Disagreement and Learning: Dynamic Patterns of Trade

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Abstract

The empirical evidence on investor disagreement and trading volume is difficult to reconcile in rational expectations models. We develop a dynamic model of trading volume in which investors have different interpretations of public information. We obtain a closed-form linear equilibrium that allows us to study what restrictions on the disagreement process yield empirically observed patterns in volume and returns. We show that when investors have infrequent but major disagreements, the model generates a positive autocorrelation in volume and a positive correlation between volume and price volatility. We also derive novel empirical predictions that relate the degree and frequency of disagreement to the level and serial correlation in volume and price volatility.

1 Introduction

The empirical literature on trading volume has documented a number of regularities that cannot be easily explained by standard rational expectations (RE hereafter) models. A summary of earlier findings can be found in Karpoff [1987] and Gallant et al. [1992]. More recently, Kandel and Pearson [1995] document significant abnormal trading volume around earnings announcements, even when the announcement returns are close to zero. They also find that analyst forecasts often diverge or flip around earnings announcements, which they argue is inconsistent with models in which analysts agree on the interpretation of public information. Chae [2005] documents that abnormal volume before an earnings announcement is low, but spikes on the announcement date and decreases slowly over the next few days. While noisy RE models can generate similar patterns using very particular stochastic endowment or noise trading processes, such explanations are completely driven by these exogenous and unobservable processes and hence do not provide many insights. This has led many to view trading volume to be the key ingredient missing from our theoretical models. For example, in a recent talk Cochrane [Nov 2007] suggested that the "Next Revolution" in asset pricing will consist of models that can explain empirically observed levels and patterns of trading volume.

In this paper we take a step in this direction and develop a dynamic model of trade. We build on the difference of opinions (DO hereafter) literature and consider a setup where agents disagree about the interpretations of public information. In contrast to RE models in which investors share common priors and disagree due to asymmetric information, investors in DO models have heterogeneous priors and so may "agree to disagree" even if they have the same information.[1] Our goal is to provide a simple and intuitive characterization for the volume process. We show that since the relative trading positions reflect the extent of differences of opinions, volume simply reflects revisions to the level of disagreement. We also show that the equilibrium price corresponds to the average valuation across investors. Based on these results, we develop several implications that relate patterns in trading volume and return volatility to investor disagreement. This is especially useful in generating empirically testable predictions since there are empirical proxies for investor disagreement (e.g. analyst forecast dispersion). While some of the model's predictions are consistent with existing empirical evidence, others that relate the dynamics of trading volume and volatility to the level and frequency of disagreement are unique to this model and have not yet been tested.

[1] These include Harrison and Kreps [1978], Harris and Raviv [1993], Kandel and Pearson [1995], Scheinkman and Xiong [2003], Basak [2004] and Cao and Ou-Yang [2008].

In particular, we show that when investors have large but infrequent disagreements, volume exhibits positive autocorrelation and is clustered around these large disagreements. When the degree of disagreement is time varying, return volatility and volume are also positively correlated over time. These relationships among disagreement, volume and return volatility seem natural. Investors agree on the interpretation of information most of the time, and periods of high disagreement are often associated with high volume and high volatility.

We then extend the analysis to an infinite-horizon model in order to analytically derive sharper and cleaner empirical predictions. Again, investors disagree about the interpretation of public signals, but now we can allow for periodic jumps in the disagreement. We show that when disagreement jumps are large, return volatility and expected volume increase in the size and frequency of jumps. However, volume autocorrelation is non-monotonic in the frequency of jumps: correlation is low when jumps are very frequent or when they are very rare, but is higher in between. Finally, if investors bear aggregate risk by holding the asset, we show that expected returns are increasing in the average level of disagreement, the size of jumps, and the frequency of jumps.

An implication of the fact that trade represents change in the level of disagreements is that volume is composed of two pieces: a convergence term and an idiosyncratic term. When agents agree on the interpretation of the current signal, but disagree on the interpretation of prior public information, Bayesian updating leads their beliefs to converge, and the corresponding volume is called belief-convergence trade. On the other hand, when agents agree on the prior information, but disagree on the interpretation of the current signal, the associated volume is called idiosyncratic trade. Note that in rational expectations equilibrium, since investors have common priors and agree on the interpretation of the signals, there is no trade of either type.

The positive autocorrelation in volume is due to belief-convergence trade. A large disagreement in the current period leads to idiosyncratic trade in the current period and belief-convergence trade in future periods. Moreover, if a large disagreement is followed by periods of low disagreement, future belief-convergence trades are relatively more important than future idiosyncratic trades. Volume spikes up when disagreement is large, but investors' beliefs converge and volume falls gradually over the next few periods. As a result, volume clusters around a period of high disagreement, and exhibits positive autocorrelation. Furthermore, when investors have more extreme interpretations (and so disagree more), price reactions to public signals are likely to be larger. Hence, periods of major disagreements are periods of higher volume and also of higher absolute price changes. This leads to positive time-series correlation between volatility and volume.

Standard RE models cannot generate these patterns easily. First, RE models are unable to generate public disagreement among investors. Even in noisy RE models, if investor disagreement is made public, beliefs would converge immediately, and there would be no trade. This implies that RE models cannot reconcile the empirical evidence that analyst earnings forecasts, despite being public, exhibit significant dispersion, and moreover, that this dispersion is related to trading volume and return dynamics. Second, trading volume is difficult to generate in RE models. The 'No-Trade Theorem' and its variants (e.g. Milgrom and Stokey [1982]) rule out trade when investors share common priors, even in the presence of asymmetric information. Noisy RE equilibrium models overcome this result by introducing noise traders, or aggregate liquidity shocks.[2] However, as He and Wang [1995] show, public information leads to trade in RE models only in the presence of private information, and usually leads to a convergence of beliefs. In contrast to what is observed empirically, trade gradually increases before a public announcement, peaks at the announcement date, and then remains low thereafter. Moreover, as Kandel and Pearson [1995] argue, it is difficult to generate large amounts of informational trading without accompanying price changes in RE models.

Still, as mentioned before, one should keep in mind that the RE framework is flexible about the aggregate noise process. For example, one can generate serial correlation in volume by assuming serial correlation in the aggregate supply shocks, or generate trade without price changes by forcing aggregate supply shocks to perfectly offset aggregate information shocks. However, this is unappealing in terms of providing insight into what generates these patterns, since the noise process is assumed to be unexplained and exogenous. Unlike the specific assumptions on the endowment process needed in a RE model, however, we remain quite agnostic about the disagreement process. The volume dynamics in our model follow from the Bayesian learning process that investors use in updating their beliefs.

The rest of the paper is organized as follows. The next section surveys some of the related literature. Section 3 describes the basic framework for the finite horizon model, discusses the assumptions, and characterizes the equilibrium. Section 4 derives the expression for volume, and analyzes the autocorrelation in volume and the relationship between volume and return in the finite horizon model. Section 5 presents the results for the infinite horizon model and derives the empirical predictions of the model that relate the size and frequency of

[2]Models that generate volume using asymmetric information and aggregate noise shocks include Grossman and Stiglitz [1980], Pfleiderer [1984], Kyle [1985], Admati and Pfleiderer [1988], Brown and Jennings [1989], Grundy and McNichols [1989], Wang [1994] and He and Wang [1995]

disagreement to volume and return dynamics. Section 6 concludes. Unless noted otherwise, proofs are in the Appendix.

2 Related Literature

A number of papers have studied volume dynamics in heterogeneous information settings. For instance, He and Wang [1995] develop a dynamic model of trading volume with private and public information, which leads to interesting patterns in trading volume. Kim and Verrecchia [1991] show that in a setup with heterogeneous private information, volume is proportional to the absolute price change and to the prior dispersion in precisions. Kim and Verrecchia [1994] present a setup in which informed investors receive private signals at the same time as the public signal is announced (which they interpret as information processing), and show that this can lead to higher disagreement in announcement periods. While these models all have interesting predictions about returns and volume around public announcements, they are unable to generate a number of empirically observed patterns in volume dynamics. In particular, in these models, there is no trading volume due to a public announcement unless investors also have private information, and there is no trade without an associated change in price.

Morris [1995] presents an excellent overview of the role of the common prior assumption in economics and its limitations, and makes a strong case for models in which agents have heterogeneous priors. With a few notable exceptions (e.g. Harris and Raviv [1993], Morris [1994], Kandel and Pearson [1995]), however, the DO literature has focused primarily on pricing implications of heterogeneous priors (e.g. Harrison and Kreps [1978], Scheinkman and Xiong [2003] and Basak [2004]). Varian [1989] studies the role of difference of opinions on prices and volume in a static model, and shows that higher disagreement leads to higher volume. Harris and Raviv [1993] is one of the earlier papers to study the effect of differences of opinions on volume, but they assume investors are risk-neutral. This leads to a binary, or "all or nothing," trading pattern in which the optimistic investors hold all of the asset and the pessimistic investors hold none. Moreover, trade only occurs when agents' beliefs flip — more specifically, agents trade exactly when their beliefs about the value of the asset cross each other and they agree. This stands in contrast to our model in which trade occurs when there is change in the level of disagreement.

Like us, other papers have explored the effect of risk-aversion on trading volume in DO models (e.g. Mayshar [1983] and Kandel and Pearson [1995]). Kandel and Pearson [1995] empirically document the relationship between volume, disagreement and return volatility around public announcements. Among others, Bamber et al. [1997] extend this empirical analysis by decomposing trading volume around earnings announcements that into components that are explained by dispersion in prior beliefs, changes in dispersion and belief jumbling, even after controlling for the announcement period price change. As Kandel and Pearson [1995] suggest, this evidence is inconsistent with standard models of rational expectations. Instead, they propose a model in which investors disagree on the interpretation of public signals, and this leads to trade. However, since investors in their model are myopic, they cannot study the dynamics of returns and volume around announcements. For instance, as we show in the appendix,when investors are myopic, there is no serial correlation in volume.

In a recent paper, Cao and Ou-Yang [2008] also examine trading in a difference of opinions model. Apart technical differences in modeling, the main difference lies in the different goals of the two papers. The focus of their paper is trade across asset classes (equities and options), while ours is on patterns in volume for a single asset. Moreover, while they allow for disagreement about the precision of public signals, this forces them to assume that the disagreement across investors about the mean of the public signal is deterministic. A result of this assumption is that trading volume in their model is linear in absolute contemporaneous price changes. In contrast, the relationship between volume and prices in our model is more subtle since the first is driven by the difference in interpretations while the second is driven by the average interpretation. In particular, this implies that our model allows for trade even in the absence of price changes - an effect that has been empirically documented (e.g. Kandel and Pearson [1995]).

In a complementary paper (Banerjee et al. [2008]), we develop a DO model in which investors disagree on the fundamentals, but condition on prices to update their beliefs on the aggregate beliefs of other investors. We use this model to study the role of higher order beliefs on the predictability of returns. However, the model in that paper is not well suited to study the relationship between disagreement and volume dynamics. In contrast to the simple and intuitive characterization of volume in the current paper, the other model is unable to generate a tractable characterization of volume that can be used to study its dynamics. We view the models in these two papers as complementary approaches to understanding the effect of difference in opinions and disagreement on different aspects of financial markets.

3 Finite Horizon Model

We examine a finite horizon model (with final period T). There are two investors (or two types of investors with equal population weights) indexed by $i \in \{1, 2\}$. Agents maximize CARA utility over final period payoff, and we set the risk aversion to 1 for notational simplicity:

$$u(W_T) = -e^{-W_T} \tag{1}$$

Agents trade two assets: a risky asset, whose final payoff D is normally distributed; and a risk free asset, which pays one unit at time T with certainty. The risky asset is assumed to be in net zero supply. This assumption simplifies the analysis. A constant aggregate supply of the risky asset would decrease the price by a deterministic risk premium term, but would not affect the dynamics of volume. Also, volume dynamics in our model are not driven by aggregate noisy supply shocks, as they are in noisy rational expectations models.[3] Instead, dynamic trading patterns in our model follow from the evolution of beliefs and disagreements.

