Discretion in Managerial Bonus Pools

Merle Ederhof  
Ross School of Business, University of Michigan  
derhof@umich.edu

Madhav V. Rajan  
Graduate School of Business, Stanford University  
mrajan@gsb.stanford.edu

Stefan Reichelstein*  
Graduate School of Business, Stanford University  
reichelstein@stanford.edu

August 2010

---

*We are grateful to Ulf Schiller for helpful comments on an earlier version of this paper. We also thank Yanruo Wang for excellent research assistance.
1 Introduction

Agency theory has for most part focused on the nature and efficiency properties of “complete” incentive contracts. Accordingly, a contract is viewed as a collection of “if-then” statements such that a specific payoff results if a particular outcome has emerged. The enforceability of such incentive contracts is usually based on the notion that the outcomes are verifiable to third parties, with the consequence that a court of law could enforce the contract provisions in case of dispute.¹

An essential characteristic of the incentive plans observed in many organizations is that they are not “hard-wired” but instead leave a considerable amount of discretion. Top-level management frequently defines a set of performance metrics and specifies certain boundary parameters for incentive compensation. Yet, at the same time management retains discretion in determining the actual rewards and compensation payments for lower level managers. The widespread use of balanced scorecards provides an illustration of discretionary performance measurement. Such scorecards usually combine a host of financial and non-financial performance indicators.² While firms make a commitment to measuring these variables for the purpose of performance evaluation, the actual aggregation (balancing) of the component variables for the purpose of determining managerial bonuses is frequently not specified contractually.

Bonus pools provide another illustration of discretionary incentive mechanisms. It is common practice in firms across a variety of industries to specify an overall bonus amount for a group of managers. While this amount frequently varies with certain high-level financial metrics, like earnings, sales revenues or return on investment (ROI), the central feature of bonus pools is that the principal (board of directors or higher-level management) retains discretion for distributing the bonus pool among the eligible agents. Some publicly traded companies disclose considerable details about the structure of their bonus pool arrangements. The following excerpt is taken from the 2007 proxy statement of Aetna corporation:

“... The Compensation Committee, after consulting with the Board, establishes specific financial and operational goals at the beginning of each performance year, and annual bonus funding is linked directly to the achievement of these annual goals. [...] For 2007, bonus pool funding under the ABP [Annual Bonus Program] depended upon performance against the following measures: Financial performance (55%), health cost management (16%), profitable

¹In contrast to this view of “complete” contracts, the literature on incomplete contracts has emphasized that in many situations the parties are constrained in the types of contracts they rely on. Certain contingencies may be difficult to describe contractually (Williamson, 1995), some variables may not be verifiable to outside parties (Tirole, 1999) or comprehensive contracts may be too costly to write (Dye, 1985; Melumad et al., 1997; Laffont and Martimort, 2001).

growth (16%), and constituent focus (13%). [...] For 2007, as a result of this performance, after applying the weightings noted above, the Compensation Committee set the 2007 ABP bonus pool funding at just above the target level. Within this pool funding, the Compensation Committee set actual bonus amounts after a subjective review of each executive officer’s individual performance for the year and consideration of recommendations from the CEO. [...] The Committee has the discretion to pay an individual executive above or below the target performance based on its assessment of individual performance.”

The description of Aetna’s compensation plan suggests that certain information variables are deemed important for assessing managerial performance, yet these variables are not specified in a formulaic fashion in the compensation scheme. This feature most likely reflects that some variables are difficult, if not impossible, to describe with sufficient precision as part of a contract. A related reason possibly is that these variables cannot be incorporated credibly into a contract because their actual realization could not be verified by outside parties. Both of these frictions may have influenced the bonus plan adopted by SWS Group in 2005, as the following excerpt suggests:

“Our incentive compensation program provides for a bonus pool, determined annually, based on our return on equity. Allocation of the bonus pool to individual executive officers is determined using objective measures of business unit performance as well as subjective measures of the executive officer’s contribution to our financial and strategic objectives”.

The objective of this article is to synthesize and integrate a growing literature that has emerged over the past 10-15 years on the use of both objective and subjective performance indicators in managerial incentive plans. Since the terms “objective” and “subjective” are not used uniformly in the literature, it is essential to clarify the meaning we attach to these terms in this article. Objective performance indicators are observed by the principal and the agent(s). These indicators are verifiable to third parties and therefore compensation arrangements can be explicitly conditioned on their realization. Accounting information, stock price and quantifiable productivity measures are prime examples in this category. In contrast, subjective performance indicators are observed by the principal, and possibly also by the agents (an issue we discuss in further detail in Section 2 below). While these variables may be quantifiable, it is assumed that their realization cannot be verified by outside (third) parties. Direct observations by the principal about an agent’s conduct or reports about this agent conveyed by other agents in the organization are leading examples of subjective information.

Without attempting a review of the literature at this stage, we mention several branches of the existing work on incentive contracting with subjective information. Bull (1987) and Baker el al. (1994) are examples of models where a principal and an agent can rely only
Contracts are entirely *implicit* in the sense that the principal is under no legal obligation to pay the amount promised under the agreement. Performance incentives can then be sustained only through the threat of terminating cooperation in future periods if the principal were to behave opportunistically in any given period and deny the agent the bonus promised under the implicit agreement.

For short-term contracting arrangements based only on subjective performance indicators, the principal faces a more severe problem in making any incentive provisions credible. MacLeod (2003) argues that one way of achieving credibility is to commit to a fixed-payment scheme, or a bonus pool. The principal will not be tempted to act opportunistically ex-post if the bonus pool is paid out in full such that any portion not paid to the agent must be diverted to a third party whose welfare is of no concern to either the principal or the agent. MacLeod demonstrates the striking result that the optimal bonus pool arrangement results in an extremely compressed incentive scheme. In particular, the agent will always receive the full bonus pool amount unless the subjective metric assumes the lowest possible outcome.

While the possibility of diverting money to a third party provides a “theoretical” solution to the credibility problem caused by subjectivity, the widespread use of bonus pools in practice suggests that it may be more efficient to combine multiple agents in one bonus pool. The agents can then serve as budget balancers for one another. An early model examining this possibility is Baiman and Rajan (1995). They show that a principal can generate a more efficient incentive structure by incorporating unverifiable information via a bonus pool that is ex-post split among the agents.

This article examines the structure of efficient bonus pools (fixed payment schemes) in the presence of subjective performance indicators. We cover a range of scenarios including single- and multi-agent settings, the interplay of objective and subjective indicators and short-term as opposed to long-term contracting relations. To synthesize the existing research, we frame our exposition around five recurring themes which collectively speak to the structure and the efficiency of incentive schemes based on subjective information.

(i) *Value of Subjective Performance Indicators*

Are subjective performance indicators valuable for contracting purposes? A fundamental result in agency theory, due to Holmstrom (1979), says that any information signal that is incrementally informative about an agent’s non-contractible action must be included in the optimal incentive scheme. We ask whether this result carries over to subjective, non-verifiable signals. The inherent tradeoff involves a balance between the informational value of the subjective signal and the constraints on the incentive scheme imposed by subjective information.

---

See also Pearce and Stacchetti (1998) and Levin (2003).
(ii) Incremental Agency Cost
Bonus pools generally entail an agency cost beyond the hypothetical benchmark in which all performance indicators are objective and verifiable for contracting purposes. We seek to identify the incremental agency cost that the principal incurs due to some performance indicators being subjective. In particular, we link the cost increment to the number of participating agents and the information content of the available signals.

(iii) Compression of Optimal Incentive Contracts
In circumstances where subjective performance indicators are valuable for contracting purposes, we continue MacLeod’s (2003) line of inquiry to determine under what circumstances optimal incentive schemes are compressed. If the principal can rely on both objective and subjective information, will an efficient bonus pool arrangement continue to have the property that the agent(s) receive identical compensation for an entire range of subjective outcomes?

(iv) Optimality of Proper Bonus Pools
If the principal constructs a joint incentive contract for multiple agents based on subjective information, will the corresponding bonus pools be proper in the sense that the entire bonus amount is always paid out to the participating agents, rather than being partially diverted to a third party? By relying on some agents to be budget balancers for others, the principal saves the cost of diverting money to parties that are external to the agency. However, if agents serve also as budget balancers, their compensation will be exposed to additional risk associated with the variability in the performance indicators of other agents. We examine whether this additional cost arising from inefficient risk-sharing dominates the savings that the principal would incur by confining attention to proper bonus pools.

(v) Value of Multiperiod Contracts
The final part of our analysis explores to what extent the efficiency of bonus pool arrangements can be improved through multi-period contracts which allow the principal to roll over parts of a current period bonus pool into future periods. We also seek to highlight the constraints imposed by subjectivity in an infinite horizon setting.

The use of subjective performance evaluation has been documented in a variety of empirical studies. Bushman et al. (1996), Ittner et al. (1997), and Hayes and Schaefer (2000) examine how bonuses for CEOs are influenced by subjective factors. These studies find evidence that subjective information plays a bigger role in environments where objective
performance signals, such as accounting information, are less informative for contracting purposes.

Murphy and Oyer (2003) provide a comprehensive description of the different ways that discretion influences the process of determining bonuses. They conclude that almost two-thirds of the companies in their sample use non-financial measures of individual performance to determine individual bonus payouts. In 42% of their sample firms, the board has discretion in determining the aggregate amount of bonuses paid. Furthermore, in 70% of their sample firms, the board has discretion in allocating the bonus pool to individuals. Finally, for approximately one third of the companies in the sample, the original bonus formula was overridden following the review of other subjective information.

Related to this last finding in Murphy and Oyer (2003), Gibbs et al. (2004) and Ederhof (2010) analyze discretionary bonus payments that were paid in addition to the bonus that was warranted according to the bonus formula in place. Gibbs et al. (2004) analyze a proprietary dataset covering department managers of auto dealerships. The authors document that discretionary bonuses are used to balance perceived flaws in quantitative performance measures and to insure managers against downside risk in their compensation. Ederhof (2010) analyzes a sample collected from companies’ forms 8-K and proxy statements that covers top-level executives who received discretionary bonuses. The discretionary bonuses paid to the executives in Ederhof’s sample are incrementally predictive of future financial performance, supporting the notion that discretionary bonuses are based on non-contractible performance measures.

Hoeppe and Moers (2010) examine two forms of discretion that may exist in determining top executive bonus payouts. They find that incentive contracts are more likely to include the option to pay a discretionary bonus if the contract is written on a single, earnings-based measure, or if the company is in an industry that experiences high levels of variability. The study also documents that the use of subjective weights on alternative performance measures is more common in companies with higher stock price volatility.

Finally, bonus pools have been investigated in several recent experimental studies, including Fisher et al. (2005), Bailey et al. (2009), and Maas et al. (2009). Supporting the theoretical findings in Baiman and Rajan (1995), Fisher et al. (2005) provide evidence that bonus pools lead to efficiency improvements when the employer has discretion over the allocation of the bonus pool, but not its size. Consistent with the empirical findings in Ittner et al. (2003), Bailey et al. (2009) document that, in allocating bonus pools, managers tend

---

4Ittner et al. (2003), Moers (2005), and Bol (2009) document that discretion can lead to biases in employee evaluation. Moers (2005) shows that subjectivity leads to performance ratings that are more lenient (“leniency bias”) and more compressed (“centrality bias”). Bol (2009) explores the two types of biases further and documents that they are driven by information gathering and confrontation costs.
to allocate the pool evenly and to overly rely on contractible performance metrics. Maas et al. (2009) document that supervisors are willing to incur personal costs in order to divide a bonus pool in an informed manner when social preferences such as fairness are taken into consideration.

The remainder of this paper is organized as follows. The next section analyzes optimal incentive contracts for an individual agent. Section 3 focuses on contracting with multiple agents and Section 4 introduces multi-period considerations. We conclude in Section 5. The material in this article draws primarily on the earlier work of Baker et al. (1994), MacLeod (2003), Rajan and Reichelstein (2006, 2009) and Ederhof (2010). Throughout this article, we provide more specific references to individual results in these papers. As a general rule, proofs of formal propositions are provided in the Appendix of this paper only when such proofs cannot be found elsewhere or we seek to emphasize a particular proof technique.

2 Incentive Contracting with a Single Agent

2.1 Subjective Performance Indicators Only

Our analysis initially considers a setting in which the principal contracts with one agent and the only signal that is informative about the action taken by the agent is a subjective performance indicator. This setting allows us to examine two of the themes identified in the Introduction: compression of the optimal incentive scheme and the incremental agency cost associated with subjectivity. The discussion in this subsection is based on MacLeod (2003).

Suppose the principal seeks to motivate the agent to take a given action, $a^h$. This action is more costly for the agent than “shirking”, i.e., to take a less productive action $a^l$. The principal only has access to a subjective metric, $y$, which has $n$ possible outcomes. The informativeness of the subjective measure is captured by:

$$q^i(a) \equiv \text{Prob}[y = y^j|a] > 0.$$ 

Throughout our analysis $q^i(a)$ is assumed to satisfy the familiar MLRP condition.\footnote{In the context of the present model, the density $q^i(a)$ satisfies the MLRP condition if $\frac{q^i(a)}{q^i(a^*)}$ is monotone decreasing in $j$.}

In order for an incentive scheme based only on subjective information to be credible, the principal is assumed to be able to commit to paying out a fixed amount, which we refer to as the bonus pool. The principal retains discretion to divert some or all of the bonus pool to a third party in case of unfavorable outcomes for the subjective indicator.
The third party could be other employees in the organization that are also included in the bonus pool. In order for the incentive scheme to be credible, however, it is essential that the principal does not derive any savings or benefit from paying these third parties. We note that in subsequent sections of this paper, which allow for both objective and subjective performance indicators and also for multiple agents, the need to divert money to a third party will frequently emerge as a “low” probability event, albeit one that is essential in order to maintain incentive compatibility.

We denote the bonus pool by $w$. For subjective outcome $y^j$, the principal “promises” compensation payment $s^j$. Let $s \equiv (s_1, ..., s_n)$. The payments $s^j$ satisfy the inequality $w \geq \max\{s^j\}$. Any difference between the bonus pool $w$ and the actual compensation payment $s^j$ is paid to the outside third party. The risk-averse agent is assumed to have additively separable preferences over wealth, $U(\cdot)$ and cost of effort, represented by $e(\cdot)$. For brevity, we denote $e(a^l) = e^l$ and $e(a^h) = e^h$. The principal’s optimization problem then becomes:

$$
P_1: \min_{\{w,s\}} w$$
subject to:

(i) $\sum_{j=1}^{n} U(s^j) \cdot q^j(a^h) - e^h \geq \bar{U}$,

(ii) $\sum_{j=1}^{n} U(s^j) \cdot q^j(a^h) - e^h \geq \sum_{j=1}^{n} U(s^j) \cdot q^j(a^l) - e^l$,

(iii) $w - s^j \geq 0$, for all $1 \leq j \leq n$.

