Dynamics of Rate-of-Return Regulation

Alexander Nezlobin
Stern School of Business, New York University
anezlobi@stern.nyu.edu

Madhav V. Rajan
Graduate School of Business, Stanford University
mrajan@gsb.stanford.edu

Stefan Reichelstein*
Graduate School of Business, Stanford University
reichelstein@stanford.edu

January 2011

*We are grateful to the Editor, an Associate Editor and three reviewers, as well as seminar participants at Berkeley, Columbia, Heidelberg, LMU (Munich), Peking University, Northwestern, Toronto, and Yale for their helpful comments and suggestions. We also thank Yanruo Wang for excellent research assistance.
Dynamics of Rate-of-Return Regulation

Abstract: Under Rate-of-Return regulation, a firm’s product prices are constrained by the requirement that investors do not earn more an allowable return on the firm’s assets. This paper examines the dynamic properties of the Rate-of-Return regulation process when the regulated firm periodically undertakes new capacity investments. Our analysis identifies prices that correspond to stationary values of the regulation process. It is shown that the underlying depreciation rules for property, plant and equipment determine whether these stationary prices will be above, equal to, or below the long-run marginal cost of providing the regulated service. We provide conditions under which the Rate-of-Return regulation process is dynamically stable so that prices indeed converge to their stationary values. The overall efficiency of this regulation method is shown to depend on how well the applicable depreciation schedule matches the productivity pattern of the assets in use.
1 Introduction

Regulatory agencies commonly rely on Rate-of-Return (RoR) regulation to determine product prices in a range of industries such as telecommunication services, electricity, gas and water.\(^1\) In its traditional form, RoR regulation sets prices so that the regulated firm’s Return on Assets attains a target value in each period. Equivalently, prices are set so that revenues cover not only current operating expenses but also an interest charge on the book value of the firm’s operating assets.\(^2\)

A frequently voiced claim in the industrial organization literature is that RoR regulation is inherently inefficient. Specifically, product prices are predicted to be too high relative to the levels that a welfare maximizing planner would choose. The alleged reason is that prices must cover the firm’s average cost, including fixed costs which frequently account for a large fraction of total costs. As a consequence, RoR regulation is not in a position to implement the prices and quantities that correspond to marginal cost pricing; see, for example, Baumol (1971), Nicholson (2005) and Schotter (2008).\(^3\)

Studies by Biglaiser and Riordan (2000) and Rogerson (2010) have examined RoR regulation in dynamic settings in which the firm makes a sequence of overlapping capacity investments. The significance of multiperiod investment settings is that it becomes possible to determine a product’s long-run marginal cost, which includes the cost of providing production capacity for a particular period.\(^4\) To the extent that the regulated firm’s fixed costs consist mainly of capacity-related depreciation charges, the efficiency of RoR regulation then hinges on how well historical cost reflects long-run marginal cost. Rogerson (2010) provides a benchmark result by identifying a particular depreciation schedule with the property that capital costs, i.e., the sum of depreciation- and interest charges on the remaining book value of the assets, are indeed equal to the long run marginal cost in each period. This depreciation schedule must not only reflect the anticipated decay of an asset’s productive

---

\(^1\)When government agencies procure unique items, such as weapon systems, prices are frequently calculated on a cost-plus basis, where the mark-up over cost is again calculated so as to give the contractor a fair return on its invested capital; see, for instance, Laffont and Tirole (1993).

\(^2\)In certain industries, such as telecommunications, traditional RoR regulation has increasingly been replaced with so-called Price Cap regulation. Accordingly, prices are set periodically so as to meet a current RoR constraint. In subsequent years, the regulated firm is then free to charge lower prices until such time as a new price cap is calculated according to an updated RoR calculation (Laffont and Tirole, 2000).

\(^3\)Parts of the regulation literature has also investigated inefficiencies related to distortions in the regulated firm’s input mix; e.g., Averch and Johnson (1962).

\(^4\)Arrow (1964) shows that despite the inherent joint cost of acquiring assets with a useful life of \(T\) periods, in a setting with overlapping investment decisions it is possible to calculate the marginal cost of providing one unit of capacity for one period of time.
capacity but also anticipated changes in future asset acquisition costs. Falling acquisition costs due to technological progress are an essential feature of some regulated industries such as telecommunications. Rogerson refers to the corresponding intertemporal cost allocation as the Relative Replacement Cost (R.R.C) rule.

Our analysis in this paper departs from the observation that straight-line depreciation is commonly used by regulatory agencies (Biglaiser and Riordan, 2000). This approach is, of course, consistent with the financial reporting practices that most unregulated firms use in connection with plant, property and equipment. The main contribution of this paper is to identify the dynamic properties of the RoR regulation process when the depreciation rules are not calibrated to the underlying fundamentals, as required in Rogerson’s (2010) benchmark result.

For a given depreciation schedule, we find that there is at most one equilibrium candidate for the product price that could emerge asymptotically as the limit of the RoR regulation process. Put differently, product prices could not possibly converge to any other price over time. The unique equilibrium price candidate is lower (higher) than the firm’s long-run marginal cost whenever the underlying depreciation schedule for capacity assets is more accelerated (decelerated) than the unbiased depreciation rule identified in Rogerson (2010). In particular, straight-line depreciation amounts to a more accelerated schedule, i.e., assets are written off more quickly early on, if the underlying asset productivity conforms to the one-hoss shay (“sudden death”) pattern, where productive capacity is undiminished for the entire useful life of the asset. On the other hand, Biglaiser and Riordan (2000) argue that the depreciation rules in regulatory practice may have deceleration bias insofar assets are kept on the books beyond their economic useful life. Our results speak to accounting biases in either direction.

An equilibrium price below the long-run marginal cost may seem at odds with the requirement that the regulated firm breaks even under RoR regulation. It must be kept in mind, though, that the break-even constraint is applied in term’s of the firm’s residual income. It is well known that for a firm with no initial assets in place the present value of all operating cash flows is equal to the present value of all residual incomes, regardless of the accounting rules (Hotelling, 1925; Preinreich, 1938). Given an accelerated (front-loaded) depreciation schedule, the resulting product prices will exceed long-run marginal cost early on before prices fall below that value and possibly settle at the long-run equilibrium price. Thus, the firm breaks even in each period in accounting terms and does so in economic terms over the entire (infinite) horizon. Of course, the temporal distortions in product prices either

---

5This identity is sometimes referred to as the conservation property of residual income. In a regulation framework, this identity was also observed by Schmalensee (1989).
above or below the long-run marginal cost do not cancel each other but instead all amount to welfare losses.

Our results show that the deviation between the long-run marginal cost and the unique price that can potentially emerge as an equilibrium of the RoR process is not only determined by the choice of depreciation schedule but also the growth rate in the product market. Specifically, we find that an expanding external product market tends to close the gap between long run marginal cost and the potential equilibrium price. To quantify these distortions, we find that for a stationary product market, the long run marginal cost of capacity exceeds the historical cost of capacity by about 20% when capacity assets have an undiminished useful life of 25 years and depreciation is calculated according to the straight line rule. Depending on the initial state of the system, the deviation between historical and marginal cost may be much larger over long periods of time until product prices approach their equilibrium values.

One might expect the RoR process to be stable in the sense that for a “reasonable” class of starting points prices do indeed converge to the unique equilibrium candidate. While we have not observed any examples to the contrary, such global stability turns out to be difficult to establish. For a fairly broad class of environments we establish a local stability result which shows that starting from an initial configuration of past investments for which prices are “close” to the candidate equilibrium price, the resulting trajectory of RoR regulation prices will indeed converge.

Global stability is shown to obtain in our model for a different class of environments in which assets have an unbounded useful life and both capacity decay and accounting depreciation follow a geometric pattern. For this class of “geometric decay” environments, one also obtains monotonic convergence: for a firm that starts out with no assets initially, prices are above long-run marginal cost in early periods, provided the depreciation schedule is accelerated relative to the R.R.C. rule. Prices then decline monotonically and approach the unique equilibrium price below marginal cost.

In terms of prior work, our analysis is most closely related to Biglaiser and Riordan (2000), Friedl (2007) and Rogerson (2010). At first glance, some of our findings may seem to contradict those in Friedl (2007) who concludes that RoR regulation will result in prices that are inefficiently high even though the regulator applies straight-line depreciation and the one-hoss shay assumption is being imposed. The key difference is that Friedl (2007) allows for only a single capacity investment decision, the acquisition cost of which is then amortized over a finite horizon. Biglaiser and Riordan (2000) focus their analysis on a setting

---

This finding applies only to accelerated depreciation schedules. Growth has the opposite effect for decelerated schedules.
in which capacity decays geometrically and book values also follow a geometric sequence. Their Theorem 2 concludes that even if the depreciation schedule is unbiased, product prices will eventually exceed the long-run marginal cost. To reconcile this finding with our results, it must be kept in mind that one distinguishing feature of their model is that, because of technical progress, incumbent assets become ultimately obsolete even though they still are still functional. Under what Biglaiser and Riordan term “naive” regulation, economically obsolete assets remain part of the rate base and this effectively creates a bias in the direction of decelerated accounting.

The plan of our paper is as follows. We present the RoR regulation model, including the evolution of product prices and investments in Section 2. The analysis of prices that can potentially emerge as stationary prices of the RoR process is contained in Section 3. There we also demonstrate how the choice of accounting rules and growth in the product market create a joint bias in the equilibrium price relative to the underlying marginal cost. Section 4 presents stability results, both local stability and a global convergence result for the class of geometric decay patterns. We conclude in Section 5. Proofs are presented in the Appendix.

2 Capacity Investments and Product Prices

We consider a single-product firm that makes its capacity investments and pricing decisions subject to a Rate-of-Return regulation constraint. In each period, the output of the firm, $q_t$, is bounded by the total capacity available in that period, $K_t$. The firm is required to operate at capacity, so that $q_t = K_t$.\footnote{In many industries of interest, such as electricity, there is a natural peak-load pattern in demand so that full capacity utilization effectively amounts to having $q_t = \Delta \cdot K_t$ for some $\Delta < 1$. The presence of peak-load demand does not alter the conclusions obtained in our model in a substantive manner.} Production capacity is generated by assets, which have a useful life of $T$ periods. Specifically, an investment expenditure of $v \cdot I_t$ at date $t$ will add capacity to produce an additional $x_\tau \cdot I_t$ units of output at date $t + \tau$, where $1 \leq \tau \leq T$. Thus, new investments come “online” with a lag of one period, yet their productivity may diminish over time. Thus we assume that $1 = x_1 \geq x_2 \geq \cdots \geq x_T > 0$. In the so-called one-hoss shay (or sudden death) scenario all $x_\tau$ are equal to one, as the asset is assumed to have undiminished productive capacity over its entire useful life.

At any date $t > T$, the current state:

$$\theta_t = (I_{t-T}, ..., I_{t-1}),$$

determines the available capacity:

$$K(\theta_t) = I_{t-T} \cdot x_T + \cdots + I_{t-1} \cdot x_1.$$
Let $P_t(K_t)$ denote the price that consumers are willing to pay in period $t$ if $K_t$ units of output are supplied. The functions $P_t(\cdot)$ are assumed to be continuous and decreasing such that $P_t(K_t) \to 0$ as $K_t \to \infty$. Furthermore, total revenue $R_t(K_t) = P_t(K_t) \cdot K_t$, is assumed to be concave in $K_t$.