Before observing any signals, the investors have prior beliefs about the final payoff D of the risky asset given by:

$$D = F + d \text{ where } d \sim N(0, \delta) \text{ and } F \sim N(v_{i,0}, \rho_0) \tag{2}$$

F is the component of the final payoff about which investors obtain signals, while d is the residual uncertainty that is not resolved till the last date.[4] In Section 5, we develop an infinite horizon version of the model where investors receive dividends in every period (similar to D), and learn about the mean dividend (i.e. F) over time.

For simplicity, we assume that investors have homogeneous, and correct, beliefs about the residual payoff d.[5] However, for F, we assume that the difference in their prior expectations is normally distributed:

$$v_{i,0} - v_{j,0} \sim N(0, \sigma_0) \tag{3}$$

This allows us to consider both the case in which investors begin with heterogeneous priors (for non-zero σ_0), and the case where investors have common priors (when $\sigma_0 = 0$).

[3]For instance, by assuming persistence in the aggregate supply shocks, one can generate serial correlation in volume and correlation between volume and absolute price changes in a rational expectations model.

[4]He and Wang [1995] use a similar payoff structure in their rational expectations model, and show that without residual uncertainty (i.e. $\delta = 0$), there is no trade unless investors receive private information.

[5]Since investors do not receive any information about d, their beliefs about it do not change over time and hence do not affect the dynamic of volume significantly.

At each date $0 < t < T$, agents observe a public signal s_t and may disagree about its interpretation. In particular, investor i believes that s_t is given by:

$$s_t = F + \varepsilon_t \text{ where } \varepsilon_t \sim N(e_{i,t}, q_t) \tag{4}$$

where $e_{i,t}$ denotes investor i's interpretation of the public signal at date t. If agent i has a higher $e_{i,t}$ then he has a more negative view of the same signal. We assume that $e_{i,t}$ are normally distributed with zero mean and are independent of any other random variables:

$$e_{i,t} \sim N(0, \lambda_t) \tag{5}$$

As a result, there is uncertainty about what the interpretation of future signals will be. At time t, each agent observes everyone's $e_{i,t}$ and so there is no asymmetry of information.

The above specification implies that each investor believes that the other investor is wrong and so ignores his interpretation. This assumption is made primarily for tractability, but also allows us to develop the intuition for a setup with pure difference of opinions. In the real world, investors are likely to agree on certain things and disagree about others. Standard asymmetric information RE models focus on aspects of the world that investors agree about and so learn about from each other. Our goal (like other models of differences of opinion) is to highlight aspects of the world that investors still disagree about after they have learned all they can from each other.[6] Moreover, while these types of beliefs can be motivated by behavioral biases or bounded rationality, they need not be. As Morris [1995] and others have pointed out, relaxing the common prior assumption does not imply or require irrationality. The fact that sophisticated rational investors (and economists) often publicly disagree is evidence of this.

While we allow investors to disagree on the mean of the public signal, we assume that they agree on the precision of the public signal, although this is allowed to vary over time. A natural question is whether this restriction can be relaxed. We were unable to allow for both heterogeneous precisions and stochastic disagreement about the mean of the public signal in a tractable manner. One possible way of allowing heterogeneous precisions while keeping the model solvable would be to make the interpretations $e_{i,t}$ deterministic. As a result of this assumption, volume and return dynamics would be driven by these exogenous, deterministic specifications.

The variability in the distribution of ε_t across time captures the notion that all the public

[6] We would like to thank the referee for this characterization.

signals need not be from the same source. For example, the signal in a given period, s_t, might be an earnings announcement, while the signal in the next period, s_{t+1}, might be an analyst report and so on. As mentioned above, the key assumption is that agents are allowed to have different interpretations for the same signal. While an earnings announcement of 10 cents is good news for one agent, it might be bad news for another. Agents are also allowed to have different interpretations across signals — a given agent might react positively to the earnings announcement, but negatively to the analyst report. Hence, we allow for the flexibility to model signals from different sources, with different precisions, without further complicating the notation.

The generality of the model allows us to consider a wide range of disagreement patterns. Since agents disagree on the meaning of public information, it is not necessary that beliefs converge in the long run. To keep the model tractable and the intuition clear, we do not complicate the investors' learning problem by explicitly allowing them to change their interpretations after learning from the others interpretation or from past signals. However, since the model allows for time variation in investors' beliefs about the precision of the public signals (i.e. q_t), it could implicitly capture such a phenomenon. By allowing time variation in λ_t, we can model periods of uncertainty and large disagreement (high λ_t) and periods of similar interpretations and learning (low λ_t).

As a result of the different interpretations, investors hold different posterior beliefs about the distribution of F. In particular, we denote investor i's conditional beliefs at date t about F as:

$$v_{i,t} = E_{i,t}\left[F\right] \text{ and } \rho_t = var_t\left[F\right] \tag{6}$$

Since their information sets are symmetric, agents disagree regarding the mean of F but agree that the variance is given by ρ_t. Finally, we use the notation $\bar{X}_t = \frac{1}{2}\left(X_{1,t} + X_{2,t}\right)$ to denote the average across investors of a random variable X_i and the notation $\Delta X_{i,t} = X_{i,t} - \bar{X}_t$ to denote the deviation of investor i in variable X from the mean.

3.1 The Two Period Case

We show that the equilibrium of the model has a simple, recursive form. In particular, we show that prices in each period are given by the average valuation of the investors (i.e. $P_t = \bar{v}_t$), and optimal demand of each investor is driven by his valuation $v_{i,t}$. To clarify the intuition for the model before presenting the main result, we explicitly derive the equilibrium for the special case where $T = 2$. We solve model using backward induction. At date 1, the optimal demand and price are given by

$$x_{i,1} = \frac{v_{i,1} - P_1}{\rho_1 + \delta} \text{ and } P_1 = \bar{v}_1$$

The price reflects the average valuation and the optimal demand of each agent reflects the difference in his valuation relative to the average valuation. Based on this we conclude that at date 0, investor i's optimal demand solves the following problem:

$$\begin{aligned}
x_{i,0} &= \arg\max_x E\left[-\exp\left\{-x\left(P_1 - P_0\right) - x_{i,1}\left(F + d - P_1\right)\right\}\right] \\
&= \arg\max_x E\left[-\exp\left\{-x\left(P_1 - P_0\right)\right\} E_1\left[\exp\left\{-x_{i,1}\left(F + d - P_1\right)\right\}\right]\right] \\
&= \arg\max_x E\left[-\exp\left\{-x\left(P_1 - P_0\right) - \frac{1}{2\left(\rho_1 + \delta\right)}\left(v_{i,1} - P_1\right)^2\right\}\right]
\end{aligned}$$

Note that the expected utility at date 0 depends on the price gain $P_1 - P_0$ and the conditional expected utility at date 1, which . At date 0, the investor forms beliefs about the value F and next period's price using Bayes Rule. We denote these beliefs by:

$$\begin{pmatrix} P_1 \\ v_{i,1} \end{pmatrix} \sim N\left(\begin{pmatrix} E_{i,0}\left[P_1\right] \\ v_{i,0} \end{pmatrix}, \begin{pmatrix} \eta_0 & \pi\rho_0 \\ \pi\rho_0 & \pi\rho_0 \end{pmatrix}\right) \text{ where } \pi = \frac{1/q_1}{1/\rho_0 + 1/q_1}$$

As a result, the solution to the above problem can be shown to have the following form:

$$\begin{aligned}
x &= \frac{\left(\rho_1 + \delta\right) E_{i,0}\left[P_1 - P_0\right] + \left(\eta_0 - \pi\rho_0\right)\left(v_{i,0} - P_0\right)}{\left(\rho_1 + \delta\right)\eta_0 + \pi\rho_0\left(\eta_0 - \pi\rho_0\right)} \\
&\equiv \omega_{1,0} E_{i,0}\left[P_1 - P_0\right] + \omega_{2,0}\left(v_{i,0} - P_0\right)
\end{aligned}$$

The optimal demand is a weighted average of two components: a speculative component given by $E_{i,0}\left[P_1 - P_0\right]$ that depends on the investors beliefs about next period's price, and a fundamental component given by $v_{i,0} - P_0$ that depends on the investor's beliefs about the final payoff F. Since we know that the price at date 1 is the average valuation (i.e. $P_1 = \bar{v}_1$), investor i's conditional expectation of the price is given by:

$$E_{i,0}\left[P_1\right] = \left(1 - \pi\right)\bar{v}_0 + \pi v_{i,0}$$

Substituting these beliefs into the optimal demand and aggregating across all investors, the market clearing condition implies that the price at date 0 is the average valuation of the asset i.e. $P_0 = \bar{v}_0$. Moreover, this implies that the speculative component of demand $E_{i,0}\left[P_1 - P_0\right]$ is a multiple of the fundamental component (i.e. $E_{i,0}\left[P_1 - P_0\right] = \pi\left(v_{i,0} - P_0\right)$)

and so investor i's optimal demand is driven by his beliefs about the fundamental value $v_{i,0}$:

$$x_{i,0} = \phi_0 (v_{i,0} - P_0)$$

We show that this generalizes to the case of $T > 2$ in the following section.

3.2 The General Case

As in the two period case, each investor uses Bayes Rule to update their beliefs about F using their own interpretation of the public signal. In particular, investor i's beliefs about F at date $t+1$ are given by

$$F \sim N (v_{i,t+1}, \rho_{t+1}) \tag{7}$$

where

$$v^i_{t+1} = (1 - \pi_t) v_t + \pi_t (s_{t+1} - e_{i,t+1}) \text{ and } \rho_{t+1} = \rho_t (1 - \pi_t) \text{ where } \pi_t = \frac{1/q_{t+1}}{1/\rho_t + 1/q_{t+1}} \tag{8}$$

Given these beliefs, we show in the appendix that we can characterize the equilibrium as follows:

Lemma 1 *For all t, and all investors i, in equilibrium:*

1. *Prices reflect average beliefs i.e. $P_t = \bar{v}_t$ for all t.*
2. *Optimal demand of investor i is of the form: $x_{it} = \phi_t (v_{i,t} - P_t) = \phi_t \Delta v_{i,t}$*
3. *Expected utility of investor i is of the form: $EU_{i,t} \propto \exp\left\{-\frac{1}{2K_t} (v_{i,t} - P_t)^2\right\}$*

where K_t and ϕ_t are recursively defined in the appendix.

In each period, investors solve a multiple period, dynamic optimization problem. The optimal demand at date t does not only depend on the investor's beliefs about the final payoff, but also on his beliefs about the price gains at each intermediate period. Given our assumptions about the conditional independence of shocks to information and interpretations, this demand takes a very simple functional form. In fact, as in the two period example from the last section, the optimal demand at date t can be decomposed into two components:

$$x_{i,t} = \omega_{1,t} E_{i,t} [P_{t+1} - P_t] + \omega_{2,t} E_{i,t} [F - P_t] \tag{9}$$

The first piece, $E_{i,t}[P_{t+1} - P_t]$, is the speculative component of demand since it depends on beliefs about next period's price P_{t+1}. The second piece, $E_{i,t}[F - P_t]$, represents a fundamental motive for trade as it depends on beliefs about the final payoff. Since the price gain $P_{t+1} - P_t$ can be expressed as:

$$
\begin{aligned}
P_{t+1} - P_t &= (1 - \pi_t)\, \bar{v}_t + \pi_t \left(s_{t+1} - \bar{e}_{t+1} \right) - P_t && (10) \\
&= (1 - \pi_t)\, \bar{v}_t + \pi_t \left(F + \varepsilon_{t+1} - \bar{e}_{t+1} \right) - P_t && (11)
\end{aligned}
$$

and since future interpretations $e_{i,t+1}$ and signal noise ε_{t+1} are independent of current information, beliefs about $P_{t+1} - P_t$ are a linear function of $E_{i,t}[F - P_t]$. In particular, we show in the appendix that:

$$E_{i,t}[P_{t+1} - P_t] = \pi_t E_{i,t}[F - P_t] \tag{12}$$

and consequently, the optimal demand at date t is linear in $v_{i,t} - P_t$. Market clearing implies that the date t price is given by the average valuation ($\bar{v}_t$), and substituting these into the objective function gives us the quadratic form for the objective function. Finally, we show in the appendix that that if there is no residual uncertainty (i.e. $\delta = 0$), the speculative component of trade is zero, and investors trade as if they are myopic.

