Diversification as a Public Good: Community Effects in Portfolio Choice

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ABSTRACT

Within a rational general equilibrium model in which agents care only about personal consumption, we consider a setting in which, due to borrowing constraints, individuals endowed with local resources under-participate in financial markets. As a result, investors "compete" for local resources through their portfolio choices. Even with complete financial markets and no aggregate risk, agents may herd into risky portfolios. This yields a Pareto dominated outcome as agents introduce "community" risk; unrelated to fundamentals. Moreover, if some agents are behaviorally biased, or cannot completely diversify their holdings, rational agents may choose more extreme portfolios and amplify the effect.

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"I've been saving like crazy. I'm expecting that when I'm 80 and need part-time nursing care, I'm going to be bidding against a lot of people for that."

(Wall Street Journal 3/4/2003 quoting a money manager)

We each consume different consumption baskets. An 80-year-old consumes nursing services that are not consumed by a 30-year-old. A person who lives in Manhattan cares about dining in a local restaurant or renting an apartment in New York; he cares less about these services in the Midwest. As a result of these taste differences, investors in a sense are divided into different communities (where a "community" can have a geographic and/or demographic interpretation). Within each community, investors compete for local resources. Anticipating this competition, should investors take into account the investment decisions of other community members when choosing their portfolios?

With perfect markets for assets, goods and services, we know from classical general equilibrium theory that the answer is no. Unfortunately this is unlikely the case; goods markets in particular are subject to numerous frictions. Moral hazard, for example, prevents forward markets for labor services. We show in this paper how such imperfections can giving merit to the quotation above, and introduce externalities in portfolio choice.

We examine how an individual's investment choices may be influenced by the investment choices of other members of his community. We show that when there are scarce local resources, competition for these resources leads investors to care about their relative wealth in the community. As a result, rational risk averse investors have an incentive to herd and choose a portfolio similar to the rest of their community.

These *community effects* have a number of important implications. First, they imply that even a small group of traders with a behavioral or other biases in their portfolio choice may have a large impact on equilibrium outcomes. Though standard intuition is that rational traders would trade in the opposite direction and offset the effects of such a bias, in our model these biased traders "pull" the entire community to trade in the same direction, amplifying the effects of the bias. In fact, in the resulting equilibrium, the bias itself can be rationalized – once the whole community is biased, it is optimal for each individual to follow.

Another important implication relates to welfare. While all agents would be better off if the community were diversified, it is in no individual investor's best interest to diversify on his or her own. Because of this externality, diversification has features of a public good. This creates a role for public policies that subsidize diversified portfolios and limit agents' ability to trade in risky securities. In particular, it suggests that there may be social benefits to preventing individuals from holding undiversified stock portfolios in 401K or social security retirement portfolios.

Finally, we show that community effects also have implications for asset prices. When a community herds into an asset class, this can drive up the price of that asset class in a way that is unrelated to aggregate consumption risk. These "price bubbles" lead to high equilibrium Sharpe ratios that cannot be explained in the context of standard asset pricing models.

Note that all of the implications described above are inconsistent with standard general equilibrium asset pricing models. In standard models, allocations are efficient and investors do not hold undiversified portfolios in equilibrium. Portfolio choices are driven by the risk/return characteristics of the individual securities, without regard to the portfolio choices of other agents; there is no "herding" in portfolio choice.

Our model differs from the standard approach in that agents are segmented into different communities. Our interpretation of a community is that of a group of people who share similar tastes. Specifically, a community is defined by the presence of local resources that are valued only by its members. This may take a geographical interpretation as well as other demographic interpretations; for example, a community may stand for individuals of a certain age group.

Because of this segmentation, competition within each community will cause the price of scarce local resources to fluctuate with the wealth of the community. These local resources represent local real estate, local labor and services, or other community specific goods. The desire to hedge this price volatility then biases portfolio choice. This implies that individual investors care about the correlation of their portfolio returns with the returns of other investors in the community.

We examine a simple version of such an economy in which financial markets are complete. The economy is a two-date economy; on the first date agents trade in financial assets, while on the second date they trade goods in the spot market and consume. In section A we examine a benchmark for our analysis. There we get the standard result that the equilibrium is unique and agents hold the market portfolio. While agents want to hedge against local price uncertainty, the trades of those whose endowments are short the local good offset the trades of those whose endowments are long the local good.

We modify this framework by assuming that agents face borrowing constraints in financial markets, and that local goods are not fully accepted as collateral.[1] This implies that there is asymmetric participation in financial markets: agents endowed primarily with local goods will be more constrained than agents endowed primarily with global goods (including financial assets).