In terms of costs, the regulated firm is allowed to set product prices so as to recover both variable operating costs and capital costs, the latter composed of depreciation charges and interest charges imputed on the book value of the firm’s capacity assets. The variable operating cost per unit of output is assumed to be constant and is denoted by $w$. Assets are capitalized in their acquisition period and then fully depreciated over their useful life, according to some schedule $d = (d_1, \ldots, d_T)$, satisfying $\sum_{\tau=1}^{T} d_{\tau} = 1$. Given the current state $\theta_t$, the total depreciation charge in period $t$ is given by:

$$D_t(\theta_t) = v \cdot (d_T \cdot I_{t-T} + \ldots + d_1 \cdot I_{t-1}).$$

Since depreciation is the only accrual in our model, income is given by revenues less variable operating costs and depreciation:

$$Inc_t = P_t(K(\theta_t)) \cdot K(\theta_t) - w \cdot K(\theta_t) - D_t(\theta_t).$$

The book value of one capacity unit acquired at date $t$ is originally recorded at its cost $v$ and then amortized over the next $T$ periods according to the depreciation schedule, $d$. Specifically the remaining book value at date $t + \tau$, $0 \leq \tau \leq T$, becomes:

$$bv_{\tau} = (1 - \sum_{i=1}^{\tau} d_i) \cdot v.$$

Given the current state, $\theta_t$, the aggregate value of capacity assets at date $t - 1$ is given by:

$$BV_{t-1}(\theta_t) = bv_{T-1} \cdot I_{t-T} + \ldots + bv_0 \cdot I_{t-1}.$$

Capital costs, which are the sum of depreciation and an imputed charge on the beginning-of-period book value, will be denoted by $C_t$:

$$C_t(\theta_t) = D_t(\theta_t) + r \cdot BV_{t-1}(\theta_t),$$

where $r$ denotes the cost of capital. The constraint imposed by Rate of Return (RoR) regulation is usually represented in terms of the firm’s accounting rate of return on assets, that is:

8The capital charge rate, $r$, will generally be determined as a weighted average of the costs of equity and debt. Our analysis treats this cost as exogenous and time invariant.
\[
\frac{Inc_t}{BV_{t-1}} \leq r. \quad (1)
\]

It has long been recognized in both the regulation and the accounting literature that a firm operating consistently under the constraint imposed by (1) will not make any positive economic profits in the sense that the present value of all cash flows is non-negative. This follows from the conservation property of residual income which states that, for a firm with no assets at its inception, the present value of cash flows is equal to the present value of all residual incomes, regardless of the applicable depreciation rules. Since the rate of return constraint in (1) is equivalent to:

\[
RI_t = Inc_t - r \cdot BV_{t-1} \leq 0,
\]

it follows that the firm will break even over its entire life time (in terms of discounted cash flows) only if the inequality constraint in (1) is met as an equality in every period. Since residual income is, by definition, equal to revenues less current operating expenses and less current capital costs, the firm is instructed to make its investment decision in period \(t\) (at date \(t - 1\)) so that if it sells the capacity available at date \(t\), its revenue will cover the full historical cost. Accordingly, a state \(\theta_t\) is said to be feasible under the Rate-of-Return regulation if:

\[
RI_t(\theta_t) \equiv Inc_t(\theta_t) - r \cdot BV_{t-1}(\theta_t) = 0. \quad (2)
\]

There may be multiple non-negative investment levels at date \(t\) which lead to a feasible state, \(\theta_{t+1}\). For the purposes of our analysis, it does not matter how the firm chooses among these alternative investment levels. On the other hand, depending on the history \(\theta_t\) and the depreciation rules in place, there may not exist an investment level \(I_t > 0\) at date \(t\) which makes \(\theta_{t+1}\) feasible. In that case, it must be that \(RI_{t+1}(\theta_{t+1}) < 0\) for all \(I_t > 0\). Our analysis takes the perspective that the investment levels up to date \(T\) are exogenously given and that the firm is subject to the RoR constraint from date \(T\) onward. In particular, new investment levels must be chosen so as maintain feasibility. If no such investment level exists, the firm is required to set \(I_t = 0\). This rule does not only have intuitive appeal, but, as argued below, setting \(I_t = 0\) when feasibility is not attainable in the short-run ultimately ensures that feasibility will be restored and maintained in the long-run.\(^9\) We refer to an infinite sequence of investments levels, \((I_0, I_1, ..., I_t, ...\)), as a trajectory \(\theta\).

\(^9\)If the sign of \(RI_{t+1}(\theta_{t+1})\) were to vary as \(I_t\) moves from 0 to large values, then by continuity of the function \(RI_{t+1}(\cdot)\) there would exist an \(I_t\) where \(RI_{t+1}(\cdot)\) is zero. To see that it is impossible to have \(RI_{t+1}(\cdot) > 0\) for all \(I_t > 0\), we note that \(P_t(K_t)\) tends to zero for large values of \(K_t\) (induced by high values of \(I_t\)). Yet, the average cost \(\frac{H_t}{K_t}\) tends to a positive number, \(w + v \cdot (d_1 + r)\) as \(I_t\) gets large.

\(^{10}\)If feasibility cannot be attained in a given period, the firm may still be able to achieve zero residual
Definition 1 A trajectory $\theta$ is compatible with Rate-of-Return regulation if for all $t \geq T$, $I_t > 0$ implies $\theta_{t+1}$ is feasible, that is, $RI_{t+1}(\theta_{t+1}) = 0$.

Rate-of-Return regulation is usually positioned as an attempt to maximize consumer welfare subject to the constraint that the firm breaks even. Leaving regulatory issues aside, we first characterize the efficient investment and capacity levels that a social planner would choose so as to optimize consumer welfare net of production costs. Consumer welfare at date $t$ is given by:

$$B_t(q_t) = \int_0^{q_t} P_t(u) du.$$  

Let $\gamma \equiv \frac{1}{1+r}$. Assuming that at date 0 the firm has no assets in place, that is, $q_0 = 0$ and $q_1 = I_0$, the regulator’s welfare maximization problem then becomes:\textsuperscript{11}

$$\max_{\{q_t; I_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left[ B_t(q_t) - w \cdot q_t - v \cdot I_t \right] \cdot \gamma^t$$

subject to:

$$q_t \leq K_t \equiv I_{t-T} \cdot x_T + \ldots + I_{t-1} \cdot x_1.$$  

A central insight in the papers of Arrow (1964) and Rogerson (2008, 2010) is that this dynamic maximization problem is intertemporally separable if market demand keeps expanding over time and therefore capacity should be added in each period. This notion is formalized in the following definition.

Assumption 1 For any $q > 0$, the inverse demand curves satisfy:

$$P_t(q) \leq P_{t+1}(q).$$

Assumption 1 will be maintained throughout our analysis. Intuitively, this assumption ensures that the welfare planner never ends up with excess capacity if in each period the investment is chosen myopically optimal so as to balance current welfare and costs. The following insight dates back to Arrow (1964).

Lemma 0 The optimal capacity levels, $K^*_t$, that solve the optimization problem in (3) are given by:

\textsuperscript{11}As will become clear, it is of no importance to the following derivation that there are no productive capacity assets in place at date 0. It only matters that the process does not commence with excess capacity.
\[ P_t(K_t^*) = w + c, \]

where

\[ c = \frac{v}{\sum_{i=1}^{T} x_i \cdot \gamma^i}. \]  

This result shows that the planner's optimization problem is intertemporally separable. Capacity investments entail an inherent jointness that results from the fact that \( v \) dollars spent today generate a \( T \)-period stream of future capacity levels. Yet, if new capacity can be added in each period, the cost \( c \) becomes effectively the marginal cost of one unit of capacity made available for one period of time.\(^{12}\) It is also useful to note that \( c \) is exactly the price that a hypothetical supplier would charge for renting out capacity for one period of time, if the rental business is constrained to make zero economic profit. Accordingly, we will also refer to \( c \) as the competitive rental price of capacity.\(^{13}\)

Lemma 0 suggests that the efficiency of Rate-of-Return regulation hinges on how well the capital cost \( C_t(\theta_t) \) approximates the economically relevant cost \( c \cdot K(\theta_t) \). Clearly, the goodness of this approximation depends on the choice of depreciation rules. Rogerson (2008, 2010) provides an important insight in this regard by focusing on the capital costs of an individual project over time. Let

\[ z_\tau \equiv v \cdot d_\tau + r \cdot bv_{\tau-1}, \]  

so that \( z_\tau \cdot I_{t-\tau} \) is the capital cost charged in period \( t \) for investment \( I_{t-\tau} \) undertaken at date \( t - \tau \). Since an investment in one unit of capacity undertaken at date \( t - \tau \) still contributes \( x_\tau \) units of capacity at date \( t \), the economically relevant capital cost at date \( t \) is:

\[ z_\tau^* = x_\tau \cdot c. \]  

\(^{12}\)For the special case of \( T = 2 \), the formula in (4) can be derived as follows. The planner can increase capacity at date \( t \) by one unit without affecting capacity levels in subsequent periods through the following “reshuffling” of future capacity acquisitions: buy one more unit of capacity at date \( t - 1 \), buy \( x_2 \) fewer units in period \( t \), buy \( (x_2)^2 \) more units in period \( t + 1 \), and so on. The cost of this variation, evaluated in terms of its present value as of date \( t - 1 \), is given by \( v \cdot [1 - \gamma \cdot x_2 + \gamma^2 \cdot x_2^2 - \gamma^3 \cdot x_2^3 + \gamma^4 \cdot x_2^4 \ldots] \). This sum is equal to: \( v \cdot [1 + \gamma \cdot x_2]^{-1} \). Finally, from the perspective of date \( t \), the present value of this variation is:

\[ (1 + r) \cdot v \cdot \frac{1}{1 + \gamma \cdot x_2} = c. \]

\(^{13}\)In Biglaiser and Riordan (2000), technical progress causes the long-run marginal cost to fall over time. Accordingly, the acquisition cost of new assets decreases over time and the variable cost of operating successive vintages of capital are assumed to decline.
We refer to the cost allocation rule in (6) as the Relative Practical Capacity (R.P.C.) rule, reflecting that the acquisition cost of a new asset is \( v \) and that the cost charge in period \( t \) is given by the proportion of current capacity relative to total discounted capacity:\(^{14}\)

\[
z^*_\tau = v \cdot \frac{x_\tau}{\sum_{i=1}^{T} x_i \cdot \gamma^i}.
\]

Earlier literature has shown that there is a one-to-one relation between depreciation schedules \( d = (d_1, ..., d_T) \) and the capital cost charges \( (z_1, ..., z_T) \). Formally, the linear mapping defined by (5) is one-to-one: for any intertemporal cost charges \( (z_1, ..., z_T) \), with the property that \( \sum_{\tau=1}^{T} z_\tau \cdot \gamma^\tau = v \), there exists a unique depreciation schedule \( d \) such that (5) is satisfied. We denote by \( d^* \) the unique depreciation schedule corresponding to the R.P.C. cost allocation rule.\(^{15}\) It is readily verified that in the one-hoss shay scenario \( (x_\tau = 1) \), the corresponding depreciation schedule is the annuity depreciation method, where \( d_\tau = (1+r) \cdot d_{\tau-1} \). The R.P.C. rule does coincide with straight-line depreciation if the productive capacity of assets capacity declines linearly over time such that \( x_\tau = 1 - \frac{r}{1+r} \cdot (\tau-1) \). Finally, if \( T = \infty \) and productive capacity declines geometrically such that \( x_\tau = (1-\alpha) \cdot x_{\tau-1} \) for \( 0 < \alpha < 1 \), the corresponding R.P.C. depreciation schedule is also geometric with \( d_\tau = (1-\alpha) \cdot d_{\tau-1} \).