Corollary 2 *If there is no residual uncertainty (i.e. $\delta = 0$) or there is no disagreement (i.e. $\lambda_t = 0$ for all t), then the optimal demand is the myopic one i.e.*

$$x_{i,t} = \frac{1}{\rho_t} \left(v_{i,t} - P_t \right) \tag{13}$$

In either of these cases, the optimal demand in each period reduces to that of myopic investors. As we show in the Appendix in these cases, there is also no serial correlation in volume.

4 Volume

Our main focus in this paper is volume and its properties. We define the signed trade of investor i between dates t and $t+1$ as the change in his position in the risky asset during that period i.e. $x_{i,t+1} - x_{i,t}$. Given our characterization of the equilibrium in Lemma 1, we know that the price at date t is the average valuation of the investors (i.e. $P_t = \bar{v}_t$) and investor i's optimal demand is given by

$$x_{i,t} = \phi_t \left(v_{i,t} - P_t \right) = \phi_t \Delta v_{i,t} \tag{14}$$

In particular, investor i's holdings depends on the difference in his valuation from the other investor's valuation. This is intuitive – if investor 1 is more optimistic than investor 2 (i.e. $\Delta v_{1,t} > 0$), then investor 1 is long in the risky asset and investor 2 is short in it. As a result, investor i's signed trade depends on the difference in current valuations and the difference in future interpretations:

$$x_{i,t+1} - x_{i,t} = \phi_{t+1} \Delta v_{i,t+1} - \phi_t \Delta v_{i,t} \tag{15}$$

Since the volume in this economy is given by the absolute value of the signed trade, we have the following result.

Proposition 3 *The volume at time $t+1$ is linear in the difference of prior beliefs and the difference in interpretation of new information, and is given by the expression*

$$Vol_{t+1} \equiv \left| x_{i,t+1} - x_{i,t} \right| = \left| \underbrace{\left(\phi_{t+1} \left(1 - \pi_t \right) - \phi_t \right) \Delta v_{i,t}}_{\textit{belief convergence term}} - \underbrace{\phi_{t+1} \pi_t \Delta e_{i,t+1}}_{\textit{idiosyncratic term}} \right| \tag{16}$$

Moreover, if $\lambda_t = \lambda$ and $q_t = q$ then $\phi_{t+1} \left(1 - \pi_t \right) - \phi_t \leq 0$.

Volume is driven by two pieces: the difference in prior beliefs about the value Δv_t, and the difference of the shocks to interpretation Δe_{t+1}. The first piece is what we refer to as the belief-convergence, or learning, term, while the second piece is referred to as the idiosyncratic term. The intuition behind these two terms is as follows. Suppose there is little difference in beliefs before time the current period, t, i.e. Δv's and Δe's have been small. Further, suppose there is a large shock to the difference in interpretations (high Δe_t). Investors update their beliefs using Bayes rule and this leads to a large difference in beliefs today (high Δv_t). The volume between period $t-1$ and t is being driven primarily by the idiosyncratic term $\phi_t \pi_{t-1} \Delta e_t$. In the next period, $t+1$, suppose further that the shock to interpretations is small (small Δe_{t+1}). Agents interpret the public signal similarly, and the Bayesian updating leads to a convergence of beliefs. In some sense, there is little uncertainty about the interpretation of the public signal, and both agents learn about the final value of the asset.[7] This learning leads to a convergence in positions and the resulting volume between periods t to $t+1$ is driven by $(\phi_{t+1}(1-\pi_t)-\phi_t)\,\Delta v_t$ — hence we call it the learning, or belief-convergence, term.

More directly, consider the following. In the event in which $\Delta e_{t+1}=0$, agents interpret the new information identically. The agents learn the same thing about the final payoff, and this leads to a convergence in beliefs about the final payoff. As their beliefs get closer to each other, the agents decrease their prior holdings (in absolute value). While hard to prove analytically, one can verify numerically that $\phi_{t+1}(1-\pi_t)-\phi_t \leq 0$ under quite general conditions. This change in positions leads to volume over time. As discussed in the next section, this is also the source for the autocorrelation in volume. On the other hand, if $\Delta v_t = 0$ (e.g. there have been no disagreements in the past), then the only source for volume is Δe_{t+1}. Agents will change their positions if they interpret the new information differently and update differently about the final payoff. As expected, $-\phi_{t+1}\pi_t \leq 0$, since a higher e_{t+1} implies a more pessimistic interpretation. The larger the difference in the interpretations, the larger the consequent difference in valuations and the larger the positions taken by the agents. Finally, since the e_{t+1} are independent over time, the idiosyncratic term cannot be directly responsible for the autocorrelation in volume.

4.1 Autocorrelation in Volume

Turning attention to the autocorrelation in volume, first recall that volume is given by the expression:

$$\mathrm{Vol}_{t+1} = \left|\left(\phi_{t+1}(1-\pi_t)-\phi_t\right)\Delta v_{i,t} - \phi_{t+1}\pi_t\Delta e_{i,t+1}\right| \tag{17}$$

where $(\phi_{t+1}(1-\pi_t)-\phi_t) \leq 0$. As discussed above, the only source of autocorrelation in volume are the $(\phi_{t+1}(1-\pi_t)-\phi_t)\,\Delta v_t$ terms, since the Δe_{t+1} terms are serially independent. For high positive autocorrelation, one would need the Δv_t terms to be positively autocorrelated, and large in comparison to the Δe_{t+1} term. Intuitively, high volume today is indicative of a large difference of interpretations today. If the shocks to the difference in interpretations (Δe's) is small in the future, this implies more convergence, and consequently more belief-convergence trade, in the future.

Note that the $(\phi_{t+1}(1-\pi_t)-\phi_t)\,\Delta v_t$ terms are autocorrelated since the Δv_t terms are

[7] When agents have similar interpretations about the public signal, and updating leads to a convergence in beliefs, we say that the agents "learn" about the final value of the asset. However, it might be the case that they are equally wrong in the interpretation of the signal, and therefore learn incorrectly.

autocorrelated. If there is a large initial realization of Δe_t followed by a series of small realizations of Δe_{t+1}, then we should observe positive autocorrelation in volume. We can generate the above pattern with occasional large jumps in λ_t (high realizations of Δe_{t+1}) followed by long periods of low λ_t's (low realizations of Δe_{t+1}). When agents disagree occasionally and agree most of the time, volume exhibits positive autocorrelation.

Proposition 4 *Expected volume is given by*

$$E\left[Vol_{t+1}\right] = E\left[\left|x_{i,t+1} - x_{i,t}\right|\right] = \sqrt{\frac{2}{\pi} var\left[x_{i,t+1} - x_{i,t}\right]} \tag{18}$$

and the serial correlation in volume is given by

$$corr\left[Vol_{t+2}, Vol_{t+1}\right] = \Psi\left(cov\left(x_{i,t+2} - x_{i,t+1}, x_{i,t+1} - x_{i,t}\right)\right) \tag{19}$$

where $\Psi(\cdot)$ is a function, symmetric around zero, defined in the appendix.

While in closed form, the expression for autocorrelation is difficult to analyze. Hence, we numerically examine the effect of the parameters on expected volume and volume autocorrelation and present the results through a set of graphs. Specifically, we examine two different effects: (i) raising a parameter in the first period while keeping it fixed in subsequent periods, and (ii) raising the level of the parameter in all periods. In the case of the variance of the signal (q), the overall level has a larger impact on the autocorrelation of volume. In the case of the dispersion of beliefs (λ), however, temporary shocks play a more important role. Hence a pattern of large occasional disagreements followed by periods of learning lead to higher volume autocorrelation. We present the results in Figures 1 and 2. The base values for the model parameters in the numerical exercise are as follows: $T = 12$, $\rho_0 = 1$, $\sigma_0 = 0$, $q_t = 0.1$, and $\lambda_t = 1$. We use a log-scale for the x-axis to show the effect of a large variation in the dependent variable. When looking at our results one should keep in mind that daily autocorrelation in volume is estimated to be around 0.3-0.4 (e.g. Llorente et al. [2002]). Results regarding the level of volume are harder to compare since in our model there is zero net supply.

For the variance of the public signal, q, the two effects are in the same directions. The top panel of Figure 1 shows that increasing the overall variance marginally decreases the expected volume and the volume autocorrelation. A higher overall variance for the public signals leads the agents to put less weight on them while updating, and this leads to smaller changes in beliefs and lower expected volume. The bottom panel shows that raising the variance of the first signal to q_1 lowers volume and autocorrelation. A high value of q_1 means that the public signal in the first period is noisier. The public signal in the second period is relatively less noisy, and updating on this new information this leads to volume and price changes between the two periods.

Figure 2 shows the effects of an overall change in the degree of disagreement versus a temporary shock to the disagreement. The top panel shows that increasing the variance of the difference of opinions (increasing λ_t) leads to higher autocorrelation in volume. However, expected volume is non-monotonic in λ_t. This is because higher levels of overall disagreement (higher λ_t) also imply higher difference in future interpretations, which leads to higher uncertainty and less aggressive trading. As a result, expected volume is increasing in λ_t for low and high values of λ_t, but is decreasing for some intermediate levels of λ_t. Again changing the overall level of λ_t has a small effect on autocorrelation while raising the overall level of disagreement to some extent contributes to the expected volume.

The bottom panel of Figure 2 captures the effect of large initial disagreements followed by periods of relative agreement and learning. As we suggested earlier, a large initial disagreement about the interpretation of the signal (due to high λ_1) leads to a divergence of opinions. This leads agents to hold more extreme positions. In the following periods, agents have more similar interpretations (relatively low λ_t), and so learn from the public signals. This leads to a convergence in beliefs and consequently leads to the observed exponential decay in volume. Also, note that the effect of a high λ_1 on volume autocorrelation has the largest magnitude as compared to the effects of the other parameters.

Finally, Figure 3 shows the effect of variation in δ and σ_0 on expected volume and volume autocorrelation. While neither variable has a large effect on the level or correlation of volume, the effects are as expected. Increasing δ increases investor uncertainty, and so decreases expected volume. However, since δ only affects the risk of the final payoff, it also increases the weight of the speculative component of demand relative to the fundamental component, and this increases the autocorrelation in volume. Increasing the prior difference in valuations (σ_0) increases the expected volume and also increases the persistence in volume as it increases the variance of $\Delta v_{i,t}$.

It is difficult to interpret the time series properties of volume since it is non-stationary. However, we confirm our intuition about the decay in volume and autocorrelation graphically in Figure 4. The time series of expected volume and volume autocorrelation are plotted for different initial levels of disagreement (λ_1). The effect of a large disagreement in the initial period is quite persistent leading to high expected volume and correlation over a number of periods.