The first constraint is the agent’s participation constraint, while the second ensures that it is incentive compatible for the agent to choose the obedient action, $a^h$. Before characterizing the solution to the agency problem identified in $P_1$, it will be instructive to consider the benchmark problem in which the signal $y^j$ is in fact contractible. The principal would then seek to solve the problem:

$$\min_{\{s^j\}} \sum_{j=1}^{n} s^j \cdot q^j(a^h),$$

subject to constraints $(i)$ and $(ii)$ in $P_1$. The MLRP condition ensures that the solution to this standard problem is monotonic in the outcome, that is, $s^{j+1} \geq s^j$ for all $j$. In general, these inequalities will be strict, that is, the optimal incentive scheme specifies $n$ distinct and strictly increasing payoffs corresponding to the different signal realizations $y^j$. 


In stark contrast, if the signal $y$ is merely a subjective performance indicator, MacLeod (2003) demonstrates the following result.\footnote{We provide a simple, direct proof of this result in the Appendix.}

**Proposition 1** The optimal one-agent incentive contract entails only two possible payoffs such that

$$s^j = \begin{cases} w, & \text{if } j > 1, \\ w - \Delta, & \text{if } j = 1. \end{cases}$$

An immediate implication of Proposition 1 is that subjectivity imposes an incremental cost on the principal because money is diverted to a third party in case of an unfavorable outcome. Furthermore, the optimality of an extremely compressed incentive scheme shows a stark departure from the optimal contract for objective performance indicators. Specifically, the results in Grossman and Hart (1983) show that with a one-dimensional contractible signal, the optimal incentive compensation paid to the agent is strictly increasing in the signal realization. In that sense, the optimal unconstrained contract is fully differentiated and does not involve any pooling of compensation payments for multiple outcomes.

One interpretation of Proposition 1 is that with subjective information alone the most efficient incentive provision is a “stick” approach, which punishes only extremely low performance. To provide some intuition for this result, consider a scenario in which the subjective indicator has three possible outcomes ($n = 3$). Suppose to the contrary, that the agent’s compensation is monotonically increasing in the outcome, with the highest level of compensation equal to the bonus pool. One possible variation of such an incentive scheme is to lower the bonus pool but pays the agent more in the intermediate state. From the principal’s perspective, such a variation obviously results in a lower payout. Given MLRP, it can be shown that the variation still implements the targeted action and satisfies the participation constraint. Thus the principal prefers to compress the compensation scheme for favorable outcomes and use the “stick” of withholding part of the bonus pool only for the least favorable outcome.

With regard to the incremental agency cost associated with subjective information, we note that this cost is small in settings where the signal $y$ is highly informative. In the extreme, this signal is fully revealing of the agent’s action so that $q^1(a^h) = 0$ and $q^1(a^l) = 1$. In this limit case of perfect subjective information, the lack of contractibility of the signal $y$ is of no consequence. To see this, we note that the principal can set $w = U^{-1}(\bar{U} + e^h)$ and $\Delta = U^{-1}(\bar{U} + e^h) - U^{-1}(\bar{U} + e^l - \epsilon)$ for some $\epsilon > 0$ (implying $\Delta > 0$). This fixed-payment scheme is feasible provided $\epsilon$ is not chosen too large. Furthermore, this payment scheme is incentive compatible for any $\epsilon > 0$ and implements first-best outcomes.
It is important at this stage to be explicit about the class of admissible incentive mechanisms. With subjective information, the parties can conceivably enter into a contract specifying payments as a function of messages that the principal and the agent send after receiving subjective information (MacLeod, 2003). However, such message-based games cannot sustain any outcome beyond those attainable by fixed-payment schemes if the subjective signal is only observed by the principal. By the Revelation Principle, equilibrium messages correspond to truthful reporting. Yet, if the principal is to report her observation truthfully, her payoff must be the same for all realizations of the subjective signal. Thus communication-based mechanisms cannot expand the set of attainable outcomes.

If both parties have access to the subjective signal $y^j$, MacLeod (2003, Proposition 4) argues that one can construct communication-based mechanisms which have equilibria that correspond to the parties receiving the same utility payoffs as if the performance indicators $y^j$ had been fully contractible. In particular, the principal can resort to a revelation mechanism that conditions the payouts to the agent on messages both parties send (to some arbiter) regarding their subjective information. Let $\{s^j_\ast\}_{j=1}^n$ denote the second-best incentive contract for the agent if hypothetically the outcome $y^j$ were verifiable for contracting purposes. To overcome the constraints imposed by subjectivity, the principal commits to a revelation mechanism according to which both parties report their observation of $y^j$. Let $m^j_P$ and $m^j_A$ denote the respective message options for the parties. If the principal reports $m^j_P$, the payoff to the agent under this revelation mechanism is $s^j_\ast$, independent of the agent’s report. The principal pays $s^j_\ast$ provided $m^j_P = m^j_A$ and $\max_j s^j_\ast + k$ for some constant $k > 0$, whenever $m^j_P \neq m^j_A$. Thus, the principal is “held accountable” for any discrepancies in the reports and the agent’s payoff is determined solely by the principal’s report. As a consequence, truthful reporting is a Nash equilibrium.\footnote{It should be observed that this mechanism is balanced on the equilibrium path, i.e., when both parties report truthfully, but is unbalanced in other cases. This lack of balance is crucial; if the principal’s payout always has to equal the agent’s compensation, truth-telling is infeasible.}

We note that truthful reporting indeed constitutes an equilibrium for the above mechanism, but so does any pair of identical reports. In fact, the weakly dominant strategy for the agent is to report that $\hat{j}$ for which $s^{\hat{j}} = \max_j s^{j_\ast}$; the principal can do no better than to match this report. Since the preferences of the parties over alternative outcomes are state-independent, there is also no possibility of eliminating the undesirable equilibria by means of additional message options without destroying the truthful equilibrium (Mookherjee and Reichelstein, 1990). The type of fixed-payment schemes examined above obviously do not suffer from such multiplicity of equilibrium problems. For the remainder of this article, we therefore disregard communication-based incentive mechanisms.
2.2 Subjective and Objective Performance Indicators

We now that for contracting purposes the principal can also rely on an objective signal in addition to the subjective performance indicator. Our results in the previous subsection can be viewed as the limit scenario of a highly uninformative objective signal. The present setting allows us to address two of the main themes mentioned in the Introduction. First, is the subjective metric always valuable for contracting purposes? Put differently, is it possible that a sufficiently informative objective performance metric may “crowd out” the subjective signal? Secondly, does the compression result in Proposition 1 extend to settings with objective and subjective performance indicators? In the presence of objective signals, it is, of course, conceivable that the degree of compression varies with the realization of the objective outcome.\(^8\)

As before, the principal seeks to induce the agent to take a given action \(a^h\). In addition to the subjective metric \(y\), the principal now also has access to an objective signal \(x\). One can think of the objective measure as financial information, including accounting numbers. For ease of exposition, we initially restrict attention to binary signals, that is, either signal realization is “high” or “low”. Naturally, a high outcome suggests greater effort on the part of the agent in the sense that:

\[
p^h \equiv \text{Prob}\{x = x^h | a = a^h\} > p^l \equiv \text{Prob}\{x = x^h | a = a^l\},
\]

and

\[
q^h \equiv \text{Prob}\{y = y^h | a = a^h\} > q^l \equiv \text{Prob}\{y = y^h | a = a^l\}.
\]

Since the principal can now also make use of an objective indicator, she can specify different bonus pools for the different objective outcomes. Based on the outcomes of the two indicators, the bonus pool corresponding to the particular objective outcome is then again divided between the agent and the third party. We denote the bonus pools corresponding to the two objective outcomes by \(w^h\) and \(w^l\), respectively. Let \(w \equiv (w^l, w^h)\). For any possible outcome \((x^j, y^k)\) the principal “promises” the compensation payment \(s^{jk}\). Consistent with the vector notation introduced above, we denote \(s = (s^{ll}, s^{lh}, s^{hl}, s^{hh})\). The compensation payments are restricted to satisfy the inequalities of a conditional fixed payment scheme:

\[
w^h \geq \max\{s^{hh}, s^{hl}\} \quad \text{and} \quad w^l \geq \max\{s^{lh}, s^{ll}\}.
\]

Finally, the agent’s expected utility from choosing action \(a^i\), exclusive of the cost of effort, will be denoted by:

\[
E[U(s)|a^i] \equiv p^i \cdot [U(s^{hh}) \cdot q^i + U(s^{hl}) \cdot (1 - q^i)] + (1 - p^i) \cdot [U(s^{lh}) \cdot q^i + U(s^{ll}) \cdot (1 - q^i)].
\]

\(^8\)This subsection draw on Sections II and III in Rajan and Reichelstein (2009).
The optimization problem for the principal then becomes:

\[ P_2: \min_{\{w,s\}} \{ w^h \cdot p^h + w^l \cdot (1 - p^h) \} \]

subject to:

(i) \( E[U(s) | a^h] - e^h \geq \bar{U} \),

(ii) \( E[U(s) | a^h] - e^h \geq E[U(s) | a^l] - e^l \),

(iii) \( w^j - s^{jk} \geq 0, \quad \text{for all } j, k. \)

In a setting with two objective performance indicators, Holmstrom (1979) has shown that a second signal is indeed incrementally valuable for contracting purposes provided the second signal is incrementally informative in a statistical sense. In particular, if hypothetically \( y \) were verifiable, it would be included in an optimal incentive scheme because the assumption of conditional independence of \( x \) and \( y \) implies that either one is incrementally informative relative to the other. In the following discussion we shall say that the subjective signal \( y \) is not valuable for contracting purposes conditional on the objective outcome \( x = x^j \), if the solution to \( P_2 \) is such that \( s^{jh} = s^{jl} = w^j \). Accordingly, we call the subjective signal valuable if it is valuable for either \( j \in \{l, h\} \).

In stating the following result, we adopt the notation: \( Q \equiv \frac{1 - q^l}{1 - q^h} \), \( P \equiv \frac{1 - p^h}{1 - p^l} \) and \( V(z) \equiv \frac{d}{d^2} U^{-1}(z) \).

**Proposition 2** The subjective metric is valuable if and only if:

\[
\frac{Q - P}{Q - 1} \cdot P < \frac{V(\bar{U} + \frac{e^h \cdot (1 - p^l) - e^l \cdot (1 - p^h)}{(p^h - p^l)})}{V(\bar{U} - \frac{e^h \cdot p^l - e^l \cdot p^h}{(p^h - p^l)})}. \tag{1}
\]

Proposition 2 says that even if a subjective performance signal is incrementally informative, its usage in the optimal contract is not assured. As one would expect, any necessary and sufficient condition for the inclusion of \( y \) must combine the agent’s preferences and the probability structures of the two signals. Intuitively, the use of the subjective signal \( y \) facilitates risk-sharing but also incurs the dead-weight loss associated with a fixed payment scheme. To obtain a better understanding for the inequality in (1), we note that \( V(\cdot) \) is increasing and both sides of the inequality are greater than one. If the subjective signal is not very informative \( (q^h \rightarrow q^l) \), the left-hand side of (1) increases without bound, ensuring that the inequality cannot hold. Conversely, if the subjective measure is highly informative \( (q^h \rightarrow 1) \), the inequality will be met because the left-hand side approaches one, while the right-hand side is unchanged.
Another implication of Proposition 2 is that a subjective signal with a given level of informativeness may be “crowded out” by a sufficiently good objective signal. To see this, note that the left-hand side of (1) is an increasing function of \((p^h - p^l)\), while the right-hand side decreases in \((p^h - p^l)\). On the other hand, if one sets \(c^l = 0\), without loss of generality, it is evident that the inequality in (1) holds if the \(e^h\) is suitably large, i.e., the subjective measure is valuable if the agency problem is sufficiently severe.

While Proposition 2 speaks to the overall value of the subjective performance indicator, the following result demonstrates that even when this signal is valuable, it comes into play only for specific realizations of the objective signal.

**Corollary 1** The solution to \(P_2\) is such that the subjective metric has no value whenever the objective outcome is favorable, that is, \(w^h = s^{hh} = s^{hl}\).

To provide a formal argument for this finding, suppose to the contrary that an optimal incentive scheme has the property that \(w^h = s^{hh} > s^{hl}\). The principal could then construct the following cheaper variation of the incentive scheme. Given the outcome \((x^h, y^l)\), the agent is paid \(s^{hl} + \Gamma\), where \(\Gamma\) is such that \(s^{hl} + \Gamma < s^{hh}\), and at the same time the payments \(s^{lh}\) and \(s^{ll}\) are reduced by \(\epsilon_1\) and \(\epsilon_2\), respectively. If the bonus pool corresponding to the low objective outcome is lowered to \(\max\{s^{lh} - \epsilon_1, s^{ll} - \epsilon_2\}\), the principal’s fixed payout is unchanged for the high objective outcome but lower for the low objective outcome. Therefore any such variation leaves the principal better off provided the agent’s incentive and participation constraints are still met. Let:

\[
U(s^{lh}) - U(s^{lh} - \epsilon_1) \equiv \Delta U_1
\]

and

\[
U(s^{ll}) - U(s^{ll} - \epsilon_2) \equiv \Delta U_2.
\]

For any given \(\Gamma\), there exist corresponding \(\epsilon_1\) and \(\epsilon_2\) such that the agent’s incentive and participation constraints are met provided:

\[
\frac{(1 - p^l)}{(1 - q^l) \cdot p^l} \cdot [\Delta U_1 \cdot q^l + \Delta U_2 \cdot (1 - q^l)] > \frac{(1 - p^h)}{(1 - q^h) \cdot p^h} \cdot [\Delta U_1 \cdot q^h + \Delta U_2 \cdot (1 - q^h)].
\]

Since \(\Delta U_1\) and \(\Delta U_2\) can assume any positive values, depending on the choices of \(\epsilon_1\) and \(\epsilon_2\), the above inequality can always be satisfied unless both:

\[
\frac{(1 - p^h)}{(1 - q^h) \cdot p^h} \cdot q^h > \frac{(1 - p^l)}{(1 - q^l) \cdot p^l} \cdot q^l
\]
and
\[
\frac{(1 - p^h)}{(1 - q^h) \cdot p^h} \cdot \frac{(1 - q^h)}{(1 - q^l) \cdot p^l} \cdot (1 - q^l) > \frac{(1 - p^l)}{(1 - q^l) \cdot p^l} \cdot (1 - q^l).
\]
Yet, the latter inequality would contradict the monotone likelihood requirement that \( p^h > p^l \). Thus, we conclude in this binary setting that the principal will always ignore the subjective metric provided the objective metric assumes the favorable outcome.\(^9\) We conclude that in the presence of an objective performance indicator the subjective performance metric is also used in a “compressed” fashion. In order to credibly use the subjective information, the principal again requires a third party as a budget-balancer.

It is natural to ask at this stage is whether the compression result in Corollary 1 generalizes beyond a setting with binary outcomes. To that end, we allow again for \( n \) possible outcomes for each of the two performance indicators. As before, we maintain the assumption that, conditional on the agent’s action, \( a \), the two signals are statistically independent. Specifically:

\[
\text{Prob}\{x = x^j, y = y^k|a\} = p^j(a) \cdot q^k(a).
\]

We maintain the vector notation \( w = (w^1, w^2, ..., w^n) \) and \( s = (s^{jk}) \) for \( 1 \leq j \leq n \) and \( 1 \leq k \leq n \). Thus:

\[
E[U(s)|a] = \sum_{j=1}^{n} \sum_{k=1}^{n} U(s^{jk}) \cdot p^j(a) \cdot q^k(a).
\]

**P3:** \( \min_{\{w,s\}} \sum_{j=1}^{n} w^j \cdot p^j(a^h) \)

subject to:

(i) \( E[U(s)|a^h] - e(a^h) \geq U \),

(ii) \( E[U(s)|a^h] - e(a^h) \geq E[U(s)|a^l] - e(a^l) \),

(iii) \( w^j - s^{jk} \geq 0 \), for all \( j, k \).

