonstrate optimality of legislators' proposal strategies given legislators' voting strategies and voters' strategies. Finally, we characterize voter beliefs and demonstrate that given these beliefs it is optimal for voters to use pork-rewarding strategies.

There are $2^6 = 64$ possible proposals, since each district can be represented by a DWM or SWM and can either receive pork or not receive pork under a given proposal. Of course not all 64 are available to a given agenda setter, e.g., if district 1 is represented by a SWM, the proposal $(D, D, 0)$ is not feasible. However, w.l.o.g. we can restrict our attention to the following types of proposals: $(0, 0, D)$, $(0, 0, S)$, $(D, D, 0)$, $(0, D, S)$, $(0, S, S)$, (S, S, S), (S, S, D), (D, D, S), (D, D, D).

The proposal $(0, 0, D)$ will be defeated because both legislators who do not receive pork will vote against it, regardless of whether they are DWMs or SWMs. The proposal $(0, 0, S)$ will likewise receive two nay votes from the legislators whose districts do not receive pork, as well as a nay vote from the SWM whose district receives pork. The proposal $(0, D, S)$ will receive a nay vote from the legislator whose district receives no pork (regardless of whether he is a DWM or SWM) and will also receive a nay vote from the SWM whose district receives pork. The proposal $(0, S, S)$ will receive nay votes from the two SWMs whose districts receive pork as well as from the other legislator (regardless of whether he is a DWM or SWM). The proposal (D, S, S) will receive three nays, as will (D, D, D) and (S, S, S).

We now consider proposals that can potentially defeat $(0, 0, 0)$. By part 11 of Lemma 2, the proposal $(D, D, 0)$ will receive two aye votes from the legislators in districts 1 and 2. The third legislator will vote nay, regardless of whether he is a DWM or SWM. Likewise, $(0, D, D)$ and $(D, 0, D)$ can defeat $(0, 0, 0)$.

The proposal (D, D, S) can defeat $(0, 0, 0)$ if and only if $b > c\left[\frac{13-12\pi^2+6\pi^3}{15-12\pi^2+6\pi^3}\right]$. If this condition holds, then DWM legislators in district 1 and 2 will support this proposal. Otherwise they will not. The SWM legislator will always vote nay. Likewise (S, D, D) and (D, S, D) can defeat $(0, 0, 0)$ if and only if $b > c\left[\frac{13-12\pi^2+6\pi^3}{15-12\pi^2+6\pi^3}\right]$.

Thus, when we move back to the proposal stage, we know that there are at most two types of proposals that can defeat $(0, 0, 0)$. These are proposals in which two DWMs receive pork, and proposals in which two DWMs and one SWM receive pork.

A SWM legislator who is selected as the agenda setter strictly prefers $(0, 0, 0)$ over any policy that enacts pork in the first period. Thus it is optimal for him to propose $(0, 0, 0)$ regardless of whether his fellow first period legislators are SWMs or DWMs.

A DWM who is selected as the agenda setter can face several different situations. First, he may be the only DWM. In this case any pork-enacting proposal will receive at least two nay votes, so the agenda setter can do no better than proposing $(0, 0, 0)$. Second, it may be the case that exactly one other legislator is a DWM. In this case a proposal, i.e., $(D, D, 0)$, $(D, 0, D)$, or $(0, D, D)$, that gives pork to the agenda setter and one other legislator will pass. A proposal such as (D, D, S), (D, S, D), or (S, D, D) may also be able to defeat $(0, 0, 0)$ but by part 9 of Lemma 2 a DWM agenda setter prefers a proposal that only gives pork to his district and one other DWM. Finally, it may be the case that all three legislators are DWMs. In this case any proposal that gives pork to exactly two districts will defeat $(0, 0, 0)$ and any other proposal will lose to $(0, 0, 0)$. Thus it is optimal for the agenda setter to propose pork in his district and one other district.

We now turn to the voters. A voter's strategy must be optimal in each of six information sets. The beliefs for these information sets are denoted $\mu_d(0, 0)$, $\mu_d(0, 1)$, $\mu_d(0, 2)$, $\mu_d(1, 1)$, $\mu_d(1, 2)$, and $\mu_d(1, 3)$ where the first argument represents the number of pork barrel projects that the voter's district receives and the second argument represents the total number of pork barrel projects enacted.