The numerical examples suggest that indeed a pattern of occasional large disagreements followed by learning is an important feature in generating positive correlation in volume. Furthermore, higher overall levels of precision of the public signal lead to higher levels and correlations in volume. In Section 5, we are able to present similar results more formally in an infinite horizon model. In particular, we show that larger disagreement shocks lead to higher volume and higher autocorrelation in volume. We also show that while the level of volume increases in the frequency of disagreement shocks, autocorrelation in volume is lower for extremely frequent and extremely rare shocks than it is for an intermediate frequency of shocks.

4.2 Volume and Volatility

Next, we look at correlation between volume and returns, or equivalently, price changes. Price changes in our model are given by

$$P_{t+1} - P_t = \bar{v}_{t+1} - \bar{v}_t = \pi_t \left(s_{t+1} - \bar{e}_{t+1} - \bar{v}_t \right) \tag{20}$$

As one can expect from a symmetric setup there is no correlation between (signed) return and volume, that is:

$$cov\left(\mathrm{Vol}_{t+1}, P_{t+1} - P_t\right) = cov\left(\left|x_{i,t+1} - x_{i,t}\right|, P_{t+1} - P_t\right) = 0 \tag{21}$$

This is because price changes are driven by the sequence of public signals $\{s_t\}$ and the aggregate interpretations $\{\bar{e}_t\}$ but volume is driven by the difference in interpretations $\{\Delta e_t\}$ which are independent of the s_t and $\bar{e}_t$. One could introduce a positive correlation between returns and volume by either introducing a component of trading driven by asymmetric information (as in standard RE models), or by introducing trading frictions (e.g. costly short selling) that introduce an asymmetry more directly. However, in order to keep the model tractable, we do not extend the model along these dimensions.

Hence we focus on the more robust feature of the data, namely the correlation between volume and absolute returns. Our analysis suggests another reason for why occasional, large disagreements followed by periods of relative agreement may be important characteristics of belief dynamics. Not only does this pattern generate higher levels of volume autocorrelation, it is also an important factor in generating the positive correlation between volume and absolute returns that is empirically documented. This is because, expected absolute returns depends on the variance in price changes:

$$\begin{aligned} E\left[\left|P_{t+1}-P_t\right|\right] &= \sqrt{\frac{2}{\pi}\operatorname{var}\left(P_{t+1}-P_t\right)} \text{ where} \\ \operatorname{var}\left(P_{t+1}-P_t\right) &= \pi_t^2\left(\operatorname{var}\left(s_{t+1}\right)+\operatorname{var}\left(\bar{v}_t\right)+\operatorname{var}\left(\bar{e}_{t+1}\right)\right) \end{aligned} \tag{22}$$

In particular, the volatility in price changes between dates t and $t+1$ depends on the variance of the average interpretation at date $t+1$ (i.e. $\operatorname{var}(\bar{e}_{t+1})$). Recall that the expected volume depends on the variance of $x_{i,t+1}-x_{i,t}$, which in turn depends on the variance of the difference in interpretations (i.e. $\operatorname{var}(\Delta e_{t+1})$). Both $\operatorname{var}(\bar{e}_{t+1})$ and $\operatorname{var}(\Delta e_{t+1})$ are given by $\lambda_{t+1}/2$, which implies that periods in which disagreement is high will have higher expected volume and higher absolute price changes. Intuitively, periods of higher disagreement not only lead to more trade, but also to higher price volatility. This implies that time-series variation in the disagreement process (i.e. in λ_{t+1}) leads to time-series correlation between price volatility and volume, despite the fact that within periods the two are uncorrelated.

5 Infinite Horizon Model and Empirical Predictions

In this section, we develop a variant of our model that is better suited to generate empirical predictions. A limitation of the finite horizon model described in Section 3 is that the equilibrium is not stationary and there is a strong time trend. This trend makes it difficult to derive some of the comparative statics results analytically. In this section, we generate predictions based on an infinite horizon model in which investors are assumed to be myopic. This behavior can be justified using a particular overlapping generations model, but is made primarily for tractability. In particular, it allows us to ignore hedging demands, and so we are able to derive the equilibrium in closed form and also provide analytic proofs for the some of the earlier comparative statics results. Moreover, as we show, this simple setup captures the essence of the fully dynamic model from the earlier section. Volume has the same form as before, and this suggests that predictions derived in this setup are also valid in the more complex model.

We show that while expected volume and return volatility are increasing in the frequency of disagreement shocks, volume autocorrelation is non-monotonically related to the frequency of these shocks. To the best of our knowledge, this is a novel prediction of our model. In Section 5.2, we also show that the frequency and size of disagreement is positively related to expected returns on the asset. This is in contrast to a number of DO models (e.g. Miller [1977]) in which higher disagreement is associated with lower returns in the presence of short sales constraints. Finally, similar to other DO models, we show that size of the disagreement shock is positively related to volume and return volatility.

The basic setup of this infinite horizon model is based on our earlier, finite horizon setup. In this model, there are two assets: a risk-free asset which pays an gross return of $R > 1$, and a risky asset which pays dividends D_{t+1}at time $t+1$. The distribution of dividends is given by:

$$D_{t+1} = F_{t+1} + d_{t+1} \text{ where } d_{t+1} \sim N(0, \delta) \tag{23}$$

and the mean dividend process F_{t+1} is unobservable and given by:

$$F_{t+1} = \alpha F_t + f_{t+1} \text{ where } f_{t+1} \sim N(0, \theta) \tag{24}$$

As before, there two investors indexed by $i \in 1, 2$. Investors maximize utility over next period's wealth:

$$x_{i,t} = \arg\max_x E_{i,t} \left[-\exp -x \left(P_{t+1} + D_{t+1} - RP_t \right) \right] \tag{25}$$

The rest of the model setup is similar to that of the finite horizon model. In addition to observing the dividend process D_t, investors also observe a public signal s_t about the mean dividend process:

$$s_t = F_t + \varepsilon_t \text{ where } \varepsilon_t \sim N(e_{i,t}, q) \text{ and } e_{i,t} \sim N(0, \lambda_t) \tag{26}$$

The degree of disagreement is given by λ_t - we assume that in general agents agree on the interpretation but periodically have disagreements:

$$\lambda_1 = \lambda_{\tau+1} = ...\lambda^* \text{ and } \lambda_s = 0 \text{ for all other } s \tag{27}$$

This allows us to derive predictions not just about the level of the disagreement shock (i.e. λ^*), but also about the frequency of disagreement shocks (i.e. $1/\tau$).[8]

To maintain notational consistency, we denote investor i's beliefs about F_{t+1} at time t by:

$$F_{t+1} \sim N(v_{i,t}, \rho_t) \tag{28}$$

[8]We can easily allow for investors to degree on average - in fact, the proofs in the appendix are for the general case of $\lambda_s = \lambda$.

The evolution of this belief process is analogous to that in the finite horizon model, and is given by:

$$v_{i,t+1} = \alpha\left[\pi_{v,t}v_{i,t} + \pi_{d,t}D_{t+1} + \pi_{s,t}\left(s_{t+1} - e_{i,t+1}\right)\right] \text{ and } \rho_{t+1} = \alpha^2\rho_t\pi_{v,t} + \theta \tag{29}$$

where $\pi_{d,t}$ and $\pi_{s,t}$ are the projection coefficients for D_{t+1} and s_{t+1}, and $\pi_{v,t} = 1 - \pi_{d,t} - \pi_{s,t}$. The stationary linear equilibrium in this model is characterized by the following lemma:

Lemma 5 *In the stationary linear equilibrium, investor i's optimal demand and price at date t are given by:*

$$x_{i,t} = \phi_t\Delta v_{i,t} \text{ and } P_t = \frac{1}{R-\alpha}\bar{v}_t \tag{30}$$

where ϕ_t is described in the appendix. .

This stationary, linear equilibrium is similar to the equilibrium of the finite horizon economy described in Lemma 1. In particular, prices are linear in the average belief about the mean dividend growth, and investor i's position depends on the disagreement between the two investors. Signed trade in this model also has a familiar form:

$$x_{i,t+1} - x_{i,t} = \underbrace{\left(\alpha\pi_v\phi_{t+1} - \phi_t\right)\Delta v_{i,t}}_{\text{belief-convergence term}} - \underbrace{\alpha\pi_s\phi_{t+1}\Delta e_{i,t+1}}_{\text{idiosyncratic term}} \tag{31}$$

As in the finite horizon model, volume depends on two components. The belief-convergence component is driven by prior difference in beliefs while the idiosyncratic term is due to different interpretations of the current period's public signal. As before, serial correlation in volume will be determined by the relative size of the learning component. From the proof of Lemma 5 we know that $\phi_t = \phi$ when there is no disagreement jump in period $t+1$, and is smaller when there is a jump. In periods when there are no disagreements (i.e. $\lambda_{t+1} = 0$), the idiosyncratic term is also zero, and volume is driven by the learning term as beliefs converge. Formally, when $\lambda_{t+1} = 0$, then we know that

$$\alpha\pi_v\phi_{t+1} - \phi_t < 0$$

and so volume is due to a convergence of beliefs. However, when there is a shock in disagreement, $\Delta e_{i,t+1}$is not zero, and both the idiosyncratic and learning components of trade drive volume.

5.1 Predictions based on Size and Frequency of Disagreement Shocks

Based on the above characterization of the equilibrium, we derive the empirical predictions of this model. The structure of the model allows us to derive empirical predictions based on the the level and frequency of disagreement shocks. These predictions are summarized in the following lemmas.

Lemma 6 *Suppose there is a jump in disagreement at date $t+1$ i.e. $\lambda_{t+1} = \lambda^*$. The return volatility at date $t+1$ is increasing in the size of the disagreement shock λ^*. Furthermore, if λ^* is large enough (as described in the appendix), then expected volume at date $t+1$ and autocovariance in volume between dates $t+1$ and $t+2$ are increasing in λ_{t+1}.*

Lemma 7 *Price volatility and expected volume increase in the frequency of disagreement shocks. i.e. the average over the τ periods of prove volatility and expected volume decreases with τ.*

When the disagreement shock λ^* is large enough, then volume autocorrelation is non-monotonic in the frequency of disagreement shocks. In particular, the average volume autocorrelation over τ periods is first increasing and then decreasing in τ.

The intuition for these results is as follows. The larger the disagreement shock λ^* at date $t+1$, the higher the uncertainty in price at date $t+1$, and as a result, the higher the price volatility. The relationship between disagreement and volume characteristics are more complicated. Intuitively, as in the finite horizon model, a large jump in disagreement leads to volume in the current period and volume in future periods. In particular, we know from (31) that a larger λ_{t+1} implies that the idiosyncratic component (i.e. $\Delta e_{i,t+1}$) of volume will be large, but also that ϕ_t will be smaller. This is because at date t, a higher λ_{t+1} implies that there is more uncertainty in the payoff at date $t+1$.

As a result, there are two effects of a large jump in disagreement (i.e. λ_{t+1}). The direct effect of a higher λ_{t+1} is a larger idiosyncratic component of signed trade at date $t+1$, which then leads to higher convergence trade at date $t+2$. This translates to larger volume at date $t+1$ and higher autocorrelation in volume between dates $t+1$ and $t+2$. The indirect effect of a higher λ_{t+1} is due to investors taking less aggressive positions at date t (i.e. lower ϕ_t) because of higher future uncertainty. When λ_{t+1} is not too large (e.g. when $\alpha\pi_v\phi - \phi_t > 0$), the effect of a lower ϕ_t is to reduce the convergence component of volume. But when λ_{t+1}

is large enough (e.g. if $\alpha\pi_v\phi - \phi_t \leq 0$), increasing λ_{t+1} decreases ϕ_t, which increases the convergence component of volume. As a result, when λ_{t+1} is large enough, expected volume and autocorrelation in volume both increase with λ_{t+1}. This relationship between the jump in disagreement, λ^*, and the level and autocorrelation in volume confirms our intuition from the plots in the bottom panel of Figure 2.