If depreciation were calculated according to the R.P.C. depreciation rule, the sum of current operating and capital costs would reflect precisely the economically relevant cost that a social planner should impute at date \( t \):

\[
w \cdot K(\theta_t) + D^*_t(\theta_t) + r \cdot BV^*_t(\theta_t) = w \cdot K(\theta_t) + z^*_T \cdot I_{t-T} + \ldots + z^*_1 \cdot I_{t-1} = w \cdot K(\theta_t) + c \cdot x_T \cdot I_{t-T} + \ldots + c \cdot x_1 \cdot I_{t-1} = (w + c) \cdot K(\theta_t).
\]

To state Rogerson’s (2010) benchmark result in the context of our model, it will be convenient to consider a setting where the firm started with no capacity investments at date

\(^{14}\)Rogerson (2010) refers to the cost allocation rule in (6) as the Relative Replacement Cost (R.R.C.) rule. In the context of his model, the concept of replacement cost is more complex than ours since the acquisition cost of new assets declines exponentially over time.

\(^{15}\)The R.P.C. depreciation schedule conforms to Hotelling’s (1925) concept of economic depreciation if one posits the existence of a competitive rental market for assets. The R.P.C. rule is also closely related to the so-called Relative Benefit Depreciation rule which has played a key role in earlier studies on managerial incentives for investment decisions; see, for example, Reichelstein (1997), Rogerson (1997), Baldenius and Ziv (2003), Bareket and Mohnen (2007) and Pfeiffer and Schneider (2007). As the name suggests, the charges under the relative benefit depreciation rule apportion the initial investment expenditure in proportion to the subsequent expected future cash inflows; the R.P.C. rule, in contrast, refers only to the decay in production capacity over time.
0. Thus \( q_1 = I_0, q_2 = I_1 + x_2 \cdot I_0 \) and so forth. A trajectory \( \theta \) is said to be feasible for \( t \geq T \) if the corresponding sequence of states \( \theta_t = (I_{t-T}, ..., I_{t-1}) \) is feasible.

**Proposition 0** Suppose \( K_0 = 0 \). If assets are depreciated consistently according to the R.P.C. rule, then Rate-of-Return regulation results in a trajectory \( \theta \) that is feasible and attains marginal cost pricing, that is, \( P_t(K(\theta_t)) = c + w \) for all \( t \geq 1 \).

This result dispels the economic logic articulated in some microeconomics textbooks (e.g., Baumol, 1971; Nicholson, 2005; Schotter, 2008) regarding the efficiency of rate of return regulation. The conventional argument is that because direct transfers to the regulated firm are politically undesirable, regulatory agencies cannot implement marginal cost pricing. In order for the firm to break even, prices must therefore cover average costs which include the (historical) fixed costs associated with the regulated firm’s investments in property, plant and equipment. Proposition 0 shows that with suitably chosen depreciation rules, the historical cost of sunk asset expenditures aligns precisely with the long-run marginal cost of capacity.\(^{16}\)

While we regard Proposition 0 as an important benchmark, the result does not speak to the efficiency of Rate-of-Return regulation if the depreciation rules differ from the R.P.C. rule. Consistent with the financial reporting practice of non-regulated firms, regulatory practice appears to calculate depreciation charges commonly according to the straight-line rule. As observed above, straight-line depreciation is consistent with R.P.C. rule in the exceptional case where the \( x_\tau \) decline linearly at a specific rate. The natural question for our analysis therefore concerns the dynamic of the rate of regulation process and the types of distortions, both in terms of direction and magnitude, one should expect to see over time as a consequence of using a depreciation policy like the straight-line rule.

### 3 Stationary Prices

This section examines the set of prices that can emerge as limit points of a trajectory generated by the Rate-of-Return regulation process. In particular, we seek to characterize how the prices that emerge as stationary values of the RoR process depend on the accounting rules underlying the regulation process and growth in market demand. For reasons of tractability, we restrict attention to so-called *proportionate growth* requiring that:

\[
P_{t+1}((1 + \mu_t) \cdot K_t) = P_t(K_t). \tag{7}
\]

\(^{16}\)Consistent with this finding, Biglaiser and Riordan (2000, p. 752) also observe that “in theory, rate of return regulation could track optimal prices.”
The parameter \( \mu_t \) then represents the growth rate of the market in period \( t \). We assume \( \mu_t < r \). We note that the proportionate growth assumption will be met by constant elasticity demand curves if the elasticity factor remains constant, yet the size of the market expands. The proportionate growth assumption can also be met by linear demand curves of the form \( P_t = a_t - b_t \cdot q_t \) if the intercept \( a_t \) remains constant, yet the slope parameter \( b_t \) contracts over time at the rate \( (1 + \mu_t)^{-1} \). As a consequence, demand will become less elastic over time. We note in passing that if an unregulated monopolist has constant marginal (long-run) cost and faces a demand curve exhibiting proportionate growth, the profit maximizing capacity levels will grow precisely at the rate \( \mu_t \) in period \( t \). Similarly, in the context of Lemma 0 above, a social planner would ideally choose capacity levels that expand at the rate \( \mu_t \).

**Definition 2** A price \( p_\infty \) is a stationary price of the Rate-of-Return regulation process if there exist a trajectory \( \theta \), that is compatible with RoR regulation and:

\[
\lim_{t \to \infty} P_t(K(\theta_t)) = p_\infty.
\]

If a price \( p_\infty \) satisfies the criterion in Definition 2, we say that \( \theta \) supports \( p_\infty \). We also consider the stronger notion of a trajectory \( \theta \) that attains \( p_\infty \) in the sense that for all \( t > T \), \( p_\infty = P_t(K(\theta_t)) \). The following result identifies a particular price level that can be attained, provided demand grows at a constant rate.

**Proposition 1** If market demand grows at some constant rate \( \mu \), then

\[
p^* = w + c \cdot \frac{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} \cdot z_\tau(d)}{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} z_\tau^*}
\]  
(8)

is a stationary price of the Rate-of-Return regulation process, provided \( P_T(0) > p^* \). This price is attained by a feasible trajectory of the form: \( I_{t+1} = I_t \cdot (1 + \mu) \).

From hereon, it will be assumed that the price \( p^* \), identified in (8) does satisfy the inequality \( P_T(0) > p^* \). Given our assumption of proportionate growth, it then follows that \( P_t(0) > p^* \) for all \( t \geq T \). We note that the result in Proposition 1 is consistent with the benchmark result in Proposition 0 insofar as \( p^* \) is equal to the long-run marginal cost \( w + c \) whenever depreciation conforms to the R.P.C. rule. We refer to the fraction:

\[
\Gamma \equiv \frac{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} \cdot z_\tau(d)}{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} \cdot z_\tau^*}
\]  
(9)

as the accounting bias. Clearly, this bias factor is a joint function of the depreciation schedule and the growth rate in the product market, \( \mu \). Before we delineate the properties of \( \Gamma \), we establish the following uniqueness result. 
Proposition 2 Suppose market demand grows at a constant rate $\mu \geq 0$. The price $p^*$, given in (8), is the unique stationary price of the Rate-of-Return regulation process, provided either (i) the productive capacity of assets decays strictly over time, that is, $x_t > x_{t+1}$, or (ii) the depreciation schedule, $d$, has the property that $z_t > z_{t+1}$ with $z_T > 0$.

It should be noted first that Proposition 2 does not say that prices will converge to $p^*$ under the Rate-of-Return regulation process. However, the result does show that no price other than $p^*$ could possibly be supported as a limit point of this regulation process. The first of the two sufficient conditions identified in Proposition 2 is not particularly restrictive, though it does rule out the one-hoss shay scenario commonly considered in the regulation literature. The alternative sufficient condition in Proposition 2 is satisfied for many commonly used depreciation schedules, including straight-line depreciation. In that case, the historical cost charges $z_t$ will decrease linearly over time.

The proof of Proposition 2 shows that any trajectory that is compatible with RoR regulation and supports a stationary price must remain feasible beyond some date $T^*$. If prices obtained under RoR regulation converge, then so must the growth-adjusted capacity levels. When each asset’s productive capacity is strictly lower over time, the presence of an infeasible state implies an absence of investment and consequently a strict drop in capacity levels. Beyond some cutoff date, however, this would be inconsistent with the convergence of the scaled capacity levels.

The uniqueness result in Proposition 2 points to the centrality of the accounting bias factor $\Gamma$ in determining the efficiency of the RoR regulation process. A natural first question is whether $\Gamma$ acts as a mark-up or discount on the long-run marginal cost of capacity, $c$. To address this question, we consider the following partial ordering on the class of depreciation schedules:

**Definition 3** A depreciation schedule $d$ is more accelerated than $d'$ if for any $\tau \leq T - 1$

$$bv_\tau(d) \leq bv_\tau(d'). \quad (10)$$

Conversely, $d$ is more decelerated than $d'$ if the inequalities in (10) are reversed. An accelerated depreciation schedule is front-loaded in comparison to the R.P.C. rule and therefore the capital costs $z_\tau$ of new investments are too high early in the life cycle of an asset. The following result shows how this bias impacts the unique stationary price under RoR regulation.

\footnote{\hspace{1em}However, it can be shown that if the periodic growth rate $\mu$ is strictly positive, then the proof of Proposition 2 can be adapted to cover the one-hoss shay scenario as well.}
Proposition 3 The stationary price $p^*$, given in (8), satisfies:

$$p^* \leq w + c$$

provided the depreciation schedule $d$ is accelerated relative to the R.P.C. rule. Conversely, $p^* \geq w + c$ if $d$ is decelerated relative to the R.P.C. rule.

The proof of this result shows that the accounting bias $\Gamma$ is less than 1 whenever the depreciation schedule is accelerated relative to the R.P.C. rule, while $\Gamma > 1$ for a decelerated depreciation rule. At first glance, the result in Proposition 3 appears implausible. How can a regulated firm break even in terms of discounted cash flows over an infinite horizon and, at the same time, consumers to obtain the product in equilibrium at a price that is below the social marginal cost $w + c$? The answer is that, provided the firm starts with no capacity at date 0, prices must early on exceed the long-run marginal cost, $w + c$, before they ultimately approach $p^* < w + c$. This point will be illustrated further in the setting examined in Section 4 below, where both depreciation charges and capacity decay are assumed to follow a geometric pattern. In order for prices emerging under RoR regulation to be consistently below $w + c$, the firm would have to start with capacity assets in place and these would effectively serve as a “subsidy” to the system.