We measure the degree of compression of an incentive scheme by the number of distinct compensation levels received by the agent. If both metrics were in fact objective and verifiable, then optimal incentive contracts would “generically” entail no compression in the sense

\(^9\) An immediate question at this point is why the above construction cannot be applied similarly to the bonus pool corresponding to the low objective outcome. In particular, it is natural to ask whether it would be cheaper to lower the bonus pool, \( w^l \), but preserve incentive feasibility by increasing the agent’s compensation \( s^{ll} \). Straightforward algebra shows that provided \( p^h > p^l \) such a variation cannot satisfy both the incentive compatibility and the participation constraint.
that the range of $s^{j k}$ would be $n^2$. On the other hand, if the subjective metric is not valuable for a particular realization of the objective metric, $x^j$, there will be partial compression since the range of $s^{j k}$ is reduced by $n - 1$ values. Formally, we define $|s^{j k}|$ to be the number of distinct elements in the set $\{ s^{j k} | 1 \leq j \leq n, 1 \leq k \leq n \}$.

**Proposition 3** The subjective metric has no value unless the objective metric assumes the lowest possible outcome. The optimal incentive scheme for $P_3$ satisfies $|s^{j k}| \leq n + 1$ such that $|s^{z k}| = 1$ for $2 \leq z \leq n$ and $|s^{1 k}| \leq 2$.

When both types of metrics have $n$ possible outcomes the optimal incentive contract is such that the subjective performance indicator is never of value unless $x = x^1$. Even in this case, the subjective information may only be used to differentiate two compensation levels. This result can be interpreted as a ‘super-compression’ result: the subjective metric is never used unless the objective metric assumes the worst possible outcome and in that scenario the principal again pays out the fixed payment entirely to the agent unless the subjective metric also assumes its lowest possible outcome. The subjective performance indicator thus comes into play only if both metrics consistently point to shirking by the agent. For that extreme outcome, the principal uses the “stick” of diverting parts of the payment $w^1$ to an outside party. Overall the cardinality of $|s^{1 k}|$ is at most $(n - 1) \cdot 1 + 2 = n + 1$. Finally, we note that consistent with Proposition 2, it is straightforward to find regions where the subjective metric is never used and therefore $|s^{1 k}| = n$.

### 2.3 Correlated Performance Indicators

Our finding in Proposition 3 is reminiscent of the concept of conditional variance investigations in managerial accounting. The basic idea of a conditional variance investigation policy is that the principal seeks additional information if a primary information signal yields an outcome below a certain threshold. In agency models analyzing variance investigation policies, it is usually assumed that the principal commits in advance to incur a cost in order to generate an additional verifiable signal, $y$, conditional on the realization of an initially informative signal, $x$ (Baiman and Demski, 1980; Dye, 1986; and Fagart and Sinclair-Desgagne, 2007). We note that in these models the additional information is costly to acquire, while in the context of our model the subjective signal is costly to use due to the balancing constraint inherent in bonus pools.

A central point of interest in the variance investigation literature is whether the investigation policy is lower-tailed, upper-tailed, or two-tailed in nature. For instance, Baiman and Demski (1980) arrive at the result that the optimal investigation is one-tailed but that the
additional signal may be used at the lower or at the upper tail. Lambert (1985) and Young (1986) show that when the assumption of independent signals or the assumption of a HARA utility function is relaxed, the optimal investigation policy may in fact be two-tailed.

To relate our framework to the earlier findings in the variance investigation literature, we recall that Proposition 2 above has shown that the second (subjective) information signal will never be used unless the objective outcome is “unacceptable”. Stated differently, our results so far indicate that compression in the payoff function takes on the specific form of the same compensation payment unless the objective outcome is the worst possible. This subsection demonstrates that if we relax the assumption of stochastic independence of the subjective and objective indicators (conditional on the agent’s action), the optimal incentive scheme will be compressed unless the objective outcome is the best possible.\(^{10}\)

For ease of notation, we return to the binary outcome setting examined in connection with Proposition 2. As before, the informativeness of the contractible performance indicator is captured by the inequality:

\[
p^h \equiv \text{Prob}\left[x = x^h | a = a^h\right] > p^l \equiv \text{Prob}\left[x = x^h | a = a^l\right].
\]

The probability that the outcome of the subjective performance measure is high, given that the agent took action \(a^i\) and that the outcome of the objective performance measure is \(x^j\), will now be captured by:

\[
q^{ij} \equiv \text{Prob}\left[y = y^h | a = a^i, x = x^j\right].
\]

In this setting, the optimal incentive contract still entails compression in the sense that the subjective indicator is used at most for one of the objective outcomes. Put differently, the investigation policy in connection with the subjective indicator continues to be one-tailed. However, in contrast to the results obtained in connection with conditionally independent signals, the following result shows that the subjective metric may now come into play for high outcomes of the objective signal. In stating the following result, we adopt the following modification of the notation used in connection with Proposition 2:

\[
Q^i \equiv \frac{1 - q^{lj}}{1 - q^{hj}}.
\]

Furthermore, we denote:

\(^{10}\)The material in this subsection draws on Ederhof (2010).
\[ P^* = \frac{1 - T \cdot P \cdot \frac{p^l}{p^h}}{1 - T}, \]

where \( P = \frac{1 - p^h}{1 - p^l} \) is as defined in the previous subsection and

\[ T = \frac{V \left( \frac{\epsilon^h \cdot p^h - \epsilon^h \cdot p^l}{p^h - p^l} \right)}{V \left( \frac{\epsilon^l \cdot (1 - p^l) - \epsilon^l \cdot (1 - p^h)}{p^h - p^l} \right)}. \tag{2} \]

**Proposition 4** The subjective metric is valuable for high outcomes of the objective metric provided \( P^* \) satisfies:

\[ Q^l < P^* < Q^h \cdot P \cdot \frac{p^l}{p^h} \tag{3} \]

The informativeness of the subjective indicator is captured by \( Q^h \) for high objective outcomes and by \( Q^l \) for low objective outcomes. We can observe that, holding the quality of the objective performance measure fixed, the conditions in (3) are satisfied if the informativeness of the subjective indicator for the low objective outcome is below a certain level and the informativeness is high enough at the high objective outcome. If this is the case, the additional agency cost that the principal incurs by using the subjective metric outweighs the benefits that she could achieve for the low objective outcome. Conversely, when the objective measure assumes the high outcome, the informativeness of the subjective signal is now so high that it is attractive for the principal to incur the additional cost of using the subjective information. Ederhof (2010) goes on to analyze the setting where the objective indicator has three possible outcomes and finds that, as long as certain conditions are satisfied, the subjective indicator will be used for high and low objective outcomes, but not for the intermediate outcome of the objective signal.

The empirical analysis in Ederhof (2010) applies the present model framework to top-executive compensation. Specifically, the study analyzes discretionary bonus payments made to the top five executives in public companies. Starting in August 2004, the Security and Exchange Commission has required companies to disclose in either a Form 8-K or proxy statement payments of material cash bonuses if the bonuses are not paid in accordance with the company’s bonus formula.

The first part in Ederhof (2010) supports the notion that discretionary bonus payments are indeed affected by subjective information.\(^\text{11}\) Due to data limitations, the study pursues

\(^{11}\)See Baiman and Rajan (1995).
an indirect approach in order to address the question. Specifically, the study follows the approach of Hayes and Schaefer (2000), regressing future financial performance on current financial performance and components of current compensation. The idea is that current compensation that is incrementally informative about future financial performance is likely to be based, at least in part, on forward-looking performance measures that are only observable to the contracting parties. Such performance indicators are likely to be subjective in nature. Ederhof (2010) applies the approach to her setting by splitting the total bonus payment into the discretionary and the formula-based portion. She documents that only the discretionary, but not the formula-based part, is incrementally informative about future financial performance. This result is consistent with the notion that discretionary bonus payments are based on subjective performance indicators and that formula-based bonuses are based on objective signals such as accounting earnings.

The second part of the analysis in Ederhof (2010) analyzes whether discretionary bonus payments are indeed more likely to occur when the objective performance of the company is either low or high. This prediction, which is based on the theoretical model developed in the paper, is contrasted with hypotheses that are based on other theories. In particular, the analysis of an incentive contract involving an objective and a subjective metric in a repeated-interaction setting in Baker et al. (1994) suggests that discretionary bonuses are more likely to occur when the objective performance measure is more susceptible to manipulation. Moreover, managerial power theory (e.g., Bebchuk et al., 2002) suggests that discretionary bonuses constitute a form of rent extraction and that they are more likely in companies where management has considerable power.

Using a sample of 234 firm-years that is hand-collected from companies’ Forms 8-K and proxy statements, it is shown that discretionary bonuses are indeed more likely when the company’s objective performance is either in the bottom or top tail of the distribution. As previously discussed, the formula-based portion of the bonus payment largely reflects the company’s performance along objective measures such as accounting earnings or accounting-based return measures. In contrast, the empirical tests do not support the predictions based on the model in Baker et al. (1994) and on managerial power theory.

In summary, our analysis of single-agent contracts has shown that the constraints imposed by subjective performance indicators lead to fundamental departures from traditional findings in agency theory. Specifically, given an objective performance measure, a second signal that is incrementally informative but subjective in nature may not be used in the optimal contract. Secondly, if the subjective indicator is included at all, it will be included only for extreme realizations of the objective signal (one-tailed investigation policies). Finally, if the subjective indicator is indeed valuable, optimal incentive contracts entail compression in
the sense that the agent’s compensation is the same for most of the subjective outcomes.

3 Incentive Contracts with Multiple Agents

3.1 Subjective Performance Indicators Only

We now expand our analysis to incentive contracts for multiple agents. One would expect the agency costs associated with subjective information to be mitigated in multi-agent settings since the principal now has more flexibility in structuring a joint fixed-payment scheme for a group of agents. In contrast to the one-agent scenario considered in the previous section, the immediate plausible hypothesis now is that optimal incentive schemes based on subjective information no longer have the property that money is diverted to third parties under certain contingencies. Instead, the principal can use other agents as “budget balancers” to absorb the payment that is withheld from an agent whose subjective performance indicator suggests low effort. If optimal fixed-payment schemes have this feature, we refer to them as proper bonus pools.\textsuperscript{12}

Extending the earlier model setup, the principal seeks to induce a given effort level, $a_i^h$, from each of two agents, $1 \leq i \leq 2$. Conditional on each agent’s unobservable action choice, the principal observes a subjective signal, $y_i$, which is informative about the action taken by agent $i$. For ease of exposition, we will confine the following analysis to a two-agent setting with simultaneous action choices. The subjective signals $y_i$ available to the principal have their support on the finite sets \{$y_i^1, y_i^2, \ldots, y_i^n$\}, respectively. Manager 1’s action, $a_1 \in \{a_1^l, a_1^h\}$, induces the density $q_1^j(a_1)$ while manager 2’s action, $a_2 \in \{a_2^l, a_2^h\}$, induces the density $q_2^j(a_2)$ on the respective sets of possible signal realizations. Both densities satisfy the MLRP condition. Initially, the information signals are assumed to be stochastically independent of each other, given the underlying actions.

Both agents are risk-averse, with preferences that are additively separable over wealth, $U_i(\cdot)$, and the cost of effort, $e_i(\cdot)$. We again use the notation $w$ to refer to the fixed payment, while the vector of compensation payments will be denoted by $s = (s_1, s_2)$. For the joint subjective outcome, $(y_1^j, y_2^k)$, the agents are “promised” $s_{jk}^1$ and $s_{jk}^2$ in compensation. For given actions $(a_1, a_2)$, the expected utility of agent $i$ becomes:\textsuperscript{13}

\textsuperscript{12}The material in this subsection draws primarily on Rajan and Reichelstein (2006, Section 2).
\textsuperscript{13}We use the common notation “−$i$” to denote the agent who is not $i$. 

18
\[ E[U_i(s^{jk}_i)|a_i, a_{-i}] \equiv \sum_{j=1}^{n} \sum_{k=1}^{n} U_i(s^{jk}_i) \cdot q^j_1(a_1) \cdot q^k_2(a_2) - c_i(a_i). \]

As a benchmark, suppose the subjective signals reveal each agent’s action perfectly. For a single agent, we found that the principal can achieve first-best efficiency with perfect information by threatening to divert the bonus pool to a third party if the low action is chosen. Obviously, this mechanism can be employed for each agent separately. An interesting question is whether the presence of a second agent obviates the need for a third-party budget-balancer. A possible downside with relying only on internal agents is that the balancing requirement for subjective signals prevents the principal from punishing both agents for not taking the desired action. However, Table 1 shows that proper bonus pools can indeed implement first-best performance in dominant strategies when subjective information is perfect. Here \( \bar{s}_i \) denote agent \( i \)'s first-best compensation payment.

<table>
<thead>
<tr>
<th>Agent 1’s Action</th>
<th>Agent 2’s Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^h )</td>
<td>( a^h )</td>
</tr>
<tr>
<td>( (s_1 + \Delta, \bar{s}_2 - \Delta) )</td>
<td>( (s_1, \bar{s}_2 + \Delta) )</td>
</tr>
<tr>
<td>( s_1 - \Delta, \bar{s}_2 + \Delta )</td>
<td>( (s_1 - \Delta, \bar{s}_2) )</td>
</tr>
</tbody>
</table>

Table 1: The First-Best in Dominant Strategies with Perfect Subjective Information

The construction in Table 1 ignores possible bounds on the punishments that are potentially inflicted on agents. By raising the value of \( \Delta \), the principal makes it increasingly costly for either agent to consider shirking, regardless of the conjectured action of the other agent.\(^{14}\) Since the principal receives perfect information, there is no risk that an agent would be falsely accused of shirking, and so no risk premium is required to compensate for that possibility. A downside of this construction, though, is that the chosen \( \Delta \) might need to be very large.

Suppose now the agents’ payoffs are restricted to be non-negative. In this case, the principal can still implement the first-best outcome provided the solution concept is weakened. Specifically, obedience can be implemented as the unique outcome to survive the iterated elimination of strictly dominated strategies. The corresponding mechanism is shown in Table 2.