Moreover, note that for a given level for the disagreement shock λ^*, increasing the frequency of shocks (i.e. decreasing τ) increases the average price volatility and expected volume over these τ periods. When the disagreement shock is large (i.e. $\alpha\pi_v\phi_{t+1} - \phi_t \leq 0$), extremely frequent disagreement shocks actually lead to a decrease in volume autocorrelation since the idiosyncratic component of volume dominates the learning component. Figure 5 shows an instance of these results when the steady state disagreement level is not zero. Again, increasing τ decreases price volatility (as measured by absolute returns), expected volume and serial correlation in volume. However, since the underlying disagreement shock is large, when disagreement shocks are extremely frequent (e.g. $\tau = 2, 3$), volume autocorrelation is lower than when they are less frequent. This confirms our intuition from the finite horizon model in which we argue that large, relatively infrequent disagreements lead to high autocorrelation in volume.

The results of Lemmas 6 and 7 provide event-based empirical tests of the model. In particular, these predictions can be used to test for the effect of disagreement on volume and return characteristics around information events such as earnings announcements, which are likely to be associated with large jumps in disagreement. There is some empirical evidence consistent with our results. For instance, Atiase and Bamber [1994] show that volume around earnings announcements are increasing in the level of pre-disclosure analyst forecast dispersion, and Chae [2005] documents that volume jumps on an earnings announcement and then gradual decays over the next few days. Finally, Ball and Kothari [1991] document that return volatility spikes on the date of an earnings announcement, but decays rapidly afterwards. Our predictions are finer, since they relate the size of the disagreement shock to these return and volume characteristics, and could potentially be used to identify which events lead to more disagreement among investors, and which lead to less. Finally, Lemma 7 provides novel predictions relating return and volume characteristics with the frequency of disagreement, which have not been tested to the best of our knowledge.

5.2 Disagreement and Expected Returns

As our primary focus in the paper has been the relationship between disagreement and volume and return dynamics, we have assumed that the aggregate supply of the risky asset is zero. If the aggregate supply of the asset was a positive constant Q instead, then the expected return in steady state equilibrium of the model is given by:

$$E\left(R_{t+1}\right) = \text{var}_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right] Q \tag{32}$$

where $\text{var}_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right]$ is the variance of payoffs conditional on the investors' time t information. Note that since the investors have symmetric information sets, the conditional variance is the same across both investors.

From the proof of Lemma 5, we know that the conditional variance $\text{var}_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right]$ is linear in the disagreement λ_{t+1} at date $t+1$. This is because a higher level of disagreement at date $t+1$ implies that for investors at date t, the payoff is more uncertain. However, this immediately implies the following comparative statics results analogous to those in the last three subsections.

Lemma 8 *Suppose the aggregate supply of the risky asset is given by Q. Then:*

1. *In the steady state equilibrium, expected returns are increasing in the overall level of disagreement λ.*

2. *Suppose the level of disagreement jumps to $\lambda^* > \lambda$ in period $t+1$, and then reverts back to λ. Then the expected return $E\left(R_{t+1}\right)$ is increasing in the magnitude of the disagreement shock λ^*.*

3. *Suppose the steady state level of disagreement is $\lambda = 0$, but there is a jump in disagreement to λ^* every τ periods. Then the expected return increases in the frequency of disagreement shocks.*

In particular, this model suggests that expected returns increase with the level of disagreement. Note that this is in contrast to other difference of opinions models (e.g. Miller [1977]) which predict a negative relationship between the two. The reason for the positive relationship in our model is because a higher level of disagreement in the future leads to more uncertainty in payoffs today, and this increase in risk leads investors to require a higher expected return to compensate them. Moreover, the relationship between expected returns and disagreement is empirically unclear. While some papers claim to document a negative relationship (e.g. Diether et al. [2002] and Hong and Kubik [2003]), others find a positive relationship between the two (e.g. Qu et al. [2004] and Banerjee [2008]). Also, Ball and Kothari [1991] document a spike in abnormal returns on the date of the announcement, which is consistent with the predictions of our model.

6 Conclusions

We develop a dynamic DO model to analyze the relationship between the disagreement and trading volume around public announcements. We show that infrequent but major disagreements among agents lead to patterns in volume and returns that are empirically observed. In particular, this leads to volume clustering even when there is no persistence in fundamentals, and time-series correlation between volume and volatility. We also develop new predictions that relate the size and frequency of disagreement shocks to volume and return dynamics.

The differences of opinion framework is an interesting alternative to the standard asymmetric information models. The view that agents may “agree to disagree” not only seems plausible, but also seem potentially better suited than models of asymmetric information to address some the empirical evidence involving trading volume. However, one feature that is lacking from this framework is the informational role of prices. The typical assumption that agents’ opinions are common knowledge seems to be unrealistic. Hence, an interesting area for research are models in which both asymmetric information and differences of opinion are in play. In such models, investors condition on prices as a source of information about other investors’ views (e.g. Banerjee et al. [2008]).

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Appendix

Proof for Lemma 1. We will prove this by induction.
Base Step: At date $t = T-1$, we know that the above holds, since

$$x_{i,T-1} = \frac{v_{i,T-1} - P_{T-1}}{\rho_{T-1} + \delta},\ P_{T-1} = \bar{v}_{T-1} \text{ and } EU_{i,T-1} = -\exp\left\{-\frac{\left(v_{i,T-1} - P_{T-1}\right)^2}{2\left(\rho_{T-1} + \delta\right)}\right\} \tag{33}$$

In particular, this implies that $\phi_{T-1} = \frac{1}{K_{T-1}} = \frac{1}{\rho_{T-1}+\delta}$.
Iterative Step: Suppose the conjecture holds for all $\tau > t$. Then we will show it holds for t. Note that beliefs of investor i can be written as:

$$\begin{aligned} s_{t+1} - e_{i,t+1} &= V + \theta_{i,t+1} \\ s_{t+1} - \bar{e}_{t+1} &= V + \theta_{i,t+1} + \frac{1}{2}\left(e_{i,t+1} - e_{j,t+1}\right) \end{aligned}$$

where $\theta_{i,t+1} \sim N\left(0, q_{t+1}\right)$. This implies that investor i's beliefs about $v_{i,t+1}$ and P_{t+1} are given by:

$$\begin{aligned} Z \equiv \begin{pmatrix} P_{t+1} \\ v_{i,t+1} \end{pmatrix} &= N\left(\begin{pmatrix} \left(1-\pi_t\right)\bar{v}_t + \pi_t v_{i,t} \\ v_{i,t} \end{pmatrix}, \begin{pmatrix} \pi_t^2\left(\rho_t + q_{t+1} + \frac{1}{2}\lambda_{t+1}\right) & \pi_t^2\left(\rho_t + q_{t+1}\right) \\ \pi_t^2\left(\rho_t + q_{t+1}\right) & \pi_t^2\left(\rho_t + q_{t+1}\right) \end{pmatrix}\right) \\ &= N\left(\begin{pmatrix} m_{i,t} \\ v_{i,t} \end{pmatrix}, \begin{pmatrix} \eta_t & \pi_t\rho_t \\ \pi_t\rho_t & \pi_t\rho_t \end{pmatrix}\right) \end{aligned} \tag{34}$$

At date t, investor i's problem is given by

$$\begin{aligned} x_{it} &= \arg\max_x E_{i,t}\left[-\exp\left\{-x\left(P_{t+1} - P_t\right) - x_{i,t+1}\left(P_{t+2} - P_{t+1}\right) \ldots - x_{i,T-1}\left(F - P_{T-1}\right)\right\}\right] \\ &= \arg\max_x E_{i,t}\left[-\exp\left\{-x\left(P_{t+1} - P_t\right) - \frac{1}{2K_{t+1}}\left(v_{i,t+1} - P_{t+1}\right)^2\right\}\right] \\ &= \arg\max_x E_{i.t}\left[-\exp\left\{c + b'Z + Z'AZ\right\}\right] \\ &= \arg\max_x \exp\left\{-\frac{1}{2}\left(\mu_Z'\Sigma_Z^{-1}\mu_Z - 2c\right) + \frac{1}{2}\left(\mu_Z + \Sigma_Z b\right)'\left(I - 2A\Sigma_Z\right)^{-1}\Sigma_Z^{-1}\left(\mu_Z + \Sigma_Z b\right)\right\} \end{aligned}$$

where $c = xP_t$, $b' = (-x, 0)$, and $A = -\frac{1}{2K_{t+1}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$. The relevant first order condition is given by:

$$\frac{\partial c}{\partial x} + \left(\mu_Z + \Sigma_Z b\right)'\left(I - 2A\Sigma_Z\right)^{-1}\frac{\partial b}{\partial x} = 0 \tag{35}$$

which reduces to:

$$\left(m_{i,t} - P_t - x\eta_t\right)K_{t+1} + \left(v_{i,t} - P_t - x\pi_t\rho_t\right)\left(\eta_t - \pi_t\rho_t\right) = 0 \tag{36}$$

Solving for x, we get:

$$x_{i,t} = \frac{K_{t+1}\left(m_{i,t}-P_t\right)+\left(\eta_t-\pi_t\rho_t\right)\left(v_{i,t}-P_t\right)}{K_{t+1}\eta_t+\left(\pi_t\rho_t\right)\left(\eta_t-\pi_t\rho_t\right)} \tag{37}$$

Aggregating over all investors, we have

$$\int x_{i,t} = 0 \quad \Rightarrow \quad P_t = \bar{v}_t \tag{38}$$

which implies that

$$m_{i,t} - P_t = \pi_t\left(v_{i,t}-P_t\right)$$

This implies that:

$$x_{i,t} = \frac{K_{t+1}\pi_t+\left(\eta_t-\pi_t\rho_t\right)}{K_{t+1}\eta_t+\pi_t\rho_t\left(\eta_t-\pi_t\rho_t\right)}\left(v_{i,t}-P_t\right) = \phi_t\left(v_{i,t}-P_t\right) \tag{39}$$

Finally, the expected utility at time t is proportional to:

$$\begin{aligned} EU_t &\propto \exp\left\{-\frac{1}{2}\left(\mu_Z'\Sigma_Z^{-1}\mu_Z-2c\right)+\frac{1}{2}\left(\mu_Z+\Sigma_Z b\right)'\left(I-2A\Sigma_Z\right)^{-1}\Sigma_Z^{-1}\left(\mu_Z+\Sigma_Z b\right)\right\} \\ &= \exp\left\{-\frac{1}{2}\frac{\left(K_{t+1}\pi_t^2+\pi_t\rho_t\left(1-\pi_t\right)^2+\left(\eta_t-\pi_t\rho_t\right)\right)}{K_{t+1}\eta_t+\pi_t\rho_t\left(\eta_t-\pi_t\rho_t\right)}\left(v_{i,t}-P_t\right)^2\right\} \end{aligned} \tag{40}$$

which implies that

$$K_t = \frac{K_{t+1}\eta_t+\pi_t\rho_t\left(\eta_t-\pi_t\rho_t\right)}{K_{t+1}\pi_t^2+\pi_t\rho_t\left(1-\pi_t\right)^2+\left(\eta_t-\pi_t\rho_t\right)} \tag{41}$$

This completes the inductive step.