Our finding in Proposition 3 stands in contrast to the results obtained by Friedl (2007). He concludes that if assets are depreciated under the straight-line rule and the productivity pattern corresponds to the one-hoss shay scenario, then the resulting capacity level will be too low and the resulting product prices too high, at least in the early periods, relative to the socially efficient levels. Essential to this result, however, is that the regulated firm is assumed to make only a single capacity investment at date 0. As a consequence, prices never reach a steady state level. In fact, since capital costs fall over time due to straight line depreciation, product prices decrease. Friedl’s model predicts excess consumer demand at the regulated prices in later periods, yet the firm is assumed not to undertake new capacity investments.

We now turn to a further examination of the accounting bias factor $\Gamma$. Clearly, this factor is a function of both the depreciation schedule and the rate of growth in the product market, $\mu$. To assess the impact of $\mu$ on $\Gamma$, we consider a stronger notion of accelerated depreciation. The notion put forth in Definition 3 is essentially a “second-order dominance” condition referring to the cumulative amount amortized at a particular point in time. The following criterion provides a more stringent “first-order dominance” criterion.

Definition 4 A depreciation schedule $d$ is uniformly more accelerated than the R.P.C. rule if $\frac{z_\tau(d)}{x_\tau}$ is monotonically decreasing in $\tau$. 
By definition, the depreciation schedule corresponding to the R.P.C. rule, \( d^* \), has the property that \( z^*_\tau = x_\tau \cdot c \). In contrast, a depreciation schedule that is uniformly accelerated relative to the R.P.C. rule starts out with a ratio \( \frac{z^*_1}{x_1} > c \) and then intersects the value \( c \) at some point in time \( \tilde{\tau} < T \).

**Proposition 4** If depreciation is uniformly more accelerated (decelerated) than the R.P.C. rule, the stationary price \( p^* \) is increasing (decreasing) in \( \mu \).

To further illustrate Propositions 3 and 4, we provide a numerical characterization of the bias factor \( \Gamma \) in a scenario where depreciation is held fixed at the straight-line rule. The productivity factors \( x_\tau \) are assumed to decline linearly over time, so that \( x_\tau = 1 - \beta (\tau - 1) \). Therefore \( \beta = 0 \) corresponds to the one-hoss shay scenario. Figure 1 shows the level sets of the function \( \Gamma(\mu, \beta|r, T) \) for values of \( r = .12 \) and \( T = 25 \). As observed earlier, when \( \beta = \frac{r}{1+rT} \), straight-line depreciation amounts to the R.P.C. rule. As a consequence, \( \Gamma = 1 \) for \( \beta = \frac{r}{1+rT} = 0.03 \). Consistent with the above results, we note that \( \Gamma(\cdot) \) assumes values below (above) 1 whenever \( \beta < (>)0.03 \). Furthermore, it can be verified that the depreciation schedule is in fact uniformly accelerated for \( \beta \leq \frac{r}{1+rT} \). As predicted by Proposition 4, we observe that \( \Gamma(\cdot) \) is increasing in \( \mu \) when \( \beta < 0.03 \).

Figure 1 shows that there is no accounting bias if the growth rate in the product market happens to equal the cost of capital, \( r \). This observation follows directly from the observation that

\[
\sum_{\tau=1}^{T} (1 + r)^{T-\tau} \cdot z_\tau(d) = v \cdot \gamma^{-T},
\]

for all depreciation schedules \( d \).

With accelerated depreciation (the lower half of the picture), the largest accounting bias occurs in the South-West corner of Figure 1, corresponding to no growth and one-hoss shay. The accounting bias then becomes \( \Gamma(0, 0|r = .12, T = 25) = .803 \). While this factor may not seem to entail that significant a distortion, we recall that it applies only to the asymptotic equilibrium price. To gauge the overall inefficiency associated with accounting biases under RoR regulation, one must consider the losses in the present value of all consumer surpluses resulting from the use of some biased depreciation rule as opposed to the unbiased R.P.C. rule.

---

\[18\] Directly related to this observation, Rajan and Reichelstein (2009) show that historical cost and marginal cost coincide, regardless of the accounting rules, whenever new investments grow at the firm’s cost of capital.
Figure 1: Level sets (isoquants) of the bias function $\Gamma(\mu, \beta | r = .12, T = 25)$ under straight-line depreciation. Here, $\mu$ represents growth in the product market while $\beta$ captures the periodic decline in productive capacity.

4 Stability of the RoR Regulation Process

The preceding characterization of the unique equilibrium price candidate, $p^*$, raises the following questions about the asymptotic properties of the RoR regulation process. First, is the process locally stable in the sense that it converges if the initial state (and corresponding prices) is sufficiently close to a state that supports $p^*$? Secondly, and more ambitiously, is the RoR process globally stable in the sense that prices will converge to $p^*$ for any initial state?

While we are not aware of examples where global stability does not obtain, we are also not in a position to address the convergence issues at the level of generality maintained in the previous sections. The following analysis will therefore consider specific combinations of productivity patterns and depreciation schedules. We start by assuming that productive capacity follows the one-hoss shay pattern and depreciation is calculated according to the straight-line rule. It is also assumed that demand is stationary over time so that $P_t(\cdot) = P(\cdot)$ for all $t$. Proposition 2 established that under these conditions $p^*$ is the unique stationary price. Let $\theta^* = (I^*, I^*, I^*, \ldots)$ denote the constant investment trajectory attaining $p^*$, that
is \( P(I^* \cdot T) = p^* \). Finally, let \( \xi(\cdot, \cdot) \) denote the Euclidean distance in \( \mathbb{R}^T \).

**Definition 5** The trajectory \( \theta^* \) is said to provide a locally stable implementation of \( p^* \), if for some \( \epsilon > 0 \), any trajectory \( \theta \) that is compatible with RoR regulation, \( \theta_T \) and satisfies \( \xi(\theta^*_T, \theta_T) < \epsilon \) also satisfies:

\[
\lim_{t \to \infty} P_t(K(\theta_t)) = p^*.
\]

A locally stable implementation requires that if any other trajectory is initially close to \( \theta^* \), that trajectory must result in prices that also converge to \( p^* \). We have the following result.

**Proposition 5** Given one-hoss shay productivity, straight-line depreciation and stationary demand, the trajectory \( \theta^* \) provides a locally stable implementation of \( p^* \).

The proof of Proposition 5 principally relies on the so-called Jury stability criteria, which provide sufficient conditions for local stability.\(^{19}\)

For the remainder of this section, we consider a setting that will be referred to as the geometric setting. Assets are infinitely lived and both productive capacity and accounting book values decay geometrically. Formally, the geometric setting satisfies the following specifications:

- The price in the product market is a linear function of firm output (or capacity). The intercept, \( a \), is stable over time, while the slope declines over time at the rate \((1 + \mu)^{-1}\), where \( \mu \) is the constant rate at which the overall market grows.\(^{20}\) Formally:

\[
P_t(q) = a - b_t \cdot q
\]

where \( b_t = \frac{b}{(1 + \mu)^T}, \mu \geq 0. \)

- Assets live for infinitely many periods, but the productive capacity of any investment declines geometrically over time at rate \( \alpha \), with \( 0 < \alpha < 1 \). As before, we normalize \( x_1 \) to equal 1. Productivity in future periods is then obtained recursively as:

\[
x_{\tau + 1} = (1 - \alpha) x_{\tau}.
\]

\(^{19}\)The proof of this result is long and tedious. It has therefore been relegated to a separate online companion paper, Nezlobin et al. (2010).

\(^{20}\)This is consistent with the proportionate growth assumption made earlier and implies that demand becomes more inelastic over time. As indicated in connection with Proposition 6 below, some of our results in this section can also be obtained with a constant elasticity demand curve.
Depreciation follows a declining-balance rule, represented by a parameter $\delta$, $0 < \delta < 1$. Thus, depreciation expense in each period is a constant proportion $\delta$ of the beginning of the period book value. This implies that the book value of one unit of capacity acquired at date 0 equals the acquisition cost, $v$, while the remaining book value at any future date $\tau \geq 0$, is given by:

$$bv_\tau = v \cdot (1 - \delta)^\tau$$

Given this specification, the relationship between the depreciation rate and beginning book value can be expressed as

$$d_{\tau+1} = \delta \cdot \frac{bv_\tau}{v}$$

Taken together, these assumptions imply that book values, depreciation charges, and capital costs all follow a geometric pattern over time:

$$bv_{\tau+1} = (1 - \delta) bv_\tau;$$

$$d_{\tau+1} = (1 - \delta) d_\tau;$$

and, therefore,

$$z_{\tau+1} = (1 - \delta) z_\tau.$$

If investments remain productive indefinitely, rather than have a finite life of $T$ years, the definition of a state needs to be modified. In particular, the relevant history includes not just investments made in the prior $T$ periods, but the entire vector of new investments from the inception of the firm. Accordingly, we redefine $\theta_t$ as $\theta_t = (I_0, ..., I_{t-1})$. Note that, because of the geometric structure of the model, the future capacity levels derived from past investments are completely defined by the current capacity level, $K_t$, and the same observation holds for the capital costs, $C_t$:

$$C_t(\theta_t) = \sum_{\tau=1}^{t} z_\tau \cdot I_{t-\tau}$$

In the geometric setting, current capacity and current capital costs are a “sufficient statistic” for the entire investment history. We again call a state $\theta_t$ feasible if the regulation constraint is satisfied as an equality in that state, that is $RI_t(\theta_t) = 0$. It is readily verified that in the geometric setting the R.P.C. rule amounts to $\delta = \alpha$, i.e., the depreciation rate matches the rate of decay in productivity. Furthermore, a value $\delta > \alpha$ corresponds to an accelerated depreciation rule.

In the more general settings examined in Sections 2 and 3, a technical concern was the possibility that there might exist infeasible states along the trajectory of investments, i.e.,
periods in which no positive investment level would satisfy the rate of return constraint. We show next that this issue does not arise in the geometric decay setting. Starting with a feasible state, there exists one, and only one, positive investment level that meets the following period’s Rate-of-Return constraint as an equality.

**Lemma 1** In the geometric setting, suppose depreciation is more accelerated than the R.P.C. rule. Starting from any feasible \( \theta_t \), there exists a unique \( I_t > 0 \) such that \( \theta_{t+1} \) is feasible.

We also note that if the firm starts with no capacity assets at date 0, there will be a unique \( I_0 \) such that \( \theta_1 \) is feasible. By induction, it follows then that when a firm enters the market with no assets in place, there is a unique chain of future investment levels that lead to period-by-period feasibility under rate of return regulation.