Here, \( w \) denotes the sum of \( \bar{s}_1 \) and \( \bar{s}_2 \). The key to observing the limited liability constraint is the asymmetric construction of the payoffs. The first agent is incentivized to be obedient

\(^{14}\)For technical details, see the proof of Proposition 3 and Corollary 1 in Rajan and Reichelstein (2006).
since he receives at least his reservation payoff from obedience and at most his minimum payoff otherwise. In response, the second agent then has an incentive to choose the right effort to avoid getting his lowest payoff.\footnote{This result extends to settings with any number of agents. One simple mechanism is to have the obedient agents split the bonus pool. Disobedient agents get 0, with the exception that agent \( n \) is treated asymmetrically and gets the entire pool if everyone shirks. It is easily verified that when strictly dominated strategies are eliminated, obedience is the unique equilibrium.}

When the subjective measures are a noisy reflection of agents’ efforts, we formulate the incentive compatibility constraint as one requiring the agents to choose the desired actions as a Nash equilibrium. The principal’s optimization problem can then be represented as follows.

\[ \mathbf{P}_4: \min_{\{w, s\}} w \]

subject to:

(i) \( E[U_i(s_{jk}^h)|a_i^h, a_{-i}^h] \geq U_i \) for \( 1 \leq i \leq 2 \),

(ii) \( E[U_i(s_{jk}^h)|a_i^h, a_{-i}^h] \geq E[U_i(s_{jk}^l)|a_i^l, a_{-i}^h] \) for \( 1 \leq i \leq 2 \),

(iii) \( w - s_{jk}^1 - s_{jk}^2 \geq 0 \) for all \( 1 \leq j, k \leq n \).

A bonus pool \((w, s)\) will be called proper if \( w \) is paid out in its entirety to the agents for all possible outcomes. Thus, the inequalities in (iii) will then have to hold as equalities, that is, i.e., \( s_{jk}^1 + s_{jk}^2 = w \) for all \( j, k \).

**Proposition 5** With subjective performance indicators, an optimal bonus pool arrangement is proper.\footnote{See the Appendix for a proof.}

Table 2: First-Best with Perfect Subjective Information and Limited Liability

<table>
<thead>
<tr>
<th>Agent 1’s Action</th>
<th>Agent 2’s Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^h )</td>
<td>( (s_1^h, s_2^h) )</td>
</tr>
<tr>
<td>( a^l )</td>
<td>( (w, 0) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( a^h )</th>
<th>( (0, w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^l )</td>
<td>( (0, w) )</td>
</tr>
</tbody>
</table>

Compensation to (Agent 1, Agent 2)
In contrast to the single-agent case, Proposition 5 establishes that with multiple agents it is always efficient for the principal to pay out the entire bonus pool to the agents collectively for all signal realizations. With multiple agents, there is no longer a need for the principal to “waste” money on a third party to make the incentive scheme credible. The cost of eliminating the third party, however, is that each agent acts as the other agent’s budget-balancer, and so is effectively compensated on the basis of a risky measure (the other agent’s signal realizations) that he does not influence. Proposition 5 says that the saving in payoffs from not diverting money to the external party always outweighs the cost of imposing additional risk on the two budget-balancing agents.\footnote{This result extends to settings with any arbitrary number of agents. Intuitively, the cost of pooling risk goes down with the number of participating agents; as a result, proper bonus pools continue to be optimal when there are more than two agents.}

It will be instructive to consider a numerical example of an optimal bonus pool when there are two symmetric agents. For concreteness, assume binary outcomes and suppose that: \( U(s) = \frac{5}{3} s^{0.6} \); \( e(a^h) = 3; e(a^l) = 0; q^h = 0.7; q^l = 0.3; \) and \( \bar{U} = 10 \). Table 4 illustrates the optimal bonus pool.

<table>
<thead>
<tr>
<th>Bonus Pool</th>
<th>( w = 65.70 )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Agent 1’s Outcome</th>
<th>Agent 2’s Outcome</th>
<th>Compensation to (Agent 1, Agent 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>( h )</td>
<td>(32.85, 32.85)</td>
</tr>
<tr>
<td>( l )</td>
<td>( l )</td>
<td>(58.90, 6.80)</td>
</tr>
<tr>
<td>( h )</td>
<td>( l )</td>
<td>(6.80, 58.90)</td>
</tr>
<tr>
<td>( l )</td>
<td>( h )</td>
<td>(32.85, 32.85)</td>
</tr>
</tbody>
</table>

Table 3: Optimal Bonus Pool with Symmetric Agents

The example illustrates the general result that, holding fixed the other agent’s outcome realization, each agent’s payoff increases in his own outcome. Of course, the fixed bonus pool implies that each agent’s payoff decreases as a function of the other agent’s performance. The example also points to a problem with the use of bonus pool mechanisms. Specifically, note that the (symmetric) agents receive the same pay when they both achieve the same outcome, regardless of whether the common outcome is high or low. This raises the question of whether the agents might prefer to aim for both achieving the low outcome by choosing the disobedient action vector, and thereby each saves on the marginal disutility from the obedient action choice. This turns out to be the case.

Table 4 summarizes the expected utility for each agent given the optimal bonus pool mechanism. Observe that if the other agent is conjectured to act obediently, each agent
is indifferent between obedience and disobedience; that is, constraint (ii) in Program P4 is binding. The agents each receive their reservation utility in this case because constraint (i) in Program P4 also binds. Note, however, that if the other agent is conjectured to choose the low effort $a_l$, then it is in each agent’s strict best interest to also choose $a_l$. Further, each agent gets a higher expected utility in this equilibrium (13 versus 10, reflecting the marginal disutility of obedience over disobedience). Therefore, while $(a_h, a_h)$ is a Nash equilibrium, the alternative action vector $(a_l, a_l)$ is also a strict Nash equilibrium and, moreover, a Pareto-dominant one.\footnote{This implementation problem is reminiscent of the multiple equilibrium identified by Demski and Sappington (1984), albeit in a different setting with deterministic outputs and correlated private information held by two agents.}

We now show that the observation made in conjunction with the preceding example is part of a general pattern.\footnote{The proof is derived in the Appendix.} In particular:

**Corollary 2** With binary outcomes and symmetric agents, the solution to $P_4$ is such that shirking $(a_i = a_l)$ always constitutes a second Nash equilibrium that Pareto-dominates the obedient equilibrium from the agents’ perspective.

How pervasive is the problem of multiple equilibria in optimal bonus pools? We are not in a position to give a comprehensive answer but note that the negative finding in the above corollary lacks robustness in several regards. First, suppose that we extend the current example to consider three symmetric agents. Shirking by all agents is then no longer an equilibrium. Intuitively, as the bonus pool increases in size to accommodate more agents, the marginal benefit to each agent of obedience when he suspects other agents of disobedience rises, thereby negating complete disobedience as a possible equilibrium. In fact, under the parameters of the present example, regardless of whether an agent conjectures that one or both of the other agents plan to choose the low action, it is in his strict best interest to choose the obedient action, $a_h$. A second feature that calls the robustness of Corollary 2 into question is the assumed symmetry of the agents. To illustrate, suppose that the second
agent is identical to the first, with two exceptions: his utility is given by \( U_2(s) = \frac{4}{3} \cdot s^{0.75} \) and his reservation utility is \( \bar{U}_2 = 8 \). In this case, disobedience (that is, shirking) is not a Nash equilibrium. Moreover, it is a weakly dominant strategy for the second agent to choose \( a_2^h \) and of course a best response for the first agent to be obedient in turn.

In circumstances where a multiple equilibrium problem does exist, the principal can tighten the incentive properties of a bonus pool mechanism by requiring a stronger notion of incentive compatibility. In particular, she could choose the bonus pool and the associated payoffs to ensure that obedience is a dominant strategy for both agents. This leads to the following variant of the optimization program in \( P_4 \).

\[
P'_4: \min_{\{w, s\}} w \\
\text{subject to:} \\
\begin{align*}
(i) & \quad E[U_i(s^{jk}_i)|a_i^h, a_{-i}^h] \geq \bar{U}_i, \quad \text{for } 1 \leq i \leq 2 \\
(ii) & \quad E[U_i(s^{jk}_i)|a_i^l, a_{-i}] \geq E[U_i(s^{jk}_i)|a_i^l, a_{-i}] \quad \text{for } 1 \leq i \leq 2 \text{ and } a_{-i} \in \{a_i^l, a_i^h\} \\
(iii) & \quad w - s^{jk}_1 - s^{jk}_2 \geq 0, \quad \text{for all } 1 \leq j, k \leq n.
\end{align*}
\]

The following result shows that optimal bonus pool arrangements need not be proper if the game-theoretic solution concept is dominant strategies.

**Proposition 5’** With dominant strategies, the optimal fixed payment scheme amounts to a proper bonus pool, with a possible exception occurring when both subjective performance indicators assume their lowest possible realization.\(^{20}\)

If incentive compatibility has to hold in a dominant strategy sense, the optimal incentive scheme may be structured such that the principal diverts money to a third party when the subjective indicators both assume the lowest possible outcome. We note that Proposition 5’ only identifies the possibility that the principal may employ an improper bonus pool. To verify this possibility, we return to the numerical example above and now require that the agents implement the desired actions in dominant strategies. Table 5 illustrates the optimal bonus pool arrangement under dominant strategy implementation for the same symmetric setting considered earlier.

Dominant strategy implementation is obviously more expensive than Nash implementation: the bonus pool increases from 65.70 to 66.52. Furthermore, the bonus pool is still proper in every contingency, with the exception of the case where both agents perform poorly. Since the goal of dominant strategy implementation is to eliminate the equilibrium

\(^{20}\)See the Appendix for a proof.
where both agents shirk, the optimal solution is to divert a portion of the payoffs when both subjective measures hit their lowest realization to a third party. In the above example, the principal diverts 16.54 to the third party in this situation.

As emphasized in the statement of Proposition 5, dominant strategy implementation does not necessitate the use of “improper” bonus pools. To illustrate, consider the following slight modification of our example: the agents are symmetric, each has the utility function $U(s) = \frac{4}{3} \cdot s^{0.75}$, and the common reservation utility is $\bar{U} = 10$. In this case, it can be shown that the optimal bonus pool is 43.18 and is proper, i.e., the entire sum is paid out in all contingencies.\(^{21}\)

### 3.2 Subjective and Objective Performance Indicators

Suppose now that the principal observes the outcomes of both objective performance indicators. It is plausible that in multi-agent settings the constraint imposed by subjectivity is sufficiently unobtrusive so that the principal will no longer ignore subjective information in certain contingencies, or seek to compress the agents’ payout schemes. Specifically, we examine whether Propositions 2 and 3 indicated above continue to hold once the bonus pool covers two or more agents.\(^{22}\)

As a starting observation, we note that the result on proper bonus pools obtained in Proposition 5 above extend to settings where the principal also observes objective signals. In particular, if we require the desired actions to be implemented in a Nash equilibrium, it can be shown that for any objective outcome, the entire bonus pool is optimally entirely paid out to the two agents. Similarly, the finding with respect to dominant strategy implementation that we obtained in Proposition 5’ carries over to the setting where the principal also has

\(^{21}\)We have identified a condition that is necessary and sufficient for dominant strategy implementation to involve third-party payouts. Unfortunately, the condition (available from the authors upon request) is a technical one with limited economic intuition behind it.

\(^{22}\)The following results draw from Section IV in Rajan and Reichelstein (2009).
The first agent’s signals by \((x_1^j, y_1^k)\) for \(1 \leq j, k \leq n\), and those of the second agent by \((x_2^u, y_2^v)\) for \(1 \leq u, v \leq n\). We maintain the assumption of stochastic independence across agents. In addition, the objective and subjective signals for each agent are assumed to be independent, conditionally on the agent’s action. Thus the joint distribution of the four performance measures is given by:

\[
Prob[x_1 = x_1^j, y_1 = y_1^k, x_2 = x_2^u, y_2 = y_2^v|a_1, a_2] = p^j_1(a_1) \cdot q^k_1(a_1) \cdot p^u_2(a_2) \cdot q^v_2(a_2).
\]

Bonus pools can now be indexed to the objective outcome pair \((x_1^j, x_2^u)\) for \(1 \leq j, u \leq n\). Accordingly, we use the notation \(w^{ju}\). Agent \(i\)’s compensation scheme takes the form \(s_i(x_1^j, y_1^k, x_2^u, y_2^v)\). With the caveats identified in the previous subsection in mind, we require the targeted effort levels, \(a_i^h\), to constitute a Nash-equilibrium. Given a pair of actions, the expected utility of Agent \(i\) will be denoted by:

\[
E[U_i(s_i(x_1^j, y_1^k, x_2^u, y_2^v))|a_i, a_{-i}] = \sum_{j,k=1}^{n} \sum_{u,v=1}^{n} U_i(s_i(x_1^j, y_1^k, x_2^u, y_2^v)) \cdot p^j_1(a_1) \cdot q^k_1(a_1) \cdot p^u_2(a_2) \cdot q^v_2(a_2) - e_i(a_i)
\]

\[
P_5: \min_{\{w, s\}} \sum_{j=1}^{n} \sum_{u,v=1}^{n} w^{ju} \cdot p^j(a_i^h) \cdot q^u(a_i^h)
\]

subject to:

(i) \(E[U_i(s_i(x_1^j, y_1^k, x_2^u, y_2^v))|a_i, a_{-i}] \geq \bar{U}_i\) for \(1 \leq i \leq 2\)

(ii) \(E[U_i(s_i(x_1^j, y_1^k, x_2^u, y_2^v))|a_i^h, a_{-i}^h] \geq E[U_i(s_i(x_1^j, y_1^k, x_2^u, y_2^v))|a_i^l, a_{-i}^l]\) for \(1 \leq i \leq 2\)

(iii) \(w^{ju} - s_1(x_1^j, y_1^k, x_2^u, y_2^v) - s_2(x_1^j, y_1^k, x_2^u, y_2^v) \geq 0\) for all \(1 \leq j, k, u, v \leq n\).

In order to examine the structure of the optimal bonus pool arrangement, we derive comparative statics for the bonus pool and the agents’ payoffs with respect to the different performance outcomes.
Proposition 6  A solution to $P_3$ is a proper bonus pool with the following properties:

(i) The bonus pools $w^{ju}$ are increasing in each of the objective metrics $x_1^j$ and $x_2^u$,

(ii) Each agent’s compensation is increasing in both his objective and subjective metric,

(iii) Each agent’s compensation is decreasing in the other agent’s subjective metric,

(iv) Neither agent’s compensation is monotonic in the other agent’s objective outcome.

Part (ii) of Proposition 6 shows that our earlier finding (Proposition 2) saying that the subjective indicator may not be valuable for contracting purposes disappears once we add a second agent who can serve as a budget balancer. Stated differently, when contracting with multiple agents, the subjective indicator is always valuable, regardless of the relative informativeness of the objective and the subjective signal.

The findings in parts (i) and (ii) show that each agent’s compensation scheme is fully differentiated with respect to the possible outcomes. Thus, the compression result obtained in the single-agent setting (Observation 1 and Proposition 3) also do not carry over to contracts for multiple agents. We note, however, that this latter finding is specific to the use of Nash-equilibrium as the solution concept. It can be shown that if the principal insists on the stronger concept of dominant strategies, there is more separation between the agents and the resulting optimal bonus pools does entail partial compression. Specifically, with $n$ possible outcomes for both the objective and the subjective metric, it turns out that the number of distinct agent payoffs is not $n^4$, but at most $n^4 - 2 \cdot n \cdot (n - 1) + 2$.\(^{23}\) If one considers the ratio of distinct compensation payments to the number of possible outcomes, then there will be relatively less compression with two agents than in the single-agent setting. This follows directly from our finding in Proposition 3 and the inequality:

$$\frac{n^4 - 2 \cdot n \cdot (n - 1) + 2}{n^4} < \frac{n + 1}{n^2}.$$

We conclude that with multiple agents the optimality of proper bonus pools varies with the principal’s standard of incentive compatibility. Even for the more demanding concept of dominant strategies, the need to divert money emerges only in the unlikely event that all objective performance indicators assume their lowest values. Similarly, the optimality of compressed incentive schemes emerges only for the dominant strategy solution concept. Yet the degree of compression is less than in a single agent scenario where the principal ignores all subjective information unless all signals point to shirking by the agent.