We know that $K_{T-1} = \rho_{T-1} + \delta$. Suppose $K_{t+1} = \rho_{t+1} + \gamma_{t+1}$. Then,

$$\begin{aligned} K_t &= \frac{\left(\rho_{t+1}+\gamma_{t+1}\right)\eta_t+\pi_t\rho_t\left(\eta_t-\pi_t\rho_t\right)}{\left(\rho_{t+1}+\gamma_{t+1}\right)\pi_t^2+\pi_t\rho_t\left(1-\pi_t\right)^2+\left(\eta_t-\pi_t\rho_t\right)} \\ &= \frac{\gamma_{t+1}\eta_t+\rho_t\left(\eta_t-\pi_t^2\rho_t\right)}{\gamma_{t+1}\pi_t^2+\left(\eta_t-\pi_t^2\rho_t\right)} = \rho_t + \frac{\gamma_{t+1}\left(\eta_t-\pi_t^2\rho_t\right)}{\gamma_{t+1}\pi_t^2+\left(\eta_t-\pi_t^2\rho_t\right)} \\ &= \rho_t + \frac{\gamma_{t+1}\left(q_{t+1}+\lambda_{t+1}/2\right)}{\gamma_{t+1}+\left(q_{t+1}+\lambda_{t+1}/2\right)} = \rho_t + \gamma_t \end{aligned}$$

where

$$\frac{1}{\gamma_t} = \frac{1}{\gamma_{t+1}} + \frac{1}{q_{t+1}+\lambda_{t+1}/2} \tag{42}$$

This also implies that

$$\phi_t = \frac{1}{\rho_t}\left(\frac{\rho_t + \gamma_t\left(\frac{q_{t+1}}{q_{t+1}+\lambda_{t+1}/2}\right)}{\rho_t+\gamma_t}\right) \tag{43}$$

■

Proof for Corollary 2. Note that if $\delta = 0$, then $\gamma_t = 0$ for all t and so $K_t = \rho_t$. This also implies that $\phi_t = \frac{1}{\rho_t}$. Similarly, when $\lambda_t = 0$, $\eta_t = \rho_t \pi_t$ which implies $x_{i,t} = \frac{1}{\rho_t}(v_{i,t} - P_t)$. ■

Proof for Proposition 3. The expression for volume follows immediately from the expression for the signed trade $T_{i,t+1}$. Since $\phi_t > 0$ and $\pi_t \geq 0$, we know that $-\phi_{t+1}\pi_t \leq 0$. When $\lambda_t = \lambda$ and $q_t = q$ for all t, then

$$\begin{aligned}
&\phi_{t+1}(1-\pi_t) - \phi_t \\
=\ & \frac{1}{\rho_t}\left(\frac{\rho_{t+1}+\gamma_{t+1}\left(\frac{q}{q+\lambda/2}\right)}{\rho_{t+1}+\gamma_{t+1}} - \frac{\rho_t+\gamma_t\left(\frac{q}{q+\lambda/2}\right)}{\rho_t+\gamma_t}\right) \\
=\ & \frac{1}{\rho_t}\left(\frac{\left(\rho_t\left(\gamma_t-\gamma_{t+1}\right)-\rho_t\pi_t\gamma_t\right)\left(1-\frac{q}{q+\lambda/2}\right)}{\left(\rho_{t+1}+\gamma_{t+1}\right)\left(\rho_t+\gamma_t\right)}\right) \leq 0
\end{aligned}$$

since $\gamma_t \leq \gamma_{t+1}$. ■

Proof of Proposition 4. Since $T_{i,t+1}$ is normally distributed, and $\text{Vol}_{t+1} = |T_{i,t+1}|$, volume has a half-normal distribution. Specifically, this implies that:

$$E\left[|\text{Vol}_{t+1}|\right] = E\left[|T_{i,t+1}|\right] = \sqrt{\frac{2}{\pi}\sigma^2_{T_{i,t+1}}}$$

$$corr\left(\text{Vol}_{t+2}, \text{Vol}_{t+1}\right) = \frac{2}{\pi-2}\left(\left(1-\rho^2\right)^{3/2} - 1 + \rho^2\sqrt{1-\rho^2} + |\rho|\arctan\left(\frac{|\rho|}{\sqrt{1-\rho^2}}\right)\right)$$

where $\rho = \frac{cov(T_{i,t+1}, T_{i,t+2})}{\sqrt{var(T_{i,t+1})var(T_{i,t+2})}}$. The variance and covariance of signed trade are given by:

$$\text{var}\left[x_{i,t+1}-x_{i,t}\right] = \left(\phi_{t+1}(1-\pi_t)-\phi_t\right)^2\text{var}\left[\Delta v_{i,t}\right] + \phi^2_{t+1}\pi^2_t\text{var}\left[\Delta e_{i,t+1}\right]$$

$$\begin{aligned}
&\text{cov}\left(x_{i,t+2}-x_{i,t+1}, x_{i,t+1}-x_{i,t}\right) \\
=\ & \left(\phi_{t+2}(1-\pi_{t+1})-\phi_{t+1}\right)\left\{\left(\phi_{t+1}(1-\pi_t)-\phi_t\right)(1-\pi_t)\text{var}\left[\Delta v_{i,t}\right] - \phi_{t+1}\pi^2_t\lambda_{t+1}/2\right\}
\end{aligned}$$

and the variance in the difference in valuations is defined recursively as:

$$\text{var}\left[\Delta v_{i,0}\right] = \frac{1}{4}\sigma_0 \text{ and } \text{var}\left[\Delta v_{i,t+1}\right] = (1-\pi_t)^2\text{var}\left[\Delta v_{i,t}\right] + \pi^2_t\lambda_{t+1}/2$$

■

Proof of Lemma 5. Conjecture a linear equilibrium with the price given by:

$$P_t = A_t \bar{v}_t \tag{44}$$

Given the price conjecture, investor i's beliefs about dollar returns are given by:

$$\begin{aligned} E_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right] &= A_{t+1}\alpha\left(\pi_{v,t}\bar{v}_{i,t} + \left(\pi_{s,t} + \pi_{d,t}\right)v_{i,t}\right) + v_{i,t} - RP_t & (45)\\ \text{var}_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right] &= \left(A_{t+1}\alpha\left(\pi_{s,t} + \pi_{d,t}\right) + 1\right)^2 \rho_t + \left(A_{t+1}\alpha\pi_{d,t} + 1\right)^2 \delta \\ &\quad + \left(A_{t+1}\alpha\pi_{s,t}\right)^2 \left(q + \lambda_{t+1}/2\right) \equiv V_{R,t} & (46)\end{aligned}$$

where the projection coefficients are given by:

$$\pi_{d,t} = \frac{1/\delta}{1/\delta + 1/\rho_t + 1/q},\ \pi_{s,t} = \frac{1/q}{1/\delta + 1/\rho_t + 1/q},\ \pi_{v,t} = 1 - \pi_{s,t} - \pi_{d,t} \tag{47}$$

Market clearing implies that

$$\int_i \frac{E_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right]}{\text{var}_{i,t}\left[P_{t+1} + D_{t+1} - RP_t\right]} = 0 \quad \Rightarrow \quad \left(A_{t+1}\alpha + 1\right)\bar{v}_t = RP_t \tag{48}$$

which implies $A_t = \frac{1}{R}\left(1 + \alpha A_t\right)$and the optimal demand is given by:

$$x_{i,t} = \phi_t \Delta v_{i,t} \text{ where } \phi_t = \frac{1}{V_{R,t}}\left(A_{t+1}\alpha\left(\pi_{s,t} + \pi_{d,t}\right) + 1\right) \tag{49}$$

The steady state of the equilibrium is characterized by $\rho = \rho_t = \rho_{t+1}$, which solves,

$$\rho = \alpha^2 \rho \pi_v + \theta \Rightarrow \rho = \frac{\alpha^2}{1/\rho + 1/\delta + 1/q} + \theta \tag{50}$$

and $A_t = A_{t+1} = 1/\left(R - \alpha\right) \equiv A$ and consequently,

$$\begin{aligned} \phi_t &= \frac{1}{V_{R,t}}\left(1 + \alpha A\left(\pi_s + \pi_d\right)\right) & (51)\\ V_{R,t} &= \left(1 + \alpha A\left(\pi_s + \pi_d\right)\right)^2 \rho + \left(1 + \alpha A \pi_d\right)^2 \delta + \left(\alpha A \pi_s\right)^2 \left(q + \lambda_{t+1}/2\right) & (52)\end{aligned}$$

■

Proof of Lemma 6. Suppose there is a shock in λ at time $t+1$ - that is $\lambda_{t+1} = \lambda^* > \lambda_s = \lambda$ for all $s \neq t+1$. This implies that for all $s \neq t$, $V_{Q,t} > V_{Q,s} = V_Q$ and consequently, $\phi_t < \phi_s = \phi$. In particular, for what follows, note that

$$\frac{\phi_t}{\phi} \leq \alpha\pi_v \Leftrightarrow \lambda^* \geq \lambda + \frac{2V_Q\left(1 - \alpha\pi_v\right)}{\alpha\pi_v\left(A\alpha\pi_s\right)^2} \tag{53}$$

Also, note that the regression coefficients π_s, π_d, and π_v , the steady state variance ρ, and the price coefficient A, do not change from their steady state.

The price volatility at date $t+1$ depends on

$$\text{var}\left(P_{t+1}-P_t\right)=A^2\left[\frac{2\alpha^2}{1+\alpha\pi_v}\left(\left(\pi_d+\pi_s\right)^2\frac{\theta}{1-\alpha^2}+\pi_d^2\delta+\pi_s^2\left(q+\lambda_{t+1}/2\right)\right)\right]$$

which is increasing in λ_{t+1}. Signed trade between dates t and $t+1$ is given by:

$$x_{i,t+1}-x_{i,t}=\left(\alpha\pi_v\phi_{t+1}-\phi_t\right)\Delta v_{i,t}-\alpha\pi_s\phi_{t+1}\Delta e_{i,t+1} \tag{54}$$

This implies that expected volume as a result of the disagreement shock is given by:

$$\begin{aligned}\text{var}\left(x_{i,t+1}-x_{i,t}\right) &= \left(\alpha\pi_v\phi_{t+1}-\phi_t\right)^2\text{var}\left(\Delta v_{i,t}\right)+\left(\alpha\pi_s\phi_{t+1}\right)^2\lambda_{t+1}/2 && (55)\\ &= \frac{1}{2}\alpha^2\pi_s^2\phi^2\left[\lambda\frac{\left(\frac{\phi_t}{\phi}-\alpha\pi_v\right)^2}{\left(1-\alpha^2\pi_v^2\right)}+\lambda_{t+1}\right] && (56)\end{aligned}$$

An increase in the size of the disagreement shock has two effects on the expected volume. The first effect is a direct increase in expected volume through the λ_{t+1} term. The second effect depends on the size of λ_{t+1}/λ - in particular, if $\phi_t\left(\lambda_{t+1}\right)>\alpha\pi_v\phi$, then an increase in λ_{t+1} decreases expected volume. But when $\phi_t\left(\lambda_{t+1}\right)\leq\alpha\pi_v\phi$, an increase in λ_{t+1} again decreases $\phi_t\left(\lambda_{t+1}\right)$ and so increases expected volume. In particular, when λ_{t+1} is large enough so that $\frac{\phi_t}{\phi}-\alpha\pi_v\leq 0$, then an increase in λ_{t+1}increases expected volume.