We now compute the marginal cost of capacity, \( c \), and the capacity level corresponding to \( p^* \) in the geometric case. Applying equation (4) and recognizing that \( T = \infty \), the marginal cost of capacity is equal to:

\[
c = \frac{v}{\sum_{\tau=1}^{\infty} (1 - \alpha)^{\tau-1} \gamma^\tau} = v (\alpha + r).
\]

The unique equilibrium price \( p^* \) is:

\[
p^* = w + c \cdot \frac{r + \delta}{r + \alpha} \cdot \frac{\sum_{\tau=1}^{\infty} (1 + \mu)^{-\tau} (1 - \delta)^{\tau-1}}{\sum_{\tau=1}^{\infty} (1 + \mu)^{-\tau} (1 - \alpha)^{\tau-1}} = w + c \cdot \Gamma,
\]

where

\[
\Gamma = \frac{(r + \delta) \cdot (\alpha + \mu)}{(r + \alpha) \cdot (\delta + \mu)}.
\]

Let \( K^* \) denote the capacity level corresponding to \( p^* \) at date 0, that is:

\[
a - bK^* = p^*
\]

Therefore,

\[
K^* = \frac{1}{b} (a - w - c \cdot \Gamma).
\]

**Proposition 6** In the geometric decay model, suppose depreciation is more accelerated than the R.P.C. rule and the firm starts from a feasible state. Then, as \( t \to \infty \),

\[
\frac{K(\theta_t)}{(1 + \mu)^t} \to K^* \text{ and } P(K(\theta_t)) \to p^*.
\]
Lemma 1 ensures that, starting from any feasible history of prior investments, there is a unique subsequent sequence of positive levels of investment that satisfy the Rate-of-Return constraint in each period. Proposition 6 shows that this investment trajectory must eventually result in prices converging to the unique equilibrium price, \( p^* \). Moreover, the levels of investment, adjusted for growth, also converge to a unique value, \( K^* \).

As indicated in the Introduction, Biglaiser and Riordan (2000, Theorem 2) establish that under “naive” Rate-of-Return regulation prices converge to a value above the long-run marginal cost. To reconcile this finding with our results, including Proposition 6, we note that the specification of naive regulation effectively introduces a bias towards decelerated accounting. By construction, old assets are becoming technically obsolete due to technical progress, which is reflected in decreasing acquisition costs for new assets and declining variable operating costs. Yet the very specification of naive RoR regulation in Biglaiser and Riordan implies that technically obsolete assets remain in the asset base beyond their economically useful life and continue to be depreciated indefinitely.

The proof of Proposition 6 also identifies the nature of the equilibrium trajectory of investment levels and prices. Specifically, convergence to the equilibrium occurs in a monotonic fashion. Given a feasible state \( \theta_t \) and the corresponding market price at date \( t \) below the equilibrium price, \( p^* \), the unique trajectory of future feasible levels of investment will result in a sequence of prices that are monotonically increasing and converge to \( p^* \).\(^{21}\) On the other hand, if the price is greater than \( p^* \), then every future price will be strictly lower and the sequence will again reach \( p^* \) in the limit. The following figure illustrates the latter scenario.

If the firm has initially no capacity assets \( (K_0 = 0) \), the RoR regulation constraint leads to market prices which are above the long-run marginal cost, \( w + c \) in early time periods. Ultimately, prices drop below the long-run marginal cost and converge to the equilibrium price, \( p^* \). Figure 2 illustrates this dynamic for the following set of parameters: \( w = 0, v = 1, \mu = 0, r = 12\%, \alpha = 5\% \), and \( P_t(q) = 1 - q \). The horizontal line represents the marginal cost of \( v = (\alpha + r) = 0.17 \). If depreciation were to conform to the R.P.C. rule, i.e., if \( \delta = \alpha = 0.05 \), this price would be realized in every period. The curve corresponding to \( \delta = 10\% \) represents an accelerated depreciation method. It intersects the marginal cost line once, from above, and then converges in a strictly decreasing manner to the equilibrium price, 0.11. The curve corresponding to \( \delta = 15\% \) depicts the impact of using an even more

\(^{21}\)We note that for convergence to be monotonic, the assumption of a linear demand curve seems essential. Numerical analysis indicates that with a constant elasticity demand curve global convergence still obtains; however, prices need not approach \( p^* \) in a monotonic fashion. Beyond the geometric decay model, for instance in the one-hoss shay scenario, it is straightforward to construct examples where prices converge to \( p^* \), albeit not monotonically.

19
accelerated depreciation schedule. In this case, the beginning price is greater than before, but the price subsequently drops much faster, cuts the marginal cost line earlier, and eventually converges to the new, lower equilibrium price of 0.09. A natural variant of the example illustrated in Figure 2 is that the firm’s initial capacity is 0.9, thereby yielding an initial market price of 0.1. Prices will then increase monotonically and converge to the equilibrium price ($p^* = 0.11$). Thus, prices stays below the marginal cost for the entire duration.

In the general setting of finitely-lived assets, Proposition 1 established the existence of a constant growth investment schedule that attains the stationary price, $p^*$. The argument requires an appropriate initialization in terms of investment choices for the first $T$ periods that can then be continued along a constant growth trajectory. Product prices will then coincide with $p^*$ for all periods from $T$ onwards. In the geometric decay setting, a similar sequence can be shown to exist. In fact, it is not necessary to specify the first $T$ investment levels in the geometric scenario. It suffices to identify an initial level of capacity assets and associated historical costs, as well as an appropriate first-period investment. Lemma 1 and the geometric structure then ensure that future investment levels not only maintain feasibility, but also grow at the rate $\mu$. As a consequence, the market price in each subsequent period remains equal to the stationary price, $p^*$.\textsuperscript{22}

\textsuperscript{22}The global convergence claim in Proposition 6 has been stated only for accelerated depreciation schedules. A corresponding result can be established for decelerated depreciation provided the demand curve intercept $a$ is sufficiently far above the stationary price $p^*$. The precise claim and its proof are reported in the online companion paper, Nezlobin et al. (2010).
From a consumer welfare perspective, one would expect that it is preferable to follow a depreciation schedule with $\delta$ closer to $\alpha$. In particular for the case where $K_0 = 0$, Figure 2 suggests that higher values of $\delta$ lead to a price path that exhibits greater variation relative to the underlying long-run marginal costs. Thus the price distortions tend to be larger both early on and asymptotically. The following result measures welfare in accordance with the objective function in (3), i.e., the present value of discounted future welfare levels.\(^{23}\)

**Conjecture:** Suppose $K_0 = 0$. In the geometric decay model, welfare is monotonically decreasing in $\delta$ provided $\delta \geq \alpha$.\(^{24}\)

This result speaks to the real effects of accounting policy. One of the fundamental issues for capital markets research in accounting is whether investors are misled by accounting rules that do not capture the underlying economics properly. If investors can “see through” the accounting rules and the reported accounting information, the resulting firm valuations and investment decisions may be nonetheless be efficient. In the current regulation context, this reasoning obviously does not apply since historical cost is a function of the accounting rules and historical cost directly determines product prices and new investments.

Figure 4 illustrates the above conjecture by plotting the welfare loss as a function of the depreciation parameter, $\delta$, under different specifications of the inverse demand function. Both plots assume that $\alpha = 0.05$ and $w = 0$. When the R.P.C. rule is used for regulatory purposes, $\delta = \alpha = 0.05$, rate of return regulation results in prices that maximize the discounted consumer welfare, so the relative welfare loss is zero. The two curves in Figure 4 show that as the depreciation schedule becomes more conservative, i.e. as $\delta$ increases, a greater share of the consumer welfare is lost. The dashed line represents a relatively smaller market (at any given price point), and the discounted welfare is more sensitive to changes in $\delta$ in that case.

\(^{23}\)If $K_0 = 0$ and the firm breaks even over the entire infinite horizon in the sense that the discounted cash flows (or the discounted residual incomes) sum up to zero.

\(^{24}\)We have not been able to prove this conjecture analytically. Even in a continuous time version of the model, the change in welfare as a function of $\delta$ leads to an intractable expression. However, we have conducted an extensive grid search in Matlab to verify the monotonicity of welfare in $\delta$. Assuming first that $\mu = 0$, we allowed for arbitrary values of $v$ and $w$ and performed the grid search for: $0.01 \leq r \leq 4; w + rv + 0.1v \leq a \leq w + rv + 100v; 0.1v^2 \leq b \leq 10v^2$ and $0 \leq \alpha \leq \delta \leq \min (1, \frac{a-w}{v} - r)$. Finally, it can be shown that if the claim is true for $\mu = 0$, it must then also hold for positive $\mu$; see the Auxiliary lemma in the proof of Lemma 1.
5 Concluding Remarks

This paper has examined the impact of accounting rules on product prices and capacity investments that emerge under Rate-of-Return regulation. We find that there exists a unique candidate stationary value to which product prices may converge over time. The relation between this stationary price and the firm's long-run marginal production cost depends on whether the depreciation schedule for fixed assets is accelerated or decelerated relative to the benchmark of neutral accounting. Current regulatory practice appears to favor straight-line depreciation which represents a form of accelerated depreciation if the productivity of assets is undiminished over their useful life (one-hoss shay scenario). Our analysis shows that the efficiency distortions of the rate-of-return process do not “wash out” over time, not even for stationary environments. On the contrary, the distortions caused by a biased depreciation schedule tend to be larger in stationary, no-growth environments.

While our analysis points to the centrality of the depreciation schedule in assessing the asymptotic properties of the Rate-of-Return mechanism, we are mindful of the fact that our model has ignored a number of factors that are likely of first-order importance in practice. For instance, this paper has treated the cost of new assets as time invariant. In certain industries, however, input prices have shown predictable price patterns over time. For instance, the cost of telecommunications equipment has been falling steadily over the past decades.
(2010) identifies unbiased depreciation rules which extend the concept of R.P.C. depreciation to settings where asset prices decline geometrically over time. A natural extension of our analysis in this paper is to study the intertemporal distortions and the limit behavior of the RoR regulation process if historical cost is calculated according to certain rules, such as straight-line depreciation, while the replacement cost of new assets is declining over time.

Secondly, our study of stationary prices has relied on the assumption that operating costs are assumed to be time-invariant and assets do not become economically obsolete until the end of their assumed useful lives. Technological innovation may make it advantageous to replace older but functional assets. As indicated above, this feature is central to the model of Biglaiser and Riordan (2000).

Finally, this paper has ignored the role of corporate income taxes. Rate-of Return regulation is commonly based on net income, that is income after taxes. Assets are commonly depreciated faster for tax purposes than for financial reporting purposes. The long-run marginal cost of production as well as the stationary price under RoR regulation will then depend on both the depreciation schedule used for regulatory purposes and the one used for tax purposes; see, for instance, İŞlegen and Reichelstein (2010). In the presence of corporate income taxes, the concept of unbiased accounting will change, that is, the R.P.C. schedule identified in our analysis will no longer lead to efficient outcomes. Studying issues of price convergence and welfare efficiency in a model with changing asset prices and corporate income taxes would bring the predictions of our analysis a significant step closer to the institutional realities of Rate-of-Return regulation.