\(^{23}\)For further details the reader is referred to Proposition 6 in Rajan and Reichelstein (2009).
3.3 A LEN-Framework

For a more detailed examination of incentive schemes that combine subjective and objective performance indicators, we now turn to a version of the so-called LEN-model: Linear contracts, Exponential utility and Normally distributed noise terms. This framework has been used extensively in the agency literature to study the aggregation of performance metrics as a function of their informativeness, their noisiness and possible correlations among the signals.\footnote{The material in this subsection is largely taken from Section 3 in Rajan and Reichelstein(2006).}

Suppose for each of $n$ agents there is one subjective performance indicator $y_i$ and one objective indicator $x_i$. In contrast to the setup that we have followed so far, the performance measures are now assumed to be described by a joint normal distribution. In particular, the stochastic outcomes are given by:

$$\tilde{x}_i = a_i + \tilde{\epsilon}_i$$

and

$$\tilde{y}_i = m_i \cdot a_i + \tilde{\mu}_i,$$

where the random disturbances $(\tilde{\epsilon}, \tilde{\mu}) \equiv (\tilde{\epsilon}_1, ..., \tilde{\epsilon}_n, \tilde{\mu}_1, ..., \tilde{\mu}_n)$ have a joint normal distribution with mean zero and covariance matrix $\Sigma^o$. The parameter $m_i$ represents the sensitivity of the subjective measure $y_i$ to agent $i$’s unobservable action, $a_i \in [\bar{a}_i, \tilde{a}_i]$. Without loss of generality, the sensitivity of $x_i$ with regard to $a_i$ is normalized to one. By construction, the LEN framework restricts attention to incentive schemes that are linear in the entire vector of information variables. In particular, the *explicit* bonus for agent $i$ is required to be a linear combination of the verifiable information variables, i.e., $\sum_{j=1}^n u_{ij} \cdot x_j$.

To incorporate subjective information signals in this setting, the principal can set up a (linear) bonus pool arrangement specifying a total amount $w$ to be distributed based on the subjective variables $(y_1, ..., y_n)$. We note that in the LEN framework any bonus pool arrangement must be proper. Since the support of each $y_i$ is the entire real line and compensation schemes are restricted to be linear, it is impossible to guarantee a third party residual payments that are always non-negative. Thus, the effective compensation scheme for agent $i$ takes the form:

$$s_i = w_i + \sum_{j=1}^n [u_{ij} \cdot x_j + w_{ij} \cdot y_j],$$

subject to the bonus pool balancing requirements: $\sum_{i=1}^n w_i = w$ and $\sum_{i=1}^n w_{ij} = 0$ for all $1 \leq
\( j \leq n. \) We note that the linearity restriction in the compensation schemes implies that compression of the incentive scheme is an issue only to the extent that the coefficient \( w_{ii} \) could be set to zero.

With regard to preferences, the LEN framework restricts attention to exponential utility functions of the form:

\[
U_i(s_i, a_i) = -\exp\left\{-k_i \cdot (s_i - e_i(a_i))\right\}.
\]

Conditional on the action vector \( a \), the certainty equivalent of each agent’s expected utility is then given by the following mean-variance expression:

\[
CE_i(a) = E[\tilde{s}_i(a)] - e_i(a) - k_i \cdot \beta'_i \cdot \Sigma^o \cdot \beta_i,
\]

where \( k_i \equiv \frac{\hat{k}_i}{2} \) and \( \beta_i = (u_{i1}, \ldots, u_{in}, w_{i1}, \ldots, w_{in}) \).

The expression for an agent’s certainty equivalent in (4) shows immediately that each agent has a dominant strategy incentive with regard to his action choice. This feature reflects the linearity of the compensation schemes and the fact that the risk borne by agent \( i \) is independent of the actions taken by any other agent. Suppose the principal seeks to implement the fixed action vector \( a^o = (a^o_1, \ldots, a^o_n) \). Then, the participation constraint for agent \( i \) becomes:

\[
E[U_i(\tilde{s}_i(a^o), a^o_i)] \geq \bar{U}_i.
\]

Without loss of generality, \( \bar{U}_i \) can be normalized so that \( CE_i(a^o) \geq 0 \). The agents must be compensated for their cost of effort and the risk imposed on them. The risk premium for each agent depends on the choice of \( \beta_i \). To minimize total expected compensation, the principal’s optimization therefore seeks coefficients \( \beta_i \) so as to minimize the sum of the risk premia, subject to the individual incentive compatibility constraints and the balancing requirement for each subjective performance indicator.

\[
P_6: \min_{\{\beta_i\}} \sum_{i=1}^{n} k_i \cdot \beta'_i \cdot \Sigma^o \cdot \beta_i
\]

subject to:

(i) \( m_i \cdot w_{ii} + u_{ii} = e'_i(a^o_i), \quad \text{for } 1 \leq i \leq n; \)

(ii) \( \sum_{i=1}^{n} w_{ij} = 0, \quad \text{for } 1 \leq j \leq n. \)

Suppose initially that all information variables are stochastically independent with \( Var(\tilde{\epsilon}_i) = \sigma_i^2 \) and \( Var(\tilde{\mu}_i) = \eta_i^2 \). As a consequence, the agents are only connected through the bonus
pool balancing condition. In the absence of correlation among the performance measures, there is clearly no need for relative performance evaluation with regard to the verifiable signals, i.e., \( u_{ij} = 0 \) for \( i \neq j \). Moreover, if the realization of \((y_1, \ldots, y_n)\) were hypothetically verifiable, the agents’ payoffs would be completely separated in that agent \( i \)'s compensation would depend only on \( x_i \) and \( y_i \).

The first question to be addressed is whether the subjective performance indicators are valuable for contracting purposes in this setting. The following result shows that, despite the constraints imposed by subjectivity, an optimal contract never puts zero weight on any \( y_i \).

**Proposition 7** The optimal coefficients in program \( P_6 \) are such that for each agent \( i \), \( w_{ii} \neq 0 \).

The intuition underlying Proposition 7 can be seen as follows. With linear compensation schemes, the balancing requirement inherent in a bonus pool requires that any choice of bonus coefficient \( w_{ii} \) must be absorbed collectively by the other \((n-1)\) agents. The principal faces a quadratic cost minimization problem in the variables \( \{u_{ij}, w_{ij}\} \), subject to \( 2n \) linear constraints. The quadratic objective function implies that the marginal cost of the signal \( y_i \), which can be interpreted as the marginal risk premium, is zero at \( w_{ii} = 0 \). Thus, it is never optimal to put zero weight on any \( w_{ii} \). We conclude that Proposition 7 reinforces the notion that with multiple agents the constraint imposed by subjectivity is sufficiently mitigated so that the principal will not disregard an incrementally informative performance signal.

One conceptual advantage of the LEN framework is that one can unambiguously assess the magnitude of the weight placed on a particular signal. A familiar result by Banker and Datar (1989) shows that in a world where all performance signals are contractible, the relative weights placed on two variables would be given by their relative signal-to-noise ratios, i.e., the ratio of the product of their sensitivities and precisions. In our model, this ratio is given by:

\[
\frac{w_{ii}}{u_{ii}} = \left( \frac{m_i}{\eta_i^2} \right) / \left( \frac{1}{\sigma_i^2} \right) = m_i \cdot \frac{\sigma_i^2}{\eta_i^2}.
\]

One would expect the non-verifiability of the signals \( y_i \) to lead to a lower ratio. To explore the attendant tradeoffs, it will convenient to focus on a scenario of \( n \) symmetric agents. Thus, \( k_i = k \), \( a_i^o = a^o \), \( m_i = m \), \( \sigma_i^2 = \sigma^2 \), and \( \eta_i^2 = \eta^2 \) for \( 1 \leq i \leq n \). By \( C(n) \) we denote the principal’s cost in \( P_6 \). It is readily verified that \( C(n) \) is equal to:

\[
\min_{\{w_{ij}\}} k \cdot \sum_{i=1}^{n} \left[ \sigma^2 \cdot (e'(a^o) - m \cdot w_{ii})^2 + \sum_{j=1}^{n} \eta_j^2 \cdot w_{ij}^2 \right] + n \cdot e(a^o)
\]

29
subject to: \( \sum_{i=1}^{n} w_{ij} = 0, \quad \text{for} \quad 1 \leq j \leq n. \)

In stating the following results, we interpret \( \frac{C(n)}{n} \) as the per-capita agency cost that the principal incurs when choosing an optimal bonus pool. This per-capita cost always exceeds the benchmark cost of \( C^* \) that would be attainable as the cost of contracting with one agent assuming that both \( x_i \) and \( y_j \) are contractible.

**Proposition 8** With mutually independent performance measures and identical agents, the optimal incentive scheme that solves \( P_0 \) has the following properties:

(i) \( \frac{w_{ii}^*}{u_{ii}^*} = \frac{n-1}{n} \cdot \left[ \frac{m}{\sigma^2} \right] \cdot \left[ \frac{\eta^2}{\sigma^2} \right], \)

(ii) \( \frac{C(n)}{n} > \frac{C(n+1)}{n+1}, \)

(iii) \( \left| \frac{C(n)}{n} - C^* \right| \in O \left( \frac{1}{n} \right). \)

Part (i) in Proposition 8 formalizes the intuition that, because the restriction to bonus pools entails an externality in risk bearing, subjective performance indicators receive less weight in optimal contracts than objective, contractible indicators. With \( n \)-identical agents the principal seeks to spread the bonus pool risk associated with the subjective variables, \( y_i \), equally among the other \((n-1)\) agents. As the number of agents gets larger, the subjectivity of the variables \( (y_1, \ldots, y_n) \) becomes less significant and the resulting performance measures rapidly approach the ones corresponding to the traditional signal-to-noise ratio. As one would expect, the distortion in the relative weighting of the two signals is largest for the case of two agents. The weight put on the subjective indicator is then just one half of what would it have been if \( y_i \) had been verifiable.

Results (ii) and (iii) in Proposition 8 state that the per-capita cost of using a bonus pool monotonically converges to the benchmark second-best cost at the rate of \( \frac{1}{n} \). Put differently, the difference between the per capita cost of a bonus pool and the benchmark cost has order \( \frac{1}{n} \).\textsuperscript{25} Intuitively, the per-capita cost approaches the benchmark cost \( C^* \) at the rate of \( \frac{1}{n} \) because the incremental risk premium required by each agent has order \( \left( \frac{1}{n} \right)^2 \) for each of the subjective signals and therefore the incremental risk premium per capita is of the order \( \frac{1}{n} \).

\textsuperscript{25}A function \( f(n) \) is said to be of order \( \frac{1}{n} \), denoted as \( f(n) \in O \left( \frac{1}{n} \right) \), if for some constant, \( K \), and \( n \geq n_0, |f(n)| \leq \frac{K}{n} \).
So far we have assumed that the performance measures are mutually independent. To explore the impact of correlation, we focus on the case of two agents who are identical in terms of their productivity and preferences. We first note that, absent any subjective performance indicators, correlation among the objective signals always reduces the overall cost to the principal. Specifically, with perfect positive (negative) correlation among the objective measures, the principal would not incur any agency costs by setting \( u_{ii} = -u_{ij} \) (\( u_{ii} = u_{ij} \)). Similarly, any level of imperfect correlation between the two objective signals can be exploited by a relative performance evaluation scheme. As reflected in the solid curve in Figure 1, the principal’s cost is quadratic, and it does not matter whether the correlation is positive or negative.

Now consider the case where the principal only has access to correlated subjective indicators. We note that with perfect positive correlation between \( y_1 \) and \( y_2 \), the restriction to bonus pools would not matter to the principal. For the purpose of relative performance evaluation, the principal seeks to set \( w_{ii} = -w_{ij} \) so as to eliminate the entire noise and this choice conforms precisely to a bonus pool. Conversely, in order to take advantage of perfect negative correlation between the subjective indicators, the principal would have to measure each agent’s performance by a linear combination of \( y_1 \) and \( y_2 \) that places the same positive weight on both signals. Yet the restriction to bonus pools renders this impossible. This observation suggests that the principal is worse off by any negative correlation among the subjective performance indicators.

Suppose now that each (symmetric) agent has an objective and a subjective performance measure. The error terms of the objective indicators are independent from all other signals, while the error terms of the unverifiable signals are correlated. Specifically, let the correlation coefficient between \( \mu_1 \) and \( \mu_2 \) be denoted by \( \theta \). For any choice of coefficients \( (u_{ij}, w_{ij}) \), the principal’s cost then becomes \( C(2|\theta) \), given by:

\[
\min_{\{w_{ij}\}} k \cdot \sum_{i=1}^{2} \left[ \sigma^2 \cdot (e'(a^o) - m \cdot w_{ii})^2 + \sum_{j=1}^{2} \eta^2 \cdot (w_{ii}^2 + 2 \cdot w_{ii} \cdot w_{ij} \cdot \theta + w_{ij}^2) \right] + 2 \cdot e(a^o),
\]

subject to the bonus pool condition, \( w_{1i} + w_{2i} = 0 \), for \( 1 \leq i \leq n \). Let \( R_i(\theta) \) denote the ratio of \( w_{ii}^*(\theta) \) to \( u_{ii}^*(\theta) \), i.e., the relative optimal weights attached to the subjective and objective indicator for agent \( i \).
Proposition 9 With independent objective performance indicators but correlated subjective indicators:

(i) $C(2|\theta)$ is monotonically decreasing in $\theta$,

(ii) $R_i(\theta)$ is monotonically increasing in $\theta$, and $R_i(\theta) \to \infty$ as $\theta \to 1$,

(iii) $R_i(-1) = \frac{1}{4} \cdot m \cdot \frac{\sigma^2}{\eta^2}$.

As conjectured above, the agency cost associated with subjective information is mitigated by positive correlation among the signals, but is exacerbated by negative correlation. As a consequence, the principal will rely more on the objective performance measures when the subjective measures are negatively correlated. Specifically, for perfect negative correlation among the subjective performance indicators, i.e., $\theta = -1$, the optimal relative weight on the subjective signal is only one quarter of the signal-to-noise ratio of the subjective and objective indicators. Comparing this result to finding (i) in Proposition 8, we note that perfect negative correlation reduces the optimal relative weight on the subjective indicator by a factor of two, compared to the case of independent signals. With positive correlation, in contrast, the principal will increasingly deemphasize the objective performance indicators for larger values of $\theta$. Figure 1 illustrates that correlation among the observed performance indicators has a fundamentally different impact on agency costs depending on whether the correlation pertains to objective or to subjective information.