The autocovariance in volume depends on the absolute value of the serial covariance in signed trade:

$$\begin{aligned}&\left|\text{cov}\left(x_{i,t+2}-x_{i,t+1},x_{i,t+1}-x_{i,t}\right)\right|\\ =\ &\left|\alpha\left(\alpha\pi_v\phi_{t+2}-\phi_{t+1}\right)\left[\pi_v\left(\alpha\pi_v\phi_{t+1}-\phi_t\right)\text{var}\left(\Delta v_{i,t}\right)+\alpha\pi_s^2\phi_{t+1}\lambda_{t+1}/2\right]\right| && (57)\\ =\ &\frac{1}{2}\alpha^2\phi^2\pi_s^2\lambda\left(1-\alpha\pi_v\right)\left|\frac{\alpha\pi_v\left(\frac{\phi_t}{\phi}-\alpha\pi_v\right)}{\left(1-\alpha^2\pi_v^2\right)}-\frac{\lambda_{t+1}}{\lambda}\right| && (58)\end{aligned}$$

In this case, when λ_{t+1} is large enough so that $\phi_t-\alpha\pi_v\phi\leq 0$, increasing λ_{t+1}increases the autocovariance. On the other hand, when

$$\frac{\alpha\pi_v\left(\frac{\phi_t}{\phi}-\alpha\pi_v\right)}{\left(1-\alpha^2\pi_v^2\right)}-\frac{\lambda_{t+1}}{\lambda}>0$$

then increasing λ_{t+1} decreases the autocovariance in volume. ■

Proof of Lemma 7. Suppose the disagreement shock occurs at $t+1$. This implies that

$$\text{var}\left(\Delta v_{i,t+1}\right)=\alpha^2\pi_v^2\text{var}\left(\Delta v_{i,t}\right)+\alpha^2\pi_s^2\lambda^*/2 \text{ and } \text{var}\left(\Delta v_{i,t+s+1}\right)=\left(\alpha^2\pi_v^2\right)^s\text{var}\left(\Delta v_{i,t+1}\right) \tag{59}$$

Moreover, in the steady state of this equilibrium, since there is a disagreement shock every

τ periods, we know that

$$\text{var}\left(\Delta v_{i,t+\tau}\right) = \text{var}\left(\Delta v_{i,t}\right) = \left(\alpha^2\pi_v^2\right)^{\tau-1}\left(\alpha^2\pi_v^2\text{var}\left(\Delta v_{i,t}\right) + \alpha^2\pi_s^2\lambda^*/2\right)$$

which implies

$$\text{var}\left(\Delta v_{i,t}\right) = \frac{\left(\alpha^2\pi_v^2\right)^{\tau-1}\alpha^2\pi_s^2\lambda^*/2}{1-\left(\alpha^2\pi_v^2\right)^{\tau}} \equiv \sigma_\tau^2 \tag{60}$$

and note that $\sigma_\tau^2 \geq \sigma_{\tau'}^2$ for $\tau' > \tau$.

Volume at date $t+1$ depends on

$$\text{var}\left(x_{i,t+1} - x_{i,t}\right) = \left(\alpha\pi_v\phi - \phi_t\right)^2\sigma_\tau^2 + \left(\alpha\pi_s\phi\right)^2\lambda^*/2$$

and volume on the following dates $t+2$,...$t+s$ depend on:

$$\text{var}\left(x_{i,t+s+1} - x_{i,t+s}\right) = \phi^2\left(1-\alpha\pi_v\right)^2\left(\alpha^2\pi_v^2\right)^s\left(\alpha^2\pi_v^2\sigma_\tau^2 + \alpha^2\pi_s^2\lambda^*/2\right) \tag{61}$$

The average volume over τ periods is given by:

$$\frac{1}{\tau}\sum_{s=0}^{\tau}E\left[\left|x_{i,t+s+1} - x_{i,t+s}\right|\right] = \frac{1}{\tau}\sum_{s=1}^{\tau}\sqrt{\frac{2}{\pi}\text{var}\left(x_{i,t+s+1} - x_{i,t+s}\right)} + \frac{1}{\tau}\sqrt{\frac{2}{\pi}\text{var}\left(x_{i,s+1} - x_{i,s}\right)}$$

which implies that expected volume decreases as τ increases.

Similarly, when $\tau > 1$, then covariance between volume at $t+1$ and $t+2$ depends on

$$\begin{aligned} & \left|\text{cov}\left(x_{i,t+2} - x_{i,t+1}, x_{i,t+1} - x_{i,t}\right)\right| \\ = & \left|\alpha\left(\alpha\pi_v\phi_{t+2} - \phi_{t+1}\right)\left[\pi_v\left(\alpha\pi_v\phi_{t+1} - \phi_t\right)\text{var}\left(\Delta v_{i,t}\right) + \alpha\pi_s^2\phi_{t+1}\lambda^*/2\right]\right| \\ = & \left|\alpha\phi\left(\alpha\pi_v - 1\right)\left[\pi_v\left(\alpha\pi_v\phi - \phi_t\right)\sigma_\tau^2 + \alpha\pi_s^2\phi\lambda^*/2\right]\right| \end{aligned}$$

and on the following days, it depends on

$$\begin{aligned} & \left|\text{cov}\left(x_{i,t+s+2} - x_{i,t+s+1}, x_{i,t+s+1} - x_{i,t+s}\right)\right| \\ = & \left|\alpha\phi\pi_v\left(\alpha\pi_v - 1\right)^2\left[\left(\alpha^2\pi_v^2\right)^{s-1}\left(\alpha^2\pi_v^2\sigma_\tau^2 + \alpha^2\pi_s^2\lambda^*/2\right)\right]\right| \end{aligned}$$

When λ^* is small enough so that $\alpha\pi_v\phi - \phi_t > 0$, then increasing τ leads to a decrease in the average volume autocorrelation. However, when λ^* is large enough so that $\alpha\pi_v\phi - \phi_t < 0$, then decreasing τ will decrease the covariance in the first period (i.e. $\text{cov}\left(x_{i,t+2} - x_{i,t+1}, x_{i,t+1} - x_{i,t}\right)$) , and consequently decrease average correlation if τ is small enough. However, when τ is large enough (and so σ_τ^2 is small enough) the remaining terms dominate, and the overall effect is to decrease the average volume autocorrelation with an increase in τ. ■

Myopic optimization

A typical assumption in the literature is that agents are naive in that they ignore the future trading possibilities. That is, they act as if in the next period the value of the risky asset is realized. This can serve as a useful benchmark that highlights the effect of dynamic optimization. Moreover, as we show in Corollary 2, when there is no residual uncertainty ($\delta = 0$) or when there is no disagreement ($\lambda_t = 0$), optimal demand in the general model reduces to the myopic one.

When demand is myopic, the optimal demand coefficient is given by:

$$x_{i,t} = \phi_t \left(v_{i,t} - P_t\right) = \frac{1}{\rho_t} \left(v_{i,t} - P_t\right) \tag{62}$$

This implies that volume is given by

$$\text{Vol}_{t+1} = |x_{i,t+1} - x_{i,t}| = |\beta_t \Delta e_{i,t+1}| \tag{63}$$

This is because the coefficient α_t is zero since:

$$\alpha_t \;\; = \;\; \phi_{t+1} \left(1 - \pi_t\right) - \phi_t = \frac{1 - \pi_t}{\rho_{t+1}} - \frac{1}{\rho_t} = \frac{1 - \pi_t}{\rho_t \left(1 - \pi_t\right)} - \frac{1}{\rho_t} = 0 \tag{64}$$

Thus when optimal demand is myopic, there is volume on average but there is zero serial correlation in volume since $\Delta e_{i,t+1}$ is independent over time. The main difference is that in forming their demand, agents use the conditional variance of F and not the conditional variance of next period's price since they ignore future trading possibilities. This myopic behavior on the part of the agents eliminates the Δv_t component of volume, and hence eliminates the serial correlation in volume.

Figures

Figure 1: Expected Volume and Serial Correlation in Volume as a function of changes in the overall level of q_t and jumps in the first period q_1 only. Other model parameters are set to the following: $\rho_0 = 1$, $q_t = 0.1$, $\lambda_t = 1$, $\delta = 1$, $\sigma_0 = 0$. Note the x-axis is in log-scale.

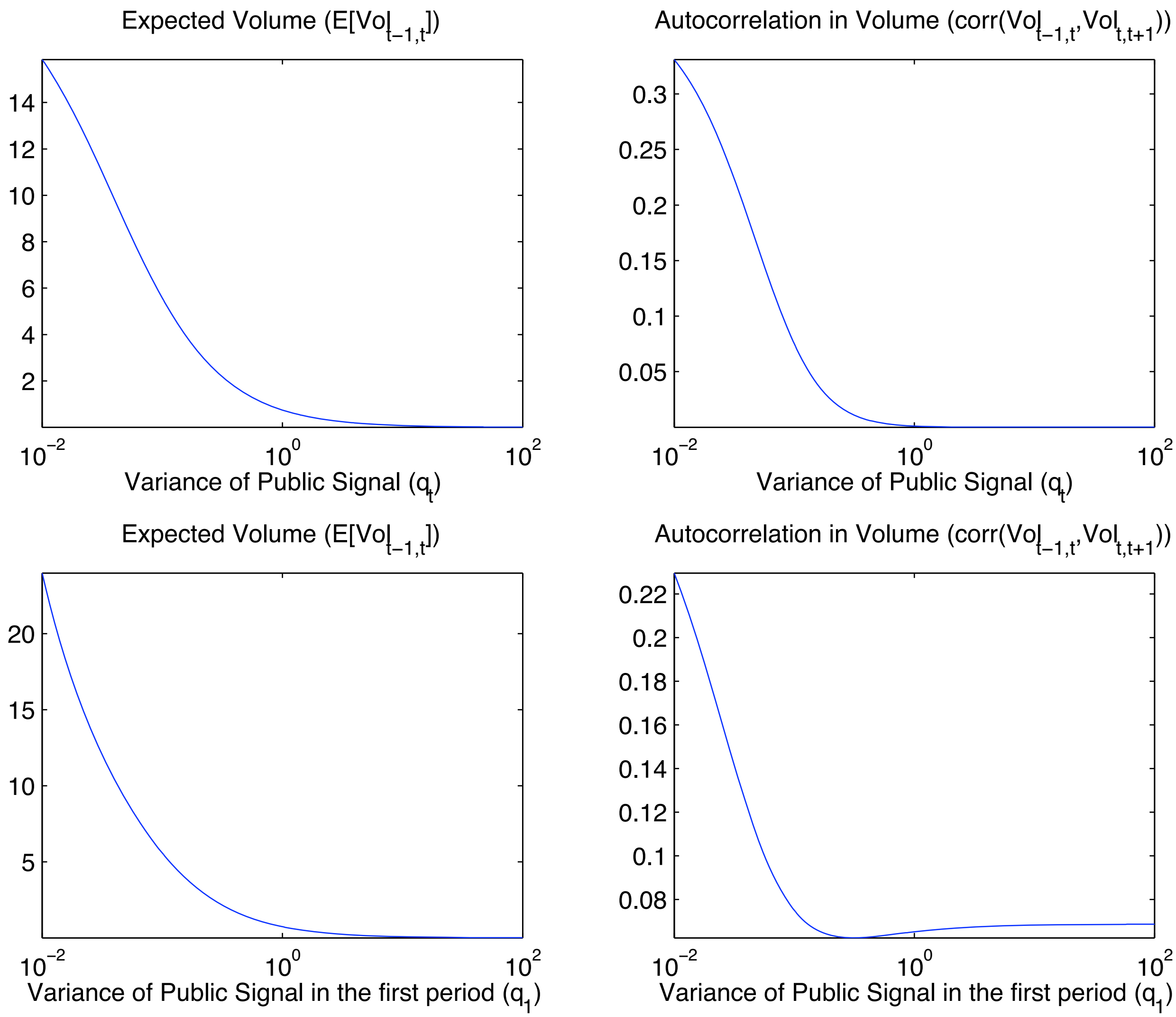

Figure 2: Expected Volume and Serial Correlation in Volume as a function of changes in the overall level of λ_t and jumps in the first period λ_1 only. Other model parameters are set to the following: $\rho_0 = 1$, $q_t = 0.1$, $\lambda_t = 1$, $\delta = 1$, $\sigma_0 = 0$. Note the x-axis is in log-scale.