6 Appendix

Proof of Proposition 1. Let $K_T$ be such that $P_T(K_T) = p^*$ and let

$$I = \frac{K_T}{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} x_{\tau}}.$$ 

Consider trajectory, $\theta$, for which investments start at $I_0 = I$ and then grow at rate $\mu$:

$$I_t = (1 + \mu)^t I.$$ 

For this trajectory, for any $t \geq T$, we have:

$$K(\theta_t) = I \cdot \sum_{\tau=1}^{T} (1 + \mu)^{t-\tau} x_{\tau} = (1 + \mu)^{t-T} K_T.$$ 

23
Therefore,
\[ P_t(K(\theta_t)) = P_T(K_T) = p^*. \]

The historical cost in period \( t \) is given by:
\[ H_t(\theta_t) = w \cdot K(\theta_t) + \sum_{\tau=1}^{T} (1 + \mu)^{t-\tau} z_{\tau}(d) \cdot I. \]

Revenues in period \( t \) are:
\[ P_t(K(\theta_t)) \cdot K(\theta_t) = p^* \cdot K(\theta_t) = w \cdot K(\theta_t) + \frac{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} z_{\tau}(d)}{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} z^*_\tau} \cdot c \cdot K(\theta_t). \]

To prove that the trajectory is compatible, it suffices to show that
\[ I = \frac{c \cdot K(\theta_t) \cdot (1 + \mu)^{T-t}}{\sum_{\tau=1}^{T} (1 + \mu)^{T-\tau} z^*_\tau}. \]

But this equality holds because \( K(\theta_t) \cdot (1 + \mu)^{T-t} = K_T \) and \( z^*_\tau = c \cdot x_\tau \) (see equation 6).

**Proof of Proposition 2.** Assume that
\[ \lim_{t \to \infty} P_t(K(\theta_t)) = p_\infty \]
for some trajectory \( \theta \) that is compatible with RoR regulation. Let \( K_\infty = P_0^{-1}(p_\infty) \). Since prices converge to \( p_\infty \), the growth-adjusted capacity levels must converge to \( K_\infty \):
\[ \lim_{t \to \infty} \frac{K_t}{(1 + \mu)^t} = K_\infty. \]

We establish three results which taken together imply that \( p_\infty = p^* \).

**Step A.** Under the conditions of Proposition 2, if the trajectory \( \theta \) results in convergent prices, then this trajectory can have at most a finite number of infeasible states.

**Proof of Step A.** Assume first that condition (i) of Proposition 2 is met. Let \( \theta_{t+1} = (I_{t-T+1}, ..., I_t) \) be an infeasible state in trajectory \( \theta \). Then, it has to be that \( I_t = 0 \) and
\[ K_{t+1} = x_2 I_{t-1} + ... + x_T I_{t-T+1}. \]

By definition,
\[ K_t = x_1 I_{t-1} + ... + x_T I_{t-T}. \] (11)
Therefore, we have:

\[ K_{t+1} = K_t - (x_1 - x_2) I_{t-1} - \ldots - (x_T - x_{T+1}) I_{t-T}, \]  

(12)

where we defined \( x_{T+1} \) to be zero. All the terms that are subtracted from \( K_t \) in the right-hand-side of (12) are non-negative since \( x_{\tau} \)'s are decreasing. Let

\[ \tau_0 = \arg \max_{\tau=1..T} x_{\tau} I_{t-\tau}. \]

Equation (11) implies that \( x_{\tau_0} I_{t-\tau_0} \geq \frac{K_t}{T} \). Therefore,

\[ (x_{\tau_0} - x_{\tau_0+1}) I_{t-\tau_0} \geq \frac{(x_{\tau_0} - x_{\tau_0+1})}{Tx_{\tau_0}} K_t. \]

Substituting this inequality into (12), we obtain:

\[ K_{t+1} \leq K_t - \frac{(x_{\tau_0} - x_{\tau_0+1})}{Tx_{\tau_0}} K_t. \]

Let

\[ \psi = \frac{1}{2} \min_{\tau=1..T} \left( \frac{x_{\tau} - x_{\tau+1}}{Tx_{\tau}} \right). \]

The strict monotoncity of the \( x \)'s implies that \( \psi \) is greater than zero. We have shown that

\[ K_{t+1} < (1 - \psi) K_t \]

for any infeasible state \( \theta_{t+1} \) and some constant \( \psi > 0 \). Since \( \mu \) is non-negative and growth-adjusted capacity levels converge to \( K_\infty \), there can only be a finite number of states such that \( K_{t+1} < (1 - \psi) K_t \), i.e. only a finite number of infeasible states.

Assume now that condition (ii) of Proposition 2 is met. The preceding arguments establish that

\[ C_{t+1} < (1 - \psi) C_t \]

for any infeasible state \( \theta_{t+1} \) and some positive \( \psi \). Since prices and growth-adjusted capacity levels converge, it must be that:

\[ \lim_{t \to \infty} \frac{(P_t (K_t) - w)K_t}{(1 + \mu)^t} = (p_\infty - w) K_\infty. \]

Therefore, for some sufficiently large \( t_0 \) and any \( t > t_0 \)

\[ (P_{t+1} (K_{t+1}) - w)K_{t+1} > \left( 1 - \frac{\psi}{2} \right) (P_t (K_t) - w)K_t. \]
If there are infinitely many infeasible states, then there will exist $t > t_0$ such that $\theta_t$ is feasible and $\theta_{t+1}$ is not (recall that feasible states happen at least every $T$ periods because $P_0(0) > w + z_1$). For this value of $t$, we have:

$$(P_{t+1} (K_{t+1}) - w)K_{t+1} > \left(1 - \frac{\psi}{2}\right)(P_t (K_t) - w)K_t = \left(1 - \frac{\psi}{2}\right)C_t > C_{t+1}.$$  

This contradicts the fact that residual income has to be negative in every infeasible state. Therefore, trajectory $\theta$ passes only through a finite number of infeasible states.

**Step B.** Under the conditions of Proposition 2, if $p_\infty$ is a stationary price, then there must exist a feasible trajectory $\theta'$ that attains $p_\infty$, i.e. for for all $t \geq T$

$$P_t (K (\theta'_t)) = p_\infty.$$  

**Proof of Step B.** Recall that $\theta_t = (I_{t-T}, ..., I_{t-1})$ and consider the following sequence of $T$-dimensional vectors. Define $s(t)$ to be the zero vector for $t < T$ and

$$s(t) = \frac{\theta_t}{(1 + \mu)^t}$$  

for $t \geq T$. Letting $s_\tau(t)$ denote the $\tau$-th component of $s(t)$, we have:

$$s_\tau(t) = \frac{I_{t-T-1+\tau}}{(1 + \mu)^t}.$$  

Note that $K(s(t)) = (1 + \mu)^{-t} K(\theta_t)$. The fact that the market grows at rate $\mu$ implies:

$$P_0 (K(s(t))) = P_t ((1 + \mu)^t K(s(t))) = P_t (K(\theta_t)).$$  

Therefore,

$$P_0 (K(s(t))) \to p_\infty.$$  

Since $P_0(\cdot)$ is monotonic and continuous, the equation above implies

$$K(s(t)) \to K_\infty.$$  

where $K_\infty = P_0^{-1}(p_\infty)$. Hence,

$$P_0 (K(s(t))) K(s(t)) \to p_\infty K_\infty.$$  

Step A established that the trajectory $\theta$ can have at most a finite number of infeasible states. Therefore, for values of $t$ greater than some $t_0$, all states $\theta_t$ are feasible. For $t > t_0$, we have:

$$P_0 (K(s(t))) K(s(t)) = (1 + \mu)^{-t} P_t (K(\theta_t)) K(\theta_t) = (1 + \mu)^{-t} H(\theta_t) = H(s(t)).$$
Applying (15), we obtain:

\[ H(s(t)) \to p_\infty K_\infty. \]  

We now show that the vectors \( s(t) \) are bounded. Recall that

\[ K(s(t)) = x_1 s_T(t) + \ldots + x_T s_1(t). \]

All \( x \)'s are strictly greater than zero, so condition (14) implies that \( s_\tau(t) \) are bounded for every \( \tau \).

We next construct a trajectory \( \theta' = (I_0', I_1', \ldots, I_t', \ldots) \) such that \( \theta'_t \) is feasible for every \( t \geq T \) and

\[ P_T(K(\theta'_t)) = p_\infty \]

for every \( t \geq T \). Since all vectors \( s(t) \) are drawn from a bounded set, there must exist a converging subsequence \( s(t_i) \). Let

\[ \theta'_T = (1 + \mu)^T \lim_{i \to \infty} s(t_i). \]

Applying the linearity of \( K(\cdot) \) and equation (14), we can compute capacity and revenues in state \( \theta'_T \) as follows:

\[ K(\theta'_T) = (1 + \mu)^T K \left( \lim_{i \to \infty} s(t_i) \right) = (1 + \mu)^T \lim_{i \to \infty} K(s(t_i)) = (1 + \mu)^T K_\infty, \]

\[ P_T(K(\theta'_T)) K(\theta'_T) = P_T \left( (1 + \mu)^T K_\infty \right) (1 + \mu)^T K_\infty = (1 + \mu)^T P_0(K_\infty) K_\infty. \]

Cost function is also linear in investments, and therefore:

\[ H(\theta'_T) = (1 + \mu)^T H \left( \lim_{i \to \infty} s(t_i) \right) = (1 + \mu)^T \lim_{i \to \infty} H(s(t_i)) = (1 + \mu)^T p_\infty K_\infty. \]

From these equations, one can see that state \( \theta'_T \) is feasible and \( P_T(K(\theta'_T)) = p_\infty \). It remains to show that this trajectory can be continued indefinitely into the future so that state feasibility and prices are preserved. We will prove the following statement.

Assume that state \( \theta'_t \) is feasible for some \( t \),  

\[ P_t(K(\theta'_t)) = p_\infty, \]

and there exists a subsequence of \( s(t), s(t_i) \), such that

\[ \theta'_t = (1 + \mu)^t \lim_{i \to \infty} s(t_i). \]  

(17)

Then there exists an investment \( I'_t \) such that state \( \theta'_{t+1} \) is feasible,

\[ P_{t+1}(K(\theta'_{t+1})) = p_\infty, \]

and there exists a subsequence of \( s(t), s(t'_i) \), such that

\[ \theta'_{t+1} = (1 + \mu)^{t+1} \lim_{i \to \infty} s(t'_i). \]  

(18)
The initial state, $\theta'_{T}$, was constructed to satisfy all of the above-mentioned conditions, so proving the preceding claim above will conclude the proof of Step B.