Several of the information sets are never reached in equilibrium. For these, we set beliefs as follows. If the voter's district is the only district to receive pork, then she assumes that her representative is a DWM, i.e., $\mu_d(1, 1) = 1$. If all three districts receive pork, then the voter assumes that her representative is a DWM, $\mu_d(1, 3) = 1$. If the voter's district did not receive pork, but exactly one of the other districts did receive pork, then the voter assumes that her representative is a SWM, $\mu_d(0, 1) = 0$. Given these beliefs it is optimal for the voter to use pork-rewarding strategies in these information sets, re-electing a legislator if and only if he delivered pork to the district.

We now use Bayes' Rule to calculate beliefs for information sets that are reached in equilibrium. First we calculate $\mu_d(0, 0)$. There are three ways for the first period policy outcome to be $x^1 = (0, 0, 0)$. It may be the case that all three legislators are SWMs, which occurs with probability $(1-\pi)^3$. It may be the case that there are two SWMs and one DWM, which occurs with probability $3\pi(1-\pi)^2$. Finally it may be the case that there are two DWMs but the other legislator, a SWM, is chosen as the agenda setter; this happens with probability $3\pi^2(1-\pi)\frac{1}{3}$.

There are two ways for a voter to be in this information set despite being represented by a DWM. First, it may be the case that neither other legislator is a DWM. This occurs with probability $\pi(1-\pi)^2$. Second, it may be the case that only one of the other legislators is a SWM and the SWM is selected as an agenda setter. This occurs with probability $2\pi^2(1-\pi)\left(\frac{1}{3}\right)$. Thus, applying Bayes' Rule, $\mu_d(0,0) = \frac{\pi(1-\pi)^2+2\pi^2(1-\pi)\left(\frac{1}{3}\right)}{(1-\pi)^3+3\pi(1-\pi)^2+3\pi^2(1-\pi)\frac{1}{3}}$. For voter strategies, which call for removal of the legislator in this information set, to be optimal it must be the case that $\mu_d(0,0)$ is less than or equal to π, i.e.,

$$\begin{aligned}
\frac{\pi(1-\pi)^2+2\pi^2(1-\pi)\left(\frac{1}{3}\right)}{(1-\pi)^3+3\pi(1-\pi)^2+3\pi^2(1-\pi)\frac{1}{3}} &\leq \pi \\
\pi(1-\pi)^2+2\pi^2(1-\pi)\left(\frac{1}{3}\right) &\leq \pi(1-\pi)^3+3\pi^2(1-\pi)^2+3\pi^3(1-\pi)\frac{1}{3} \\
0 &\leq \pi\left(\frac{4}{3}-\pi\right)
\end{aligned}$$

This condition holds for all $\pi \in (0,1)$.

We now calculate $\mu_d(0,2)$, focusing w.l.o.g. on the case $d=1$ which corresponds to $x^1 = (0, D, D)$. Note that both other legislators must be DWMs in this information set. There are two ways this proposal can be enacted. First, it may be the case that legislator 1 is a DWM, but another legislator was chosen as the agenda setter and did not choose legislator 1 as his coalition partner. This occurs with probability $\pi^3 \cdot \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$. Second, it may be the case that legislator 1 is a SWM, but another legislator was chosen as the agenda setter. This occurs with probability $(1-\pi)\pi^2\left(\frac{2}{3}\right)$. Thus, applying Bayes' Rule,

$\mu_d(0,2) = \frac{\pi^3\left(\frac{1}{3}\right)}{\pi^3\left(\frac{1}{3}\right)+(1-\pi)\pi^2\left(\frac{2}{3}\right)}$. After some algebra, it can be shown that this is strictly less than π for all $\pi \in (0,1)$. Thus it is optimal for a voter to remove her legislator in this information set.

The final belief we must specify is $\mu_d(1,2)$. Since we have shown that voter beliefs in all other information sets that are reached in equilibrium are less than or equal to the prior π, we know that $\mu_d(1,2) \geq \pi$.[2] Thus it is optimal to re-elect the legislator in this information set.■

[2] In fact the inequality holds strictly.