In summary, this section has demonstrated that the transition from one to multiple agents fundamentally alters our predictions regarding the value of subjective performance indicators and the structure of optimal bonus pool arrangements. Once there are two or more agents, subjective information is always valuable for contracting purposes provided it is incrementally informative. With regard to the incremental agency cost associated with subjectivity, our results obtained in the LEN framework suggest that this incremental cost diminishes at the rate of $\frac{1}{n}$ with an increasing number of agents. The incremental agency costs also tends to be low if there is positive correlation among the subjective performance indicators. Such positive correlation can be exploited by the principal in the confines of a bonus pool.
Correlation (θ)

Figure 1: Impact of correlation among signals on the principal’s cost. The solid curve depicts the owner’s compensation cost as a function of the correlation among the two managers’ objective measures of performance. The dashed curve plots the compensation cost as a function of the correlation among the two managers’ subjective measures of performance, when the objective metrics are uncorrelated.

4 Multiple Periods

4.1 Bonus Pools with Roll-Over Provisions

This final section explores the extent to which a multi-period setting mitigates the constraints imposed by the non-verifiability of certain performance indicators. To facilitate comparison with our earlier findings in a one-period setting, we first return to the setting considered in Section 2.1: the principal contracts with a single agent and the only information that is indicative of the agent’s actions is subjective in nature. The variant of the earlier model considered here is that the contract horizon is now two periods and the principal is assumed to have the ability to commit to a contract that specifies the agent’s compensation in both periods. Let \( y_i \in \{ y^h_i, y^l_i \} \), denote the subjective indicator in period \( i \in \{1, 2\} \). Consistent with our earlier notation, we also define

\[
\text{Prob}[y_i^h|a_i^h] \equiv q^h > \text{Prob}[y_i^l|a_i^l] \equiv q^l.
\]

The principal commits to a two-period compensation scheme \( s \equiv (s_1, s_2) \), where \( s_1 = \{ s_1^l, s_1^h \} \) applies in the first period and \( s_2 = \{ s_2^ll, s_2^lh, s_2^hl, s_2^hh \} \) in the second period. A bonus pool with roll-over provisions is an overall fixed-payment amount \( W \) which satisfies the constraints:
\[ s_1^j + s_2^{jk} \leq W, \] (5)

for \( j, k \in l, h \). The immediate question is whether the principal gains additional flexibility with such an arrangement. In response to an unfavorable outcome in the first period, the principal can now roll-over to the second period a share of the first-period compensation that would have been paid if the subjective performance indicator had indicated a success.

To examine the potential efficiency of multi-period commitments, we assume that the agent’s preferences are completely separable across periods. Furthermore, both the principal and the agent do not discount future payoffs and therefore seek to optimize the sum of their periodic utility payoffs. In particular, if the agent takes action \( a_1^j \) in period 1 and anticipates high effort in the second period, his overall expected utility is given by:

\[
E[\Sigma U(s)|a_1^j, a_2^h] \equiv [u(s_1^h) + u(s_2^{hh})q^h + u(s_2^{hh})(1 - q^h)]q^j
+ [u(s_1^l) + u(s_2^{lh})q^h + u(s_2^{lh})(1 - q^h)](1 - q^j) - e^j - e^h. \] (6)

Assuming that the parties can commit to a long-term contract, the principal’s optimization problem becomes:

\[
P_7: \begin{align*}
\min_{(s)} & \quad W \\
\text{subject to:} & \quad (i) \quad E[\Sigma U(s)|a_1^h, a_2^h] \geq 2 \cdot \bar{U}, \\
& \quad (ii) \quad E[\Sigma U(s)|a_1^l, a_2^h] \geq E[\Sigma U(s)|a_1^l, a_2^l] \\
& \quad (iii) \quad U(s_2^{hh})q^h + U(s_2^{hh})(1 - q^h) - e^h \geq U(s_2^{hh})q^l + U(s_2^{hh})(1 - q^l) - e^l, \\
& \quad (iv) \quad U(s_2^{lh})q^h + U(s_2^{lh})(1 - q^h) - e^h \geq U(s_2^{lh})q^l + U(s_2^{lh})(1 - q^l) - e^l, \\
& \quad (v) \quad s_1^j + s_2^{jk} \leq W \quad \text{for all } 1 \leq j, k \leq 2.
\end{align*}
\]

The inequalities in (ii) – (iv) represent incentive compatibility conditions. In the first-period the agent must be given an incentive to exert effort, assuming that he will do so in the second period. Furthermore, exerting effort in the second period must be in the agent’s interest regardless of the first-period outcome.

One feasible solution to the optimization problem in \( P_7 \) is to simply repeat the one period solution obtained in connection with \( P_1 \). By making the second period compensation payments independent of the first-period outcome, the principal creates a long-term contract.
which satisfies both the single participation constraint and the three incentive compatibility constraints in $P_1$. Thus the optimal $W^*$ in $P_7$ satisfies the inequality $W^* \leq 2 \cdot w^*$, where $w^*$ is the fixed payment that solves $P_1$. The following result demonstrates that the principal in fact cannot do better by creating a roll-over provision.

**Proposition 10** The optimal $W^*$ in $P_7$ satisfies $W^* = 2 \cdot w^*$. Thus, the optimal two-period bonus pool with a roll-over provision is equivalent to repeating the optimal one-period bonus pool arrangement.

From the principal’s perspective, the long-term contract entails an additional incentive compatibility constraint. On the other hand, there is only a single ex-ante participation constraint if the agent can commit to staying for both periods. Furthermore, a long-term contract requires the bonus pool constraint to hold only in the aggregate rather than in each period individually. The proof of Proposition 10 proceeds constructively by showing that a solution to $P_7$ can be modified so as to create two identical one-period bonus pools which satisfy the constraints in $P_1$ and leave the principal no worse off. We conjecture that the finding in Proposition 10 would extend to any finite repetition of the one-period contracting problem.

### 4.2 Infinite Horizon: Subjective Performance Indicators Only

We extend our analysis in the preceding subsection to an infinite horizon setting. At the same time, we dispense with the assumption that the principal can commit to pay out a pre-specified bonus pool. As a consequence, any incentive provisions are now *implicit*. The models examined here and in the next subsection build on the work of Bull (1987) and Baker et al. (1994).

Consider an infinitely repeated game in which the agent chooses an action $a \in [0, 1]$ at cost $e(a)$ in each period. The agent’s action $a$ stochastically determines his contribution to firm value $y \in \{y^l, y^h\}$, which is not verifiable and can therefore be interpreted as a subjective performance indicator. In particular, for a given action $a$, the probability of observing a high outcome of $y$ is $a$ while the probability of observing a low realization is $(1 - a)$. The agent’s compensation contract consists of a fixed portion $s$ and a bonus component $b$, which is paid out when the outcome of the subjective indicator is favorable. For tractability reasons, we assume from hereon that both parties are risk-neutral.

Without the ability to make contractual commitments, it would be impossible for the principal to provide any incentives in a finitely repeated game. In an infinite horizon setting, the usual backward induction argument no longer applies and the parties can sustain cooperation through an implicit contracting arrangement. To rule out uninteresting equilibria
in which the agent provides zero effort in perpetuity, we specify that the agent’s outside opportunity wage is sufficiently high, such that \( \bar{U} > y^l \).

We focus our examination of implicit incentives on so-called trigger strategies, where the parties begin by cooperating and continue to cooperate until one side defects. The (credible) threat of a trigger strategy is that there will be no cooperation thereafter. If the agent assumes that the principal will honor the implicit contract, the agent will choose his action so as to solve:

\[
\max_a \ s + a \cdot b - e(a),
\]

with first-order condition:

\[
e'(a^*) = b.
\]

Denote the agent’s optimal action, as a function of bonus payment \( b \), by \( a^*(b) \). Then the principal’s per-period profit expectation, as a function of the bonus payment is given by

\[
\pi(b) = y^l + a^*(b) \cdot (y^h - y^l) - [s + a^*(b) \cdot b],
\]

with the agent’s per-period participation constraint given by:

\[
s + a^*(b) \cdot b - e(a^*(b)) \geq \bar{U}.
\]

By adjusting the fixed payment \( s \), the principal can ensure that the participation constraint binds. Substituting this value of \( s \) into the principal’s profit, the per-period expected profit can be expressed as

\[
\pi(b) = y^l + a^*(b) \cdot (y^h - y^l) - \bar{U} - e(a^*(b)).
\] (7)

If the outcome of the agent’s subjective performance indicator is favorable, i.e., \( y = y^h \), the principal has to decide whether to honor the implicit contract by comparing the costs and benefits of doing so. If the principal reneges and does not pay the bonus, than the firm’s profit in the current period is \( y^h - s \), but zero in any future period since the parties are assumed to play trigger strategies. In contrast, if the principal honors the implicit contract, the firm’s profit in the current period reduces to \( y^h - s - b \), but the firm’s expected profits from future periods is equal to the discounted value of the expected profit from the relationship in perpetuity, i.e., \( \frac{\pi(b)}{r} \), where \( r \) is the firm’s discount rate. Therefore, the firm’s optimal strategy is to pay the bonus if and only if

\[
\frac{\pi(b)}{r} \geq b.
\]
Thus, in order for the principal to credibly incorporate the subjective performance indicator in this repeated-interaction framework, the present value of the expected future profit from honoring the contract has to exceed the current period bonus payment. This “reneging” constraint has parallels to the bonus pool constraint considered above.

Having established that, in order to credibly incorporate the subjective information, the contract is subject to the additional reneging constraint, we can solve for the optimal choice of the incentive parameter $b$.

$$\text{P}_8: \max_{\{b\}} \pi(b)$$

subject to:

$$\pi(b) - r \cdot b \geq 0.$$ 

To illustrate the solution to this program and the resulting comparative statics, we specify the cost function to be $e(a) = \gamma \cdot a^2$.

**Proposition 11** Let $Z = (y^h - y^l - 2r\gamma)$. The optimal subjective bonus $b^*$ is given by:

$$b^* = \begin{cases} 
\frac{y^h - y^l}{Z + \sqrt{4\gamma(y^l - U) + Z^2}} & \text{if } r \leq \frac{(y^l - U)}{(y^h - y^l)} + \frac{(y^h - y^l)}{4\gamma}; \\
0 & \text{if } \frac{(y^l - U)}{(H - L)} + \frac{(y^h - y^l)}{4\gamma} \leq r \leq \frac{y^h - y^l - \sqrt{U - y^l}}{2\gamma}; \\
\frac{y^h - y^l}{2\gamma} - \sqrt{\frac{U - y^l}{\gamma}} & \text{if } r > \frac{y^h - y^l}{2\gamma}. 
\end{cases}$$

Note first that if the discount rate is sufficiently low, the principal’s interest in continuing the agency relationship is so great that the reneging constraint does not bind. In this case, it is possible to implement the first-best solution where the agent’s share equals his marginal product, $(y^h - y^l)$. In Figure 2, this is represented by the constraint line, $r_L \cdot b$, which is strictly lower than the objective function, $\pi(b)$ at the unconstrained maximum, $b_{FB}$. At the other extreme, if the discount rate is sufficiently high, no values of the incentive parameter satisfy the reneging constraint, and the implicit contract cannot be implemented. The agency relationship breaks down as a consequence. To visualize this result, consider the constraint line, $r_H \cdot b$, which lies above the objective function everywhere. For discount rates that take on intermediate values, the reneging constraint binds, and imposes an additional restriction on the choice of the incentive parameter. The optimal incentive weight strictly decreases in the firm’s discount rate in this range (because higher values of $r_M$ lower the $b^*$ at which the constraint intersects the objective function for the second time). Overall, the optimal subjective bonus parameter, $b^*$, is weakly decreasing in $r$ everywhere.

A second comparative statics result is that the optimal incentive weight, $b^*$ is decreasing in the agent’s outside opportunity $U$. As the agent’s outside opportunity increases, the present
Figure 2: Impact of low ($r_L$), medium ($r_M$) and high ($r_H$) discount rates on the optimal choice of subjective bonus parameter, $b$. The dashed curve represents the principal’s profit curve, $\pi(b)$. 

$y_l - \bar{U}$

$b^*$

$b_{FB} = y^b - y^f$
value of the firm’s expected profit from an ongoing relationship decreases. Therefore, a higher outside opportunity has a similar effect as a high discount rate.

4.3 Infinite Horizon: Objective and Subjective Performance Indicators

We now enrich the above setting by allowing the principal to obtain an additional objective performance indicator, \( x \in \{0, 1\} \), that is informative about the agent’s action. Prior to choosing his action, suppose that the agent receives a private signal, \( \mu > 0 \), about the impact of his action on the objective measure. We normalize \( \mu \) so that \( E[\mu] = 1 \). Given \( \mu \) and agent action \( a \), the probability that \( x = 1 \) is \( \mu a \) and the probability that \( x = 0 \) is \( 1 - \mu a \).

The agent’s compensation contract now consists of the fixed portion \( s \), the subjective bonus component \( b \), and an objective bonus component, \( \beta \), which is paid out when the outcome of the objective indicator is favorable.

Analogous to the case with only subjective information, if the agent trusts the principal to honor the implicit contract, he will choose his action by solving \( \max_{\{a\}} \left\{ s + a \cdot b + \mu \beta a - \gamma a^2 \right\} \).

This defines the optimal action choice:

\[
a^* = \frac{b + \mu \beta}{2\gamma}.
\]

The agent’s per-period participation yields:

\[
s = \bar{U} - \mathbb{E}_\mu[a^* \cdot b + \mu \beta a^* - \gamma a^{*2}],
\]

and the principal’s expected per-period profit, after substituting for \( s \) is:

\[
V(b, \beta) = y^f - \bar{U} + \mathbb{E}_\mu[a^* (y^h - y^l) - \gamma a^{*2}] = y^f - \bar{U} + (y^h - y^l) \cdot \frac{(b + \beta)}{2\gamma} - \frac{(b^2 + \beta^2 \mathbb{E}(\mu^2) + 2b\beta)}{4\gamma}.
\] (8)

In the setting where the principal only has access to subjective information, the firm’s expected future profit was zero in case the principal reneges on the implicit contract since the agent will refuse to participate in any future implicit contracts. In the case where the principal can also use an objective indicator, we still assume that the agent will refuse to participate in any future implicit contracts should the principal not honor the subjective contract in the current period. However, we assume that the agent would be willing to enter into an objective contract with the principal. The firm’s per-period expected profit...
that could be achieved by contracting on objective information only is given by $V(\beta^*)$. Depending on the agent’s outside opportunity and the quality of the objective indicator, this expected future profit may be positive or negative. We will restrict our attention to the more interesting case where this expected profit is positive.