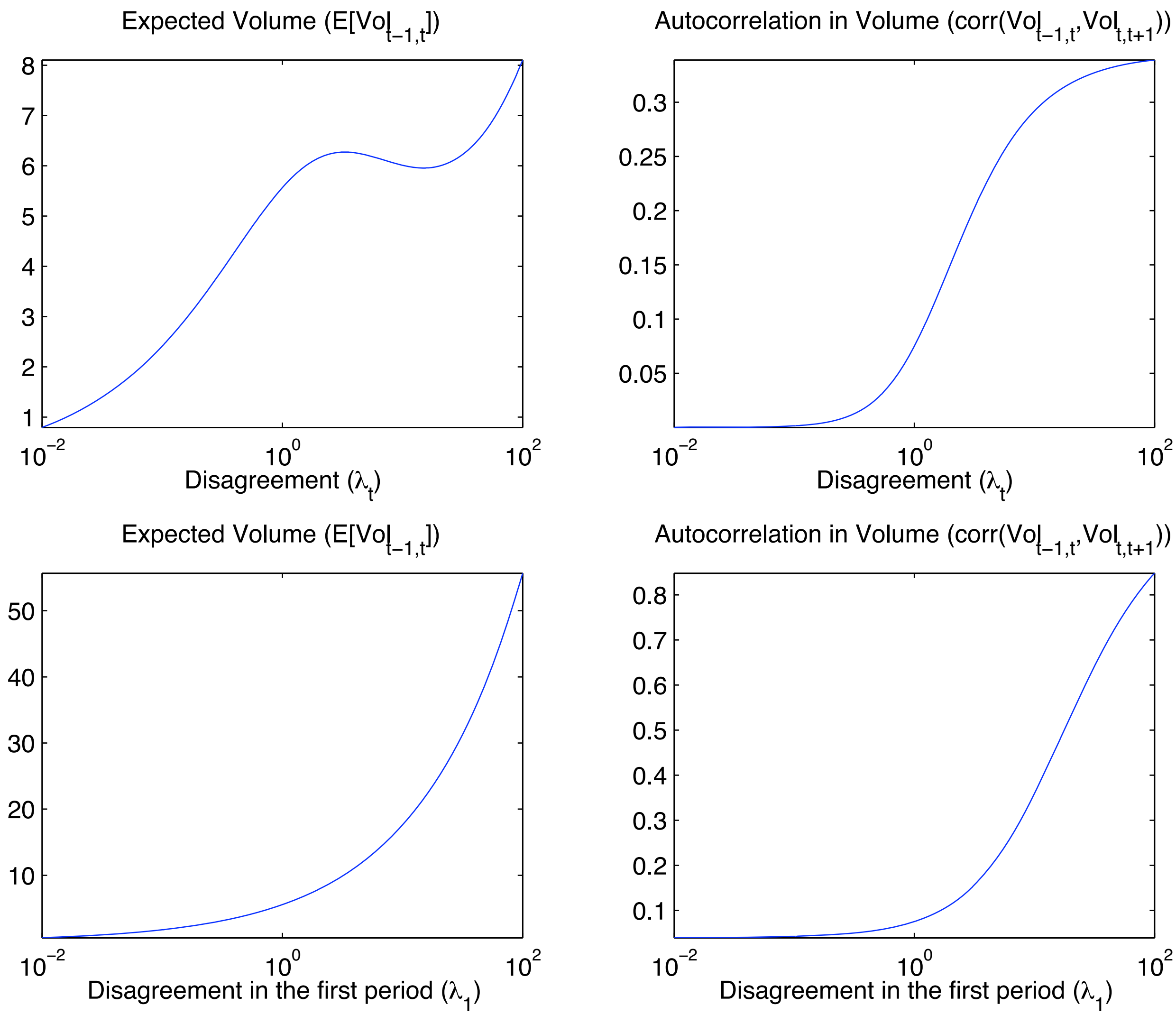

Figure 3: Expected Volume and Serial Correlation in Volume as a function of initial dispersion in beliefs σ_0 and payoff uncertainty δ. Other model parameters are set to the following: $\rho_0 = 1$, $q_t = 0.1$, $\lambda_t = 1$, $\delta = 1$, $\sigma_0 = 0$. Note the x-axis is in log-scale.

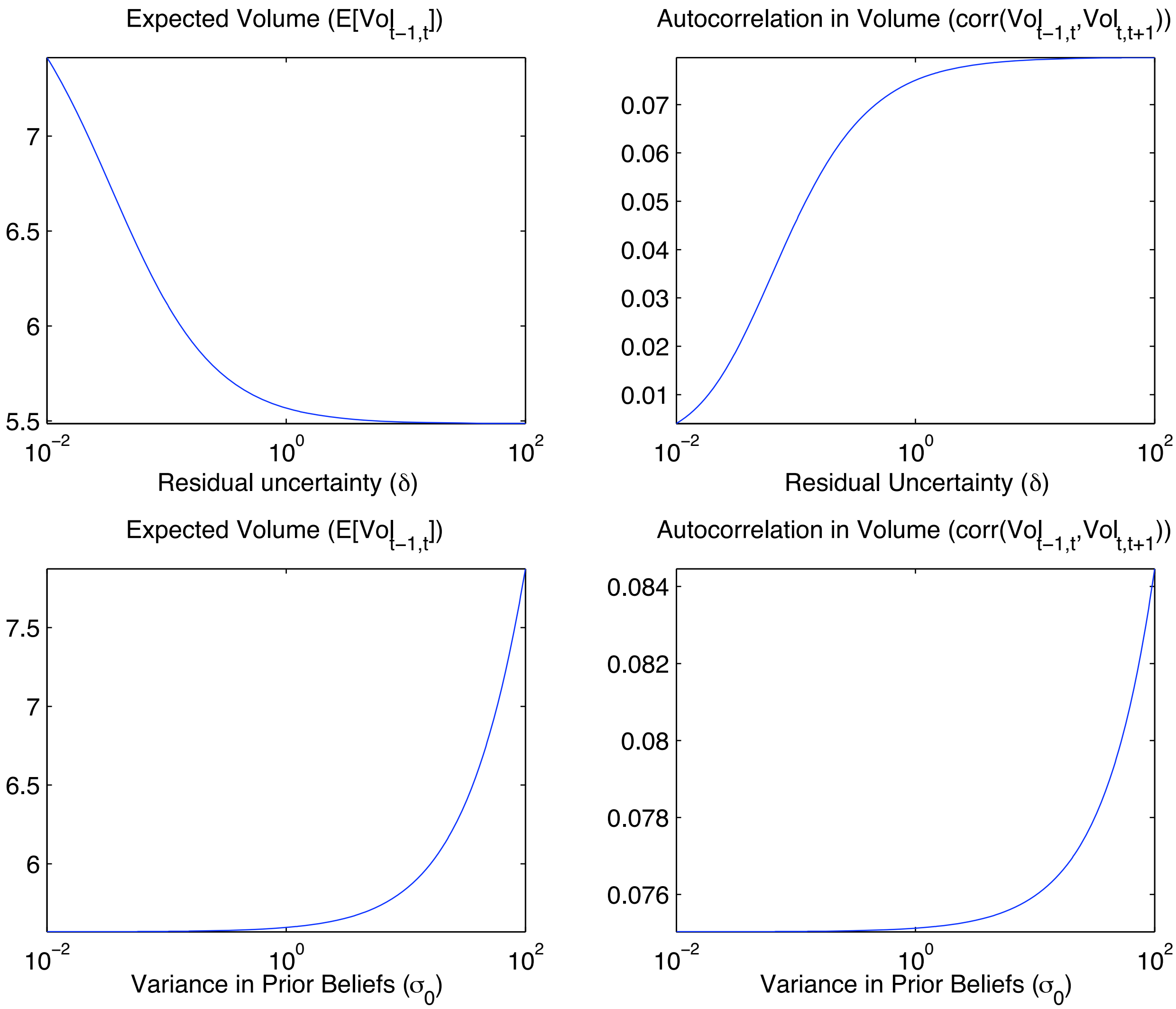

Figure 4: The Time Series Dynamics of Expected Volume and Volume Autocorrelation after a initial jump in λ_1. Other model parameters are set to the following: $\rho_0 = 1$, $q_t = 0.1$, $\lambda_t = 1$, $\delta = 1$, $\sigma_0 = 0$

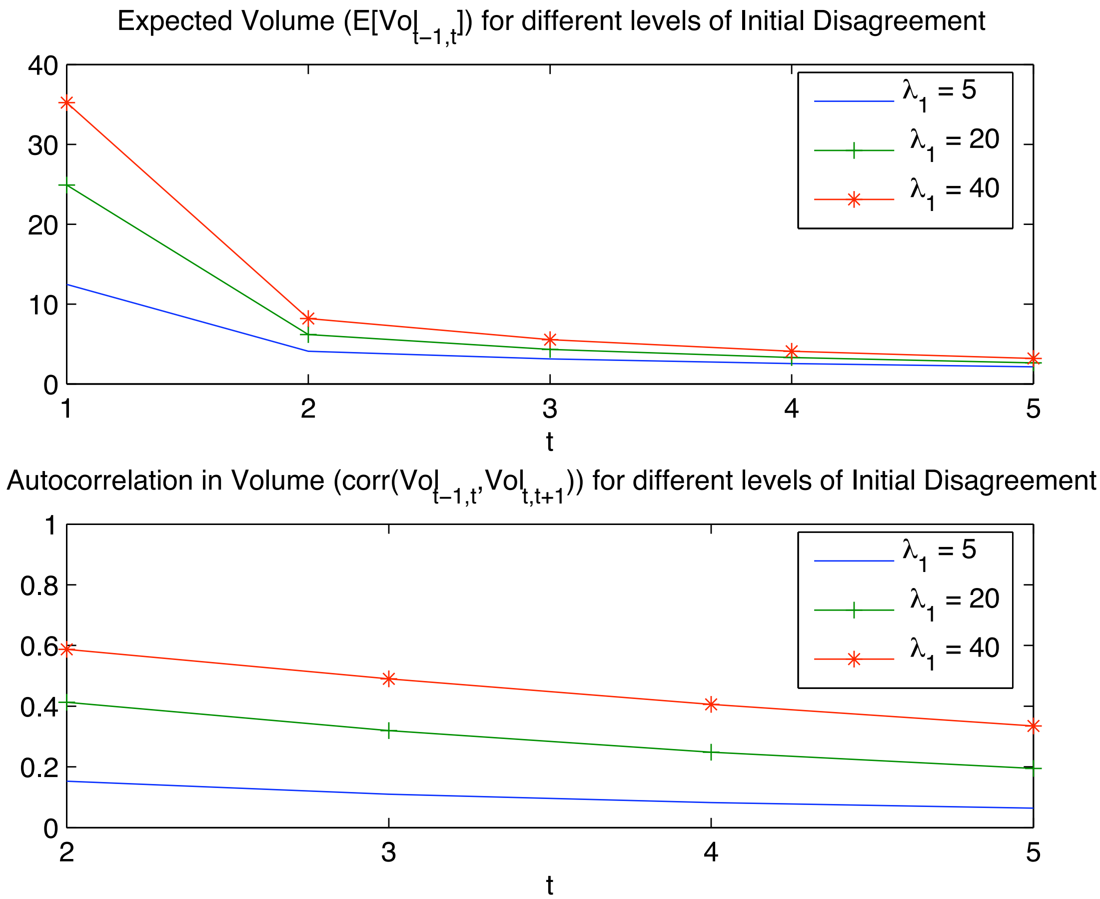

Figure 5: The effect of increasing the frequency of disagreement shock on return and volume characteristics