Consider a sequence $\{s(t_{i} + 1)\}_{i=1}^{\infty}$ which is comprised of elements of $\{s(t)\}$ immediately following the elements of $\{s(t_{i})\}$ from (17). The sequence $\{s(t_{i} + 1)\}_{i=1}^{\infty}$ must have a converging subsequence (since all vectors $s(\cdot)$ are drawn from a compact set). Let $\{s(t'_{i})\}_{i=1}^{\infty}$ denote this converging subsequence and let

$$\varphi = (1 + \mu)^{t+1} \lim_{i \to \infty} s(t'_{i}) .$$

To show that $\varphi$ is a legitimate candidate for $\theta'_{t+1}$, we need to check that the first $T - 1$ elements of $\varphi$ correspond to the last $T - 1$ elements of $\theta'_{t}$. Letting $\varphi_{\tau}$ denote the $\tau$-th coordinate of $\varphi$, we have:

$$\varphi_{\tau} = (1 + \mu)^{t+1} \lim_{i \to \infty} s_{\tau}(t'_{i}) .$$

By construction, for $\tau < T$

$$s_{\tau}(t'_{i}) = (1 + \mu)^{-1} s_{\tau+1}(t'_{i} - 1) .$$

Therefore, for $\tau < T$,

$$\varphi_{\tau} = (1 + \mu)^{t+1} \lim_{i \to \infty} s_{\tau}(t'_{i}) = (1 + \mu)^{t} \lim_{i \to \infty} s_{\tau+1}(t'_{i} - 1) = (1 + \mu)^{t} \lim_{i \to \infty} s_{\tau+1}(t_{i}) = I'_{t-T+\tau} .$$

Setting $I'_{t} = \varphi_{T}$, we will have

$$\theta'_{t+1} = \varphi = (1 + \mu)^{t+1} \lim_{i \to \infty} s(t'_{i}) .$$

It remains to check that $\theta'_{t+1}$ is feasible and that $P_{t+1}(K(\theta'_{t+1})) = p_{\infty}$. To that end, we apply the assumption of constant market growth, linearity of $K(\cdot)$ and $H(\cdot)$, and equations (14) and (16) to complete the proof:

$$K(\theta'_{t+1}) = (1 + \mu)^{t+1} K_{\infty} ,$$

$$P_{t+1}(K(\theta'_{t+1})) K(\theta'_{t+1}) = (1 + \mu)^{t+1} P_{t+1}((1 + \mu)^{t+1} K_{\infty}) K_{\infty} = (1 + \mu)^{t+1} p_{\infty} K_{\infty} ;$$

$$H(\theta'_{t+1}) = (1 + \mu)^{t+1} p_{\infty} K_{\infty} .$$

**Step C.** Under the conditions of Proposition 2, $p^*$ is the unique stationary price of the RoR regulation process.
Proof of Step C. We will prove this step assuming that condition (i) of Proposition 2 holds. Under condition *ii), the proof is analogous with \( x_\tau \) replaced by \( z_\tau \) and \( K_t \) replaced by \( C_t \).

If \( \theta \) supports \( p^\infty \), then, as was shown in the previous step, there exists a trajectory \( \theta' \) that attains \( p^\infty \). For notational simplicity, we will now redefine \( \theta \) to be the trajectory that attains \( p^\infty \). First, let us show that there exists some \( \eta \) such that

\[
I_t - \eta K_t \to s^*
\]

when \( t \to \infty \), for some constant \( s^* \). To sustain constant prices, the firm needs to grow capacity at rate \( \mu \):

\[
K_{t+1} = (1 + \mu) K_t.
\]

The latest investment can be expressed as:

\[
I_t = \frac{\mu}{x_1} K_t + \frac{x_1 - x_2}{x_1} I_{t-1} + \ldots + \frac{x_T - x_{T+1}}{x_1} I_{t-T},
\]

(19)

where \( x_{T+1} = 0 \). There exists some \( \eta \) satisfying:

\[
\frac{\mu}{x_1} = \eta - \frac{x_1 - x_2}{x_1 (1 + \mu)} \eta - \ldots - \frac{x_T - x_{T+1}}{x_1 (1 + \mu)^T} \eta.
\]

(20)

If \( \mu = 0 \), then any \( \eta \) satisfies the equation above and we will proceed with \( \eta = 0 \). If \( \mu > 0 \), then the coefficient on \( \eta \) in the right-hand-side is strictly greater than zero and the equation defines a unique (positive) \( \eta \). Since

\[
K_{t-\tau} = \frac{K_t}{(1 + \mu)^\tau},
\]

equations (19) and (20) imply:

\[
(I_t - \eta K_t) = \frac{x_1 - x_2}{x_1} (I_{t-1} - \eta K_{t-1}) + \ldots + \frac{x_T - x_{T+1}}{x_1} (I_{t-T} - \eta K_{t-T}).
\]

Let \( s_t = I_t - \eta K_t \), and let us show the sequence \( \{s_t\} \) converges. We know that

\[
s_t = \frac{x_1 - x_2}{x_1} s_{t-1} + \ldots + \frac{x_T - x_{T+1}}{x_1} s_{t-T}.
\]

(21)

All coefficients in the right-hand-side are strictly positive and add up to unity. Let

\[
\bar{s}_t = \max_{\tau=0,\ldots,T-1} s_{t-\tau}
\]

and

\[
\underline{s}_t = \min_{\tau=0,\ldots,T-1} s_{t-\tau}.
\]

29
Equation (21) implies that $s_{t-1} \leq s_t \leq \bar{s}_{t-1}$. Therefore, it has to be that $s_{t-1} \leq s_t \leq \bar{s}_t \leq \bar{s}_{t-1}$. These inequalities imply that both sequences $\{s_t\}$ and $\{ar{s}_t\}$ are monotonic and must converge to some limits, $\underline{s}^*$ and $\bar{s}^*$, respectively. Let us assume that $\underline{s}^* \neq \bar{s}^*$. Then, by equation (21),

$$s_t \geq s_{t-1} + \left( \min_{\tau=1..T} \frac{x_\tau - x_{\tau+1}}{x_1} \right) (\bar{s}_{t-1} - \underline{s}_{t-1}).$$

For sufficiently large values of $t$, this becomes

$$s_t \geq \underline{s}^* + \left( \min_{\tau=1..T} \frac{x_\tau - x_{\tau+1}}{x_1} \right) (\bar{s}^* - \underline{s}^*) - \epsilon.$$

Therefore, for large values of $t$,

$$\underline{s}_t \geq \underline{s}^* + \left( \min_{\tau=1..T} \frac{x_\tau - x_{\tau+1}}{x_1} \right) (\bar{s}^* - \underline{s}^*) - \epsilon.$$

On the other hand, $\underline{s}_t$ converges to $\underline{s}^*$, so for large $t$'s, $\underline{s}_t$ has to be not greater than $\underline{s}^* + \epsilon$. Choosing $\epsilon$ so that

$$\underline{s}^* + \epsilon < \underline{s}^* + \left( \min_{\tau=1..T} \frac{x_\tau - x_{\tau+1}}{x_1} \right) (\bar{s}^* - \underline{s}^*) - \epsilon$$

yields a contradiction. We have shown that $\underline{s}^* = \bar{s}^* \equiv s^*$ and, since $s_{t-1} \leq s_t \leq \bar{s}_t$, $s_t$ converges to $s^*$.

If $\mu = 0$, then $s_t = I_t$ and we have shown that investment levels converge. Then,

$$p_{\infty} - w = \frac{C(\theta_t)}{K(\theta_t)} = \lim_{t \to \infty} \frac{C(\theta_t)}{K(\theta_t)} = \frac{s^* \sum_{\tau=1}^T x_\tau}{s^* \sum_{\tau=1}^T x_\tau} = p^* - w.$$

Now assume that $\mu > 0$ and let $K_{\infty} = P_0^{-1} (p_{\infty})$. Then, $K_t = (1 + \mu)^t K_{\infty}$, and, therefore,

$$K_{\infty} = \frac{K_t}{(1 + \mu)^t} = \lim_{t \to \infty} \frac{K_t}{(1 + \mu)^t}.$$

Since $\lim_{t \to \infty} (I_t - \eta K_t) = 0$,

$$\lim_{t \to \infty} \frac{I_t - \eta K_t}{(1 + \mu)^t} = 0.$$

Hence,

$$\lim_{t \to \infty} \frac{I_t}{(1 + \mu)^t} = \eta K_{\infty}. \quad (22)$$

To compute $\eta$, we note that:

$$K_{\infty} = \lim_{t \to \infty} \frac{K_t}{(1 + \mu)^t} = \lim_{t \to \infty} \frac{x_1 I_{t-1} + \ldots + x_T I_{t-T}}{(1 + \mu)^t} = \left( \frac{x_1}{(1 + \mu)^t} + \ldots + \frac{x_T}{(1 + \mu)^t} \right) \eta K_{\infty},$$

$$30$$
and, therefore,
\[ \eta = \frac{1}{\sum_{\tau=1}^{T} x_{\tau} (1 + \mu)^{-\tau}}. \]
Since prices are constant, capacity grows at rate \( \mu \), and the rate-of-return constraint is satisfied in every state, capital costs must also grow at rate \( \mu \). Let \( C_{\infty} = \frac{C_t}{(1 + \mu)^t} \). Then,
\[ C_{\infty} = \frac{C_t}{(1 + \mu)^t} = \lim_{t \to \infty} \frac{C_t}{(1 + \mu)^t} = \lim_{t \to \infty} \frac{z_{1} I_{t-1} + \ldots + z_{T} I_{T-t}}{(1 + \mu)^t}. \]
Let us now apply (22):
\[ C_{\infty} = \lim_{t \to \infty} \frac{z_{1} I_{t-1} + \ldots + z_{T} I_{T-t}}{(1 + \mu)^t} = \sum_{\tau=1}^{T} \frac{z_{\tau} (1 + \mu)^{-\tau}}{(1 + \mu)^t} \eta K_{\infty}. \]
To conclude, observe that
\[ p_{\infty} - w = \frac{C_{\infty}}{K_{\infty}} = \frac{\sum_{\tau=1}^{T} z_{\tau} (1 + \mu)^{-\tau}}{\sum_{\tau=1}^{T} x_{\tau} (1 + \mu)^{-\tau}} = p^{*} - w. \]
This concludes the proof of Proposition 2.

**Proof of Proposition 3.** Define
\[ h(\mu, d) \equiv (1 + \mu)^0 \cdot z_{T}(d) + \ldots + (1 + \mu)^{T-1} \cdot z_{1}(d) \]
By definition,
\[ \Gamma \equiv \frac{h(\mu, d)}{h(\mu, d^{*})}, \tag{23} \]
where \( d^{*} \) is the unique depreciation schedule corresponding to the R.P.C. rule. In particular, \( z_{\tau}(d^{*}) \equiv z_{\tau}^{*} \). To prove the claim, it suffices to show that:
\[ h(\mu, d) < h(\mu, d^{*}), \]
whenever \( d \) is accelerated relative to \( d^{*} \) and, conversely, that the inequality is reversed if \( d \) is decelerated relative to \( d^{*} \). Since \( \mu < r \), both claims follow directly from Proposition 1 in Rajan and Reichelstein (2009).

**Proof of Proposition 4.** By construction of the R.P.C. rule,
\[ h(\mu, d^{*}) = c \cdot [(1 + \mu)^0 \cdot x_{T} + \ldots + (1 + \mu)^{T-1} \cdot x_{1}], \]
where \( h(\cdot, \cdot) \) is the same as in the proof of Proposition 3. Proposition 3 in Rajan and Reichelstein (2009) shows that
\[
\frac{(1 + \mu)^0 \cdot z_T(d) + \ldots + (1 + \mu)^{T-1} \cdot z_1(d)}{c \cdot [(1 + \mu)^0 \cdot x_T + \ldots + (1 + \mu)^{T-1} \cdot x_1]}
\]
is increasing (decreasing) in \(\mu\) provided \(\frac{z(d)}{x_t}\) is monotonically decreasing (increasing) in \(t\). That is precisely the definition of a depreciation schedule that is uniformly accelerated (decelerated) relative to the R.P.C. rule.

**Proof of Lemma 1.** First, we will establish an auxiliary lemma that will convert the case of constant market growth to the case of stationary markets \((\mu = 0)\). We subsequently prove Lemma 1 under the assumption of no growth.