At the end of each period, the principal now investigates the difference in expected future profits that she could achieve by honoring the implicit contract versus not honoring the implicit contract. In particular, the principal optimally pays the subjective bonus if and only if

$$\frac{V(b, \beta) - V(\beta^*)}{r} \geq b.$$  \hfill (9)

The principal chooses the two incentive parameters $b$ and $\beta$ so as to maximize the firm’s expected current period profit in (8) subject to the reneging constraint, (9). The solution to this optimization problem is characterized below:

**Proposition 12** Let $h(\mu) = \frac{Var(\mu)}{1 + Var(\mu)}$. If the principal contracts on both subjective and objective information, the optimal incentive parameters are given by:

(i) $b^{**} = \begin{cases} (y_h - y_l), & \text{if } r < \frac{h(\mu)(y_h - y_l)}{4\gamma}; \\ 2(y_h - y_l) - \frac{4r\gamma}{h(\mu)}, & \text{for interim values of } r; \\ 0, & \text{if } r > \frac{h(\mu)(y_h - y_l)}{2\gamma}. \end{cases}$

(ii) $\beta^{**} = \frac{y_h - y_l - b^{**}}{E(\mu^2)}$

As in the case where the principal only has access to subjective information, the subjective performance measure is set to first-best levels when $r$ is small. Further, the objective measure receives no weight in this region. As the discount rate increases above a threshold, the weight on the subjective metric decreases linearly in $r$. However, once $r$ reaches a second threshold, the subjective measure is essentially ignored because the reneging constraint proves too costly. As the solution for $\beta^{**}$ indicates, changes in the weight on the objective indicator are negatively related to changes in the subjective indicator. In other words, the subjective and objective performance indicators are substitutes with respect to changes in $r$. A similar relationship is seen in the comparative statics with respect to the distortion in the objective indicator. As $Var(\mu)$ (or, equivalently, $h(\mu)$) increases, the objective indicator becomes less attractive while the subjective bonus increases, again implying that the two performance measures are substitutes.\(^{26}\) Finally, note that the weight on the performance measures is unaffected by the agent’s reservation utility, in contrast to the finding in Proposition 11. The

\(^{26}\)In contrast to the above finding, Proposition 3 in Budde (2007) claims that in his setting of a “Balanced Scorecard”, the objective performance metric will never preclude the principal from implementing the first-
reason is that in the presence of an objective measure, both the on-equilibrium and fallback positions for the owner involve meeting the agent’s participation constraint, thus nullifying the impact of changes in $\bar{U}$.

For the different settings examined in this section, the principal can incorporate subjective information in a credible fashion either by committing to a multi-period bonus pool or by means of an infinite horizon implicit contract. Recent work by Baldenius and Glover (2010) has begun to combine these two instruments. They examine an infinite horizon setting in which a principal seeks to provide incentives for two agents. The principal can commit to bonus pools and the question is to what extent incentive provisions should be based on implicit incentives, that is, reputation, as opposed to explicit incentives in the form of bonus pools. Baldenius and Glover link this tradeoff to the notion to the underlying task structure, in particular, to the notion the agents’ actions being strategic substitutes rather than complements.

5 Conclusion

In this paper, we analyze incentive contracts that include subjective performance indicators. Since subjective signals are unverifiable, they cannot be incorporated into simple incentive contracts. Our analyses focus on the principal’s committing to a bonus pool as the enforcement mechanism that enables the credible use of incrementally informative subjective performance metrics in incentive schemes. This mechanism entails an additional agency cost relative to the benchmark in which all performance measures are verifiable since the principal may divert money to third parties in certain states. The analyses in this paper are focused on the implications of this additional agency cost. Specifically, we pose the question of whether the resulting optimal incentive contracts exhibit features that constitute departures from “traditional” incentive contracts that are exclusively based on objective information.

In a one agent setting, the findings indicate that, contrary to results that concern contractible measures, an incrementally informative subjective performance measure may not be included in the optimal incentive contract. Specifically, a sufficiently informative objective signal may render the subjective metric useless. In the limit case of a completely uninformative objective performance measure, the subjective performance measure is always included in the optimal contract. Moreover, the results show that the structure of the optimal incentive contracts in the presence of subjective signals constitute a departure from best subjective bonus parameter. Budde analyzes a setting where the agent’s action is multidimensional and where the subjective indicators induce portions of the action vector that cannot be induced by the objective metrics.
the optimal contract that results for contractible measures. Specifically, subjective signals result in “compression” in the payoff function in that the compensation to the agent is the same for a range of different performance outcomes. Under the assumption of independent performance measures, the agent is only punished when the performance measures assume the worst possible outcomes. If we allow for correlation among the subjective and objective indicators, the agent may also be punished for bad subjective performance when the objective outcome is favorable.

Our findings on the structure of optimal bonus pools change substantially once we move from single- to multiple agent settings. Broadly speaking, the constraints imposed by subjectivity tend to be much less severe and the resulting incentive provisions with subjective information are more similar to contracts based entirely on contractible signals. First, the subjective performance indicator is always part of the optimal incentive contract, even in the presence of a very informative objective signal. Second, the optimal incentive contract no longer exhibits the “compression” feature that we identified for the single-agent setting. Specifically, optimal multi-agent incentive contracts are generally fully differentiated with respect to the performance outcomes. Moreover, our results show that, in most cases, it is optimal for the principal to exhaustively distribute the bonus pool among the participating agents. Stated differently, the principal no longer has the need for a third party in order to enforce the subjective contract. Despite the similarities to optimal objective incentive contracts, the principal still incurs an additional agency cost in employing a bonus pool scheme for multiple agents. Through the lens of a LEN- framework, we show that this additional agency cost is decreasing in the number of agents participating in the contract. Consequently, the subjective indicator plays a relatively larger role when more agents participate in the bonus pool.

We contrast these findings for single- or multi-agent one-period contracts with results for multi-period settings. Specifically, we show that the additional flexibility of adding a second agent to the contract does not translate to additional flexibility for the principal as a result of writing a (single-agent) contract over two periods. Interestingly, we show that under the fairly different infinite horizon setting, we also get the result that objective and subjective measures are substitutes, and an objective signal that is of sufficient quality may “crowd out” the subjective performance measure.
6 Appendix

Proof of Proposition 1: We proceed by characterizing the solution to program $P_1$. Let $\lambda$ and $\mu$ denote the multipliers for constraints (i) and (ii) respectively. Denote by $\beta^j \geq 0$ the multiplier for the constraint $w - s^j \geq 0$ (there are $n$ such inequalities embedded in constraint (iii)). The first-order conditions for the optimal solution with regard to $s^j$ are given by:

$$U'(s^j) \cdot [\lambda \cdot q^j(a^h) + \mu \cdot (q^j(a^h) - q^j(a^l))] - \beta^j = 0,$$

or

$$\lambda + \mu \cdot \left[1 - \frac{q^j(a^l)}{q^j(a^h)}\right] = \frac{\beta^j}{U'(s^j) \cdot q^j(a^h)}. \quad (10)$$

The constraint $w - s^j \geq 0$ cannot bind for all $j$, since this would violate (ii) (the incentive constraint). So, consider some $j$ for which $w - s^j > 0$. This implies that $\beta^j = 0$, which in turn implies (from (10)) that

$$\lambda + \mu \cdot \left[1 - \frac{q^j(a^l)}{q^j(a^h)}\right] = 0. \quad (11)$$

By MLRP, the left-hand side of (11) strictly increases in $j$, implying that the equality can hold at most at one $y^j$. The optimal schedule thus involves paying $s^j < w$ for one outcome and the entire amount $w$ everywhere else. That the former must occur at the lowest outcome follows again from MLRP. If (11) were to hold at some $j > 1$, that would imply that the left-hand side of (10) was negative at $j = 1$, which would contradict the fact that the right-hand side of (10) is always non-negative.

Proof of Proposition 5: To characterize the solution to program $P_4$, let $\lambda_i$ and $\mu_i$ represent the multiplier for manager $i$’s participation and incentive compatibility constraint, respectively. Noting that constraint (v) represents a total of $M_1 \cdot M_2$ constraints, we represent the multiplier for the bonus pool constraint when the realized outcome is $(y^1_1, y^2_2)$ by $\beta_{jk}$. The first-order conditions for the Lagrangian in program $P_4$ with respect to the choice variables are:

$$w : -1 + \sum_{j=1}^{M_1} \sum_{k=1}^{M_2} \beta_{jk} = 0, \quad (12)$$

$$s^j_k : U'_1(s^j_k) \cdot [\lambda_1 \cdot q^j_1(a^h_1) \cdot q^k_2(a^h_2) + \mu_1 \cdot q^k_2(a^h_2) \cdot (q^j_1(a^h_1) - q^j_1(a^l_1))] - \beta^{jk} = 0,$$

or

$$\lambda_1 + \mu_1 \cdot \left[1 - \frac{q^j_1(a^l_1)}{q^j_1(a^h_1)}\right] = \frac{\beta^{jk}}{U'_1(s^j_k) \cdot q^j_1(a^h_1) \cdot q^k_2(a^h_2)}. \quad (13)$$
\[ s_{jk}^2 : \lambda_2 + \mu_2 \cdot \left[ 1 - \frac{q_k^h(a_k^l)}{q_a^k(a_k^h)} \right] = \frac{\beta_{jk}}{U'_2(s_{jk}^2) \cdot q_1^d(a_1^h) \cdot q_2^k(a_2^h)}. \]  

(14)

To show that proper bonus pools are optimal for all subjective outcomes \( \{y_1^j, y_2^k\} \), we consider the contrapositive and obtain a contradiction. Accordingly, suppose that there is some realization \( (y_1^j, y_2^k) \) for which

\[ s_{jk}^1 + s_{jk}^2 < w_{jk}. \]

By complementary slackness, this implies that \( \beta_{jk} = 0 \) or, using (13), that

\[ \lambda_1 + \mu_1 \cdot \left[ 1 - \frac{q_1^d(a_1^l)}{q_1^d(a_1^h)} \right] = 0. \]

(15)

But (15) is independent of \( k \), so this must hold for all realizations of \( y_2^k \), implying in turn that (from (13)),

\[ \beta_{jk} = 0 \text{ for all } k. \]

Similarly, using (14), \( \beta_{jk} = 0 \) implies that

\[ \lambda_2 + \mu_2 \cdot \left[ 1 - \frac{q_2^k(a_2^l)}{q_2^k(a_2^h)} \right] = 0. \]

(16)

Again, equation (16) is independent of \( j \), and so it must hold for all realizations of \( y_1^j \), implying (from (14)) that

\[ \beta_{jk} = 0 \text{ for all } j. \]

Therefore, it follows that

\[ \beta_{jk} = 0 \text{ for all } j, k. \]

However, that would violate the first-order condition for \( w \) in equation (12), thus establishing a contradiction. Therefore, it must be the case that \( s_{jk}^1 + s_{jk}^2 = w \) for all \( (j, k) \).

---

**Proof of Corollary 2**: We proceed by first showing that disobedience constitutes a second Nash equilibrium for program \( P_4 \). In a second step, we show that this second equilibrium Pareto-dominates the obedient equilibrium. Consider the optimal solution to program \( P_4 \). Given symmetry, \( s^{hh} = s^{ll} = \frac{w}{2} \). Since the incentive constraint to implement obedience as an equilibrium (constraint (ii)) binds, it follows that

\[ e(a^h) - e(a^l) = (q^h - q^l) \cdot [(1 - q^h) \cdot U(s^{hl}) - q^h \cdot U(s^{lh}) + (2q^h - 1) \cdot U(\frac{w}{2})] \]

(17)
For disobedience to be an equilibrium, it must be that:

\[
U(s^{hl}) \cdot q^l \cdot (1 - q^l) + U(s^{lh}) \cdot (1 - q^l) \cdot q^l + U(w) \cdot [q^l + (1 - q^l)^2] - e(a^l) \geq \\
U(s^{hl}) \cdot q^h \cdot (1 - q^l) + U(s^{lh}) \cdot (1 - q^h) \cdot q^l + U(w) \cdot [q^h \cdot q^l + (1 - q^h) \cdot (1 - q^l)] - e(a^h)
\]

or, equivalently,

\[
(q^h - q^l) \cdot [-(1 - q^l) \cdot U(s^{hl}) + q^l \cdot U(s^{lh}) + (1 - 2q^l) \cdot U(w)] + e(a^h) - e(a^l) \geq 0
\]  
(18)

Combining (17) and (18), we obtain

\[
U(s^{hl}) \cdot (q^l - q^h) + U(s^{lh}) \cdot (q^l - q^h) + U(w) \cdot 2 \cdot (q^h - q^l) \geq 0, \text{ or}
\]

\[
U(w) \geq \frac{U(s^{hl}) + U(s^{lh})}{2}.
\]  
(19)

But (19) is always true since \(s^{hl} + s^{lh} = w\) and \(U(\cdot)\) is a concave function. In fact, the inequality in (19) holds strictly, so disobedience is a strict Nash equilibrium. To demonstrate that it Pareto-dominates the obedient equilibrium, we have to show (again using the fact that the incentive constraint binds) that:

\[
U(s^{hl}) \cdot q^l \cdot (1 - q^l) + U(s^{lh}) \cdot (1 - q^l) \cdot q^l + U(w) \cdot [q^l + (1 - q^l)^2] > \\
U(s^{hl}) \cdot q^l \cdot (1 - q^h) + U(s^{lh}) \cdot (1 - q^l) \cdot q^h + U(w) \cdot [q^l \cdot q^h + (1 - q^l) \cdot (1 - q^h)]
\]

or, equivalently,

\[
q^l \cdot [U(s^{hl}) - U(w)] + (1 - q^l) \cdot [U(w) - U(s^{lh})] > 0.
\]  
(20)

But (20) follows immediately from the monotonicity of the payoffs \((s^{hl} > w > s^{lh})\), thereby completing the proof.

**Proof of Proposition 5**: We follow the proof of Proposition 5. Let \(\gamma_1\) represent the multiplier for constraint \((iii)\), agent 1’s additional constraint in \(P_1'\). Then, the first-order condition for agent 1’s payoff, \(s_k^{ij}\), is:

\[
U_1'(s_k^{ij}) \cdot [\lambda_1 \cdot q_1^i(a_1^h) \cdot q_2^k(a_2^h) + \mu_1 \cdot q_2^k(a_2^h) \cdot (q_1^j(a_1^h) - q_1^i(a_1^h)) + \gamma_1 \cdot q_2^k(a_2^h) \cdot (q_1^i(a_1^h) - q_1^i(a_1^h))] - \beta^{jk} = 0,
\]

or:

\[
\lambda_1 + \left[ \mu_1 + \gamma_1 \cdot \frac{q_2^k(a_2^h)}{q_2^k(a_2^h)} \right] \left[ 1 - \frac{q_1^j(a_1^h)}{q_1^i(a_1^h)} \right] = \frac{\beta^{jk}}{U_1'(s_k^{ij}) \cdot q_1^i(a_1^h) \cdot q_2^k(a_2^h)}
\]  
(21)
The corresponding condition for agent 2’s payment, $s_{2}^{jk}$, is given by:

$$
\lambda_{2} + \left[ \mu_{2} + \gamma_{2} \cdot \frac{q_{1}^{k}(a_{1}^{h})}{q_{1}^{k}(a_{1}^{l})} \right] \cdot \left[ 1 - \frac{q_{2}^{k}(a_{2}^{l})}{q_{2}^{k}(a_{2}^{h})} \right] = \frac{\beta^{jk}}{U_{2}^{j}(s_{2}^{jk}) \cdot q_{2}^{k}(a_{2}^{h}) \cdot q_{1}^{l}(a_{1}^{l})}.
\tag{22}
$$

To show that proper bonus pools are optimal for all subjective outcomes $(y_{1}^{j}, y_{2}^{k})$ other than $(y_{1}^{1}, y_{2}^{1})$, we again consider the contrapositive and obtain a contradiction. Accordingly, suppose that there is some realization $(y_{1}^{j}, y_{2}^{k}) \neq (y_{1}^{1}, y_{2}^{1})$ for which

$$
s_{1}^{jk} + s_{2}^{jk} < w^{jk}.
$$

By complementary slackness, this implies that $\beta_{jk} = 0$. Without loss of generality, let $j \neq 1$. From (21), it follows that

$$
\lambda_{1} + \left[ \mu_{1} + \gamma_{1} \cdot \frac{q_{2}^{k}(a_{2}^{l})}{q_{2}^{k}(a_{2}^{h})} \right] \cdot \left[ 1 - \frac{q_{1}^{l}(a_{1}^{h})}{q_{1}^{l}(a_{1}^{l})} \right] = 0.
\tag{23}
$$

The left-hand side of (23) is increasing in $j$ since $\frac{q_{1}^{l}(a_{1}^{h})}{q_{1}^{l}(a_{1}^{l})}$ is decreasing in $j$ by the MLRP. But this implies that $\beta_{j-1,k} < 0$, which is impossible. Similarly, if $k \neq 1$, we can generate a contradiction using (22). Thus, we conclude that for all subjective outcomes, $(y_{1}^{j}, y_{2}^{k}) \neq (y_{1}^{1}, y_{2}^{1})$, proper bonus pools are optimal incentive mechanisms.