**Auxiliary Lemma.** Given a model with constant market growth, consider the following model with stationary markets:

\[
P'(K'_t) = a - bK'_t,
\]

\[
x'_\tau = \frac{x_\tau}{(1 + \mu)^\tau},
\]

\[
z'_\tau = \frac{z_\tau}{(1 + \mu)^\tau}.
\]

Then, state \(\theta_t = (I_0, \ldots, I_{t-1})\) is feasible in the original constant growth model if and only if state \(\theta'_t = \left(\frac{I_0}{(1 + \mu)^0}, \ldots, \frac{I_{t-1}}{(1 + \mu)^{t-1}}\right)\) is feasible in the stationary model defined above.

**Proof.** First observe that

\[
K'_t(\theta'_t) = \sum_{\tau=1}^{t} I_{t-\tau}(1 + \mu)^{-(t-\tau)} x_\tau(1 + \mu)^{-\tau} = (1 + \mu)^{-t} K_t.
\]

Hence, \(P'(K'_t) = P_t(K_t)\) and \(P'(K'_t)K'_t = (1 + \mu)^{-t}P_t(K_t)K_t\). It remains to show that

\[
H'_t(\theta'_t) = (1 + \mu)^{-t}H_t(\theta_t).
\]

Indeed,

\[
H'_t(\theta'_t) = \sum_{\tau=1}^{t} I_{t-\tau}(1 + \mu)^{-(t-\tau)} z_\tau(1 + \mu)^{-\tau} = (1 + \mu)^{-t}H_t.
\]

It remains to establish Lemma 1 in the case of no growth. Since state \(\theta_t\) is feasible, we have:

\[
P_t(K_t) K_t = C_t + wK_t.
\]

State \(\theta_{t+1}\) is feasible if and only if

\[
P_{t+1}(K_{t+1}) K_{t+1} = C_{t+1} + wK_{t+1}.
\]
Capital costs in period $t+1$ can be decomposed as the sum of capital costs of investments prior to $I_t$ and the capital cost of the latest investment:

$$C_{t+1} = C_t (1 - \delta) + z_1 I_t.$$  

Applying (24), the first term in the RHS of the equation above can be rewritten as

$$C_t (1 - \delta) = (1 - \delta) (P_t (K_t) K_t - w K_t).$$

For declining balance depreciation, $z_1 = \delta v + rv$. Since $x_1 = 1$ and capacity declines at the rate $(1 - \alpha)$,

$$I_t = K_{t+1} - (1 - \alpha) K_t.$$  

These observations together imply that state $\theta_{t+1}$ is feasible if and only if

$$P_{t+1} (K_{t+1}) K_{t+1} = (1 - \delta) (P_t (K_t) K_t - w K_t) + (\delta + r)v (K_{t+1} - (1 - \alpha) K_t) + w K_{t+1}.$$  

Given the linearity of demand curves, this equality can be rewritten as:

$$(a - b K_{t+1}) K_{t+1} = (1 - \delta) ((a - b K_t) K_t - w K_t) + (\delta + r)v (K_{t+1} - (1 - \alpha) K_t) + w K_{t+1}.$$  

We obtain a quadratic equation in $K_{t+1}$ and it has two solutions:

$$K_{t+1}^{(1)} = \frac{1}{2b} (a - w - v (r + \delta)) + \frac{1}{2b} \sqrt{S(K_t)}$$  \hspace{1cm} (26)

and

$$K_{t+1}^{(2)} = \frac{1}{2b} (a - w - v (r + \delta)) - \frac{1}{2b} \sqrt{S(K_t)},$$  \hspace{1cm} (27)

where

$$S(K_t) \equiv (a - w - v (r + \delta))^2 + 4b(-a + rv + w - rv\alpha + (a + v - w - v\alpha)\delta)K_t + 4b^2(1 - \delta)K_t^2.$$ \hspace{1cm} (28)

Let us now recall that $I_t = K_{t+1} - (1 - \alpha) K_t$ and compute the corresponding investments:

$$I_t^{(1)} = \frac{1}{2b} (a - w - v (r + \delta) - 2 (1 - \alpha) b K_t) + \frac{1}{2b} \sqrt{S(K_t)},$$

$$I_t^{(2)} = \frac{1}{2b} (a - w - v (r + \delta) - 2 (1 - \alpha) b K_t) - \frac{1}{2b} \sqrt{S(K_t)}.$$  

First we will show that $I_t^{(2)}$ is negative and, therefore, if a positive feasible continuation exists, it is unique. Second, we will show that a positive feasible $I_t$ exists (this will imply that $I_t^{(1)}$ is positive). For the former statement, it suffices to check that

$$(a - w - v (r + \delta) - 2 (1 - \alpha) b K_t)^2 < S(K_t).$$
Expanding and subtracting the right-hand-side from the left-hand-side, the inequality above can be rewritten as:

\[ 4bK_t \left( bK_t(\alpha^2 - 2\alpha + \delta) + (a - w)(\alpha - \delta) \right) < 0. \]  

(29)

Let us assume that \( \alpha^2 - 2\alpha + \delta > 0 \). Then, the left-hand-side of (29) is less than zero if

\[ K_t < \frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)} . \]

Note that \( \delta - \alpha > \alpha^2 - 2\alpha + \delta \). Therefore,

\[ \frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)} > \frac{a - w}{b} . \]

On the other hand, we know that \( a - bK_t > w \), which implies \( K_t < \frac{a - w}{b} \), and inequality (29) has to hold. Now let us assume that \( \alpha^2 - 2\alpha + \delta < 0 \). Then, inequality (29) holds whenever

\[ K_t > \frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)} . \]

In that case, since by assumption \( \delta > \alpha \), and \( P_t(K_t) > w \), we have \( a > w \). Therefore,

\[ \frac{(a - w)(\delta - \alpha)}{b(\alpha^2 - 2\alpha + \delta)} < 0 \]

and \( K_t \) has to be greater than this number.

To show that a positive feasible \( I_t \) exists, we first observe that, if the company does not invest anything in period \( t \), its total costs will be lower than revenues:

\[ P_{t+1}(K_{t+1})K_{t+1} = (1 - \alpha)P_{t+1}((1 - \alpha)K_t)K_t > (1 - \alpha)P_t(K_t)K_t \]

\[ = (1 - \alpha)(C_t + wK_t) > (1 - \delta)C_t + (1 - \alpha)wK_t \]

\[ = C_{t+1} + wK_{t+1} . \]

On the other hand, if the company were to invest \( K_{t+1} = \frac{a}{b} \), its revenues would be equal to zero, while its costs would be positive. Therefore, there must exist a positive \( I_t \) which equates revenues to total costs.

**Proof of Proposition 6.** By the same auxiliary argument that was employed in the proof of Lemma 1, the case of constant market growth can be converted to the case of no growth by adjusting \( x_T \) and \( z_T \) appropriately. We therefore restrict attention to the case when \( \mu = 0 \).

First we will show that \( K_{t+1} > K_t \) if \( K_t < K^* \) and \( K_{t+1} < K_t \) otherwise. We know that \( K_{t+1} \) will be given by the expression for \( K^{(1)}_{t+1} \) from the proof of the previous lemma, equation (26):

\[ K_{t+1} = \frac{1}{2b} (a - w - v(r + \delta)) + \frac{1}{2b} \sqrt{S(K_t)} , \]
where $S(K_t)$ is defined by equation (28).

It is clear from this formula that, starting from $K_2$, it will be the case that

$$K_t \geq \frac{1}{2b} (a - w - v(r + \delta)).$$

Therefore, $K_{t+1}$ is greater than $K_t$ whenever

$$(a - w - v(r + \delta) - 2bK_t)^2 < S(K_t).$$

This is equivalent to

$$4bK_t ((a - w)\delta + \alpha(r + \delta) + b\delta K_t) < 0.$$ 

The latter inequality holds when

$$0 < K_t < \frac{(a - w)\delta - \alpha(r + \delta)}{b\delta} = K^*,$$

and the converse holds when $K_t > K^*$. We have shown that $K_{t+1} > K_t$ if $K_t < K^*$ and $K_{t+1} < K_t$ otherwise.

We next show that $K_{t+1} > K^*$ if $K_t > K^*$ and $K_{t+1} < K^*$ otherwise. We first observe that since $\delta > \alpha$,

$$\frac{1}{2b} (a - w - v(r + \delta)) < \frac{(a - w)\delta - \alpha(r + \delta)}{b\delta} = K^*.$$ 

Therefore, $K_{t+1} > K^*$ if and only if

$$(a - w - v(r + \delta) - 2\frac{(a - w)\delta - \alpha(r + \delta)}{\delta})^2 < S(K_t).$$

This condition is equivalent to:

$$\frac{4((w - a)\delta + \alpha(r + \delta) + bK_t\delta)(\alpha - \delta)(r + \delta) - b(1 - \delta)\delta K_t}{\delta^2} < 0.$$ 

The quadratic function in the numerator of the left-hand-side has two roots:

$$\left\{ \frac{(a - w)\delta - \alpha(r + \delta)}{b\delta}, \frac{v(\alpha - \delta)(r + \delta)}{b(1 - \delta)} \right\}.$$ 

The first root is equal to $K^*$ and the second root is negative due to the assumption, $\delta > \alpha$. The coefficient on $K_t^2$ in (30) is negative, so the function will be greater than zero for $0 < K_t < K^*$ and less than zero for $K_t > K^*$. We have shown that if $K_t < K^*$, then $K_t < K_{t+1} < K^*$, and if $K_t > K^*$, then $K_t > K_{t+1} > K^*$. These inequalities imply that
sequence \( \{K_t\} \) converges to some limit \( K_\infty \). Let us show that \( K_\infty = K^* \). Since all states \( \theta_t \) are feasible, \( P(K_t)K_t = wK_t + C_t \), it has to be that

\[
wK_t + C_t \rightarrow P(K_\infty)K_\infty.
\]

Since \( I_t = K_{t+1} - (1 - \alpha)K_t \), \( I_t \rightarrow \alpha K_\infty \). On the other hand, \( (\delta + r) v I_t = C_{t+1} - (1 - \delta)C_t \), therefore

\[
(\delta + r) v I_t \rightarrow \delta (P(K_\infty)K_\infty - wK_\infty).
\]

We conclude that:

\[
\alpha K_\infty = \frac{\delta (P(K_\infty)K_\infty - wK_\infty)}{(\delta + r) v}
\]

and

\[
P(K_\infty) = w + \frac{\alpha(\delta + r)}{\delta} v = p^*.
\]

References


MIT Press, Cambridge, MA.

bridge, MA.

Return Regulation Process,” Working Paper, GSB, Stanford University, available at:

Publishers, Boston, Massachusetts.

Controlling Investment Decisions under Sequential Private Investment,” Management
Science 53, 495-507.

Econometrica 6, 219-231.

Marginal and Historical Cost,” Journal of Accounting Research 47, 823-865.


of Political Economy 116, 931-950.

Schmalensee, R. (1989). “An Expository Note on Depreciation and Profitability under Rate-
of-Return Regulation,” Journal of Regulatory Economics 1, 293-298.

Boston, MA.