Proof of Proposition 10:

Step 1:

We first show that repeating the optimal single period solution twice is always feasible for the roll-over bonus pool. Therefore $W^{*} \leq 2w^{*}$.

Let $s_{1}^{h} = s_{2}^{h} = s_{1}^{l} = s_{2}^{l} = s_{1}^{ll} = s_{2}^{ll} = w^{*} - \Delta$. Since in the optimal one-period bonus pool, $w^{*} = s^{h}$, we set $W = 2w^{*} = 2s^{h}$. We then check that the so-constructed repeated bonus pool satisfies the constraints (i)-(v) in $P_{7}$. By construction, the bonus pool constraints are all met. Similarly, the two incentive compatibility constraints in the second period reduce to:

$$
u(w^{*})q^{h} + u(w^{*} - \Delta)(1 - q^{h}) - e^{h} \geq u(w^{*})q^{l} + u(w^{*} - \Delta)(1 - q^{l}) - e^{l},$$

which is exactly the incentive compatibility constraint in the single period problem. It also follows immediately that the participation constraint in $P_{7}$ is met. The last constraint that remains to be checked is (ii) in $P_{7}$:

46
\[
[u(w^*) + u(w^*)q^h + u(w^* - \Delta)(1 - q^l)]q^h + [u(w^* - \Delta) + u(w^*)q^h + u(w^* - \Delta)(1 - q^h)](1 - q^h) - 2e^h \\
\geq [u(w^*) + u(w^*)q^h + u(w^* - \Delta)(1 - q^l)]q^l + [u(w^* - \Delta) + u(w^*)q^h + u(w^* - \Delta)(1 - q^h)](1 - q^l) - e^h - e^l,
\]

\[
\Leftrightarrow u(w^*)q^h + u(w^* - \Delta)(1 - q^h) - e^h \geq u(w^*)q^l + u(w^* - \Delta)(1 - q^l) - e^l,
\]

which coincides with the incentive constraint in the single period problem and hence is satisfied.

**Step 2:**
Denote the optimal solution to \(P_7\) by \((s_1, s_2)\). The Kuhn-Tucker multipliers for this optimization problem are represented in chronological order by \(\lambda, \mu_1, \mu_2, \mu_3, \beta_1, \beta_2, \beta_3, \beta_4\), respectively. The first-order conditions for \(P_7\) are:

\[
\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1, \quad (24)
\]

\[
\lambda u'(\cdot)q^h + \mu_1 u'(\cdot)q^h - \mu_1 u'(\cdot)q^l - \beta_1 - \beta_2 = 0, \quad (25)
\]

\[
\lambda u'(\cdot)(1 - q^h) + \mu_1 u'(\cdot)(1 - q^h) - \mu_1 u'(\cdot)(1 - q^l) - \beta_3 - \beta_4 = 0, \quad (26)
\]

\[
\lambda + \mu_1 \frac{q^h - q^l}{q^h} + \mu_2 \frac{q^h - q^l}{(q^h)^2} = \frac{\beta_1}{u'(\cdot)(q^h)^2}, \quad (27)
\]

\[
\lambda + \mu_1 \frac{q^h - q^l}{q^h} + \mu_2 \frac{q^h - q^l}{(1 - q^h)q^h} = \frac{\beta_2}{u'(\cdot)(1 - q^h)q^h}, \quad (28)
\]

\[
\lambda + \mu_1 \frac{q^h - q^l}{(1 - q^h)} + \mu_2 \frac{q^h - q^l}{q^h(1 - q^h)} = \frac{\beta_3}{u'(\cdot)(1 - q^h)q^h}, \quad (29)
\]

\[
\lambda + \mu_1 \frac{q^h - q^l}{(1 - q^h)} + \mu_2 \frac{q^h - q^l}{(1 - q^h)^2} = \frac{\beta_1}{u'(\cdot)(1 - q^h)^2}. \quad (30)
\]

**Claim 1:** \(\beta_1 > 0\).

Otherwise \(\lambda = \mu_1 = \mu_2 = 0\), \(\Rightarrow \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0\), which would contradict (24).

**Claim 2:** \(\beta_2 = 0\).

Otherwise \(s_2^{sh} = s_2^{sl}\), which would contradict one of the second-period incentive constraints.

**Claim 3:** \(\lambda > 0\).

Since \(\beta_1 > 0\), (27) implies \(\lambda > 0\).

**Claim 4:** \(\mu_2 > 0\).

Otherwise \(\beta_2 \geq \lambda > 0\).
We next show that \( s^h = s_2^{hh}, s^l = s_2^{hl} \) is feasible for the single period bonus pool. Claims 3 and 4 imply that (i) and (iii) in \( P_7 \) must be binding. Therefore

\[
[u(s_1^h) + u(s_2^{hh})q^h + u(s_2^{hl})(1 - q^h)]q^i + [u(s_1^l) + u(s_2^{lh})q^h + u(s_2^{ll})(1 - q^h)](1 - q^i) - 2e^h = 2\bar{u}. \tag{31}
\]

Let \( A = [u(s_1^h) + u(s_2^{hh})q^h + u(s_2^{hl})(1 - q^h)], B = [u(s_1^l) + u(s_2^{lh})q^h + u(s_2^{ll})(1 - q^h)] \).

Thus, if constraint (i) in \( P_7 \) binds, we obtain:

\[
Ag^h + B(1 - q^h) - 2e^h = 2\bar{u}. \tag{32}
\]

On the other hand, if (iii) binds, then:

\[
u(s_2^{hh})q^h + u(s_2^{hl})q^h - e^h = u(s_2^{hh})q^l + u(s_2^{hl})(1 - q^l) - e^l.
\]

This means that if we set \( s^h = s_2^{hh}, s^l = s_2^{hl} \), the incentive compatibility constraint in the single-period problem is satisfied with equality.

Next, we note that (ii) in \( P_7 \) is equivalent to

\[
A - B \geq \frac{e^h - e^l}{q^h - q^l}. \tag{33}
\]

Thus

\[
B \leq A - u(s_1^h) + u(s_1^l).
\]

Substituting the above inequality into (32), we obtain:

\[
\frac{u(s_1^h)q^h + u(s_1^l)(1 - q^h) - e^h \geq \bar{u}}{}.
\]

Thus the participation constraint in the single period problem is satisfied. We have thus demonstrated that \( s^h = s_2^{hh} \) and \( s^l = s_2^{hl} \) is feasible for the single-period bonus pool. Steps 1 and 2 together imply that \( W^* = 2w^* \).

**Proof of Proposition 11:** For \( e(a) = \gamma a^2 \), \( a^*(b) = \frac{b}{2\gamma} \), and the principal’s profit function \( \pi(b) \) in (7) simplifies to:

\[
y^l + \frac{b}{2\gamma} \cdot (y^h - y^l) - \tilde{U} - \frac{b^2}{4\gamma}.
\]

This is concave in \( b \) and the unconstrained optimum is to set \( b^* = (y^h - y^l) \). This solution
satisfies the reneging constraint, \( \pi(b^*) - r \cdot b^* \geq 0 \) if and only if

\[
y^l - \bar{U} + \frac{(y^h - y^l)^2}{2\gamma} - \frac{(y^h - y^l)^2}{4\gamma} - r \cdot (y^h - y^l) \geq 0, \text{ or}
\]

\[
r \leq \frac{y^l - \bar{U}}{y^h - y^l} + \frac{y^h - y^l}{4\gamma}.
\]

For values of \( r \) in this region, we thus obtain the first-best solution, \( b^* = (y^h - y^l) \).

At the other extreme, note that for a given value of \( r \), the function \( \pi(b) - r \cdot b \) equals

\[
y^l - \bar{U} + \frac{b}{2\gamma} \cdot (y^h - y^l) - \frac{b^2}{4\gamma} - rb.
\]

This is a strictly concave function of \( b \), with a unique maximizer at \( \hat{b}(r) = y^h - y^l - 2\gamma r \).

Unless (34) is non-negative at \( b = \hat{b} \), there exists no value of \( b \) that will meet the reneging constraint. Substituting \( \hat{b}(r) \) into (34), we observe that the reneging constraint cannot be satisfied (and no positive bonus payment can be sustained in equilibrium) for

\[
r \geq \frac{y^h - y^l}{2\gamma} - \sqrt{\frac{\bar{U} - y^l}{\gamma}}.
\]

Finally, for interim values of \( r \), the reneging constraint binds. It follows that the optimal \( b \) is the higher of the two roots that satisfy the expression, (34) = 0. Solving this quadratic equation in \( b \), we obtain \( b^* = Z + \sqrt{4\gamma(y^l - \bar{U}) + Z^2} \), where \( Z = (y^h - y^l - 2r\gamma) \).

**Proof of Proposition 12:** We first characterize \( V(\beta^*) \), the principal’s profit using just the objective measure. Consider (8) with the value of \( b \) set to zero. The principal’s unconstrained optimization problem is

\[
max_{\beta} \quad y^l + \frac{\beta}{2\gamma} \cdot (y^h - y^l) - \mathbb{E}(\mu^2) \frac{\beta^2}{4\gamma} - \bar{U}
\]

The first-order condition yields the optimal incentive weight

\[
\beta^* = \frac{y^h - y^l}{\mathbb{E}(\mu^2)}
\]

which in turn provides the following representation of the principal’s profit:

\[
V(\beta^*) = y^l - \bar{U} + \frac{(y^h - y^l)^2}{4\gamma(\mathbb{E}(\mu^2))}.
\]

(35)
Now, the objective function when both measures are used is given by (8). From (8) and (35), the reneging constraint in (9) reduces to:

\[ V(b, \beta) - V(\beta^*) - rb \geq 0, \text{ or} \]

\[ \frac{b + \beta}{2\gamma} (y^h - y^l) - \frac{1}{4\gamma} \left( b^2 + \beta^2 \mathbb{E}(\mu^2) + 2b\beta \right) - \frac{(y^h - y^l)^2}{4\gamma \mathbb{E}(\mu^2)} - rb \geq 0. \]  

(36)

Let \( \lambda \) be the multiplier on this constraint. Differentiating (8), the first-order condition with respect to \( \beta \) is:

\[ \frac{\partial}{\partial \beta} = \left[ \frac{(y^h - y^l)^2}{2\gamma} - \frac{1}{4\gamma} \cdot (2\beta \mathbb{E}(\mu^2) + 2b) \right] (1 + \lambda) = 0, \]

\[ \Rightarrow \beta^{**} = \frac{y^h - y^l - b}{\mathbb{E}(\mu^2)}. \]

Substituting this expression back into (8), the objective function, \( V(b, \beta^{**}) \), equals:

\[ (y^l - w_a) + \frac{1}{4\gamma \mathbb{E}(\mu^2)} \left[ 2(y^h - y^l)(b\mathbb{E}(\mu^2) + y^h - y^l - b) - b^2 \mathbb{E}(\mu^2) - 2b(y^h - y^l - b) - (y^h - y^l - b)^2 \right]. \]  

(37)

Similarly, given \( \beta^{**} \), the reneging constraint \( V(b, \beta^{**}) - V(\beta^*) \geq rb \) in (36) is equivalent to

\[ \frac{2(y^h - y^l)(b\mathbb{E}(\mu^2) + y^h - y^l - b) - b^2 \mathbb{E}(\mu^2) - 2b(y^h - y^l - b) - (y^h - y^l - b)^2}{4\gamma \mathbb{E}(\mu^2)} \geq rb \]

or \( \frac{b(\mathbb{E}(\mu^2) - 1) [2(y^h - y^l) - b]}{4\gamma \mathbb{E}(\mu^2)} \geq rb. \)  

(38)

From the definition of \( h(\mu) \), and using the fact that \( \mathbb{E}(\mu) = 1 \), (38) reduces to

\[ b \cdot [2(y^h - y^l) - b] \geq \frac{4rb\gamma}{h(\mu)}. \]  

(39)

The efficient bonus \( b \) maximizes \( V(b, \beta^{**}) \) subject to the firm not reneging to \( V(\beta^*) \). This is the same as maximizing the difference \( V(b, \beta^{**}) - V(\beta^*) \) subject to constraint (39). Using a scaled version of the difference, from the lhs of (38), we obtain the objective function:

\[ \max_b b \cdot [2(y^h - y^l) - b] \]
This objective is concave in $b$ and reaches its max value at $b^{**} = (y^h - y^l)$. Therefore, if

$$(y^h - y^l) \geq \frac{4r\gamma}{h(\mu)}, \text{ or equivalently } r \leq \frac{(y^h - y^l)h(\mu)}{4\gamma},$$

(39) is automatically met, and the unconstrained optimum can be achieved.

On the other hand, if

$$\frac{4r\gamma}{h(\mu)} \geq 2(y^h - y^l), \text{ or } r \geq \frac{h(\mu)(y^h - y^l)}{2\gamma},$$

it is evident that no positive $b$ will satisfy (39) and the firm has to set $b = 0$.

For interim values of $r$, there is a unique $b$ that satisfies constraint (39):

$$[2(y^h - y^l) - b] = \frac{4r\gamma}{h(\mu)} \Rightarrow b^{**} = 2(y^h - y^l) - \frac{4r\gamma}{h(\mu)}.$$

Thus

$$b^{**} = \begin{cases} 
(y^h - y^l) & \text{if } r < \frac{h(\mu)(y^h - y^l)}{4\gamma}; \\
2(y^h - y^l) - \frac{4r\gamma}{h(\mu)} & \text{for interim values of } r; \\
0 & \text{if } r > \frac{h(\mu)(y^h - y^l)}{2\gamma}.
\end{cases}$$


References