Therefore, as a result of these borrowing constraints, financial markets are dominated by traders who wish to positively correlate their portfolio with the price of the local good. Since the local good price is increasing in the wealth of the community, this creates an externality in portfolio choice: if other investors hold portfolios with a high payoff in some state, then the local good price will be high in that state, and so each individual investor will also want their portfolio payoff to be high in that state. Due to this externality, when risk aversion is not too low multiple equilibria exist. While fully diversified portfolios can still exist in equilibrium, this equilibrium is not stable. The stable equilibria are ones in which investors in a given community tilt their portfolio away from a fully diversified portfolio, thereby taking on unnecessary risk.

We show that in these equilibria, agents are worse off than in a fully diversified equilibrium. The externality in portfolio choice creates a coordination problem: while all investors would be better off if all held diversified portfolios, it is not in any single investor's best interest to diversify on his or her own. This public goods aspect of portfolio diversification has important policy consequences. For example, restricting investors to hold well-diversified portfolios in retirement accounts may lead to welfare gains for all agents in the economy. On the contrary, adding new financial securities (through financial innovation) or new trading partners (through financial integration) may increase the risk investors can take in equilibrium and result in a welfare loss for all agents.

In section III, we examine the impact of agents who are subject to a behavioral or other exogenous bias in their portfolio choice. We show that if there are enough such agents, the equilibrium is unique. In this unique equilibrium, rational investors tilt their portfolios in the direction of the behavioral bias, in many instances choosing even more extreme portfolios than the behavioral traders. This response by rational agents amplifies the behavioral bias and "rationalizes" it: the biased traders are behaving optimally in equilibrium. This stands in contrast to common wisdom that suggests that rational agents exploit the behavioral bias by trading in the opposite direction.

An alternative story to the behavioral bias is that some investors are constrained to hold undiversified portfolios due to corporate control or moral hazard considerations. (For example, workers in Silicon Valley receive much of their compensation through stock-based incentive schemes which cannot be fully diversified.) More generally, agents may possess non-tradable skills or human capital whose value is positively correlated with the productivity of some sector. Similar to the behavioral bias, these constraints induce other agents to invest in the same sector.

It is important to note that the results in our model are derived in a setup for which agents have standard utility functions. That is, agents care only about their own personal consumption. Hence, the result that agents' care about other people's wealth is endogenous. One could also derive similar implications by exogenously assuming that agents care about their community's aggregate wealth or consumption. If this preference is similar to the indirect utility that we obtain then the implications are similar (though the welfare analysis may differ). We refer to this approach as an exogenous "preference for status," and discuss it in more detail in section IV, where we also evaluate the robustness of our main modelling assumptions. A weakness of the exogenous approach is that it is not clear how such preferences should be defined. As we will show, standard functional forms that have been used do *not* produce herding in equilibrium absent exogenous portfolio constraints.

In section V, we generalize the model by considering an arbitrary asymmetric setting with N communities. We demonstrate that the undiversified equilibria identified in the symmetric 2-community case is robust. Moreover, in the asymmetric setting a new feature emerges: community effects will lead to distortions in asset prices. As a result, our model's endogenously generated externality in portfolio choice can increase equilibrium Sharpe ratios, and that this can be related to the equity premia puzzle and pricing "fads."

The magnitude of the herding effects in our model depend upon the importance of the local good to consumers, and the price variability of these goods within the community. These in turn will depend upon the relative scarcity and the supply and demand elasticity of the local goods, as well as the riskiness of investors' portfolios. As a result we examine in Section VI empirical implications and evidence regarding cross-sectional differences in herding effects across communities.

Related Empirical Literature: Recently there is a growing literature studying community effects and social interaction. Duflo and Saez (2000) show that co-workers tend to choose similar portfolios. Benartzi (2001) finds that employees tend to over-invest in their company's stock in the retirement account. Since this choice is often discretionary, it implies that investors knowingly choose undiversified portfolios. Hong, Kubik and Stein (2003) show that investment decisions are related to social interaction, which is naturally linked to communities. Huberman (1999) (within the U.S.), Grinblatt and Keloharju (1999) (within Finland), and Feng and Seasholes (2002) (within China) show that investors are more likely to invest in firms that are geographically close to them. Our model of community effects can generate these patterns, and predicts them uniquely if some traders are constrained to hold local stocks (either for moral hazard reasons or due to a behavioral bias such as "familiarity").

Also related is the well-known "home bias" puzzle in international finance. Lewis (1999) surveys a large literature documenting this phenomenon. We discuss in the last section to what extent our model may explain this bias. An even more perplexing (and arguably more important) puzzle in the international literature is the lack of consumption risk sharing across countries. As Lewis (1999) notes there is little consumption co-movement across countries, which is inconsistent with standard models yielding Pareto-optimal outcomes. The fact that our model yields inefficient consumption patterns may help in explaining this puzzle as well.

Related Theoretical Literature: A related theoretical literature studies tournaments, where agents are rewarded for their relative standing. Lazear and Rosen (1981) and Green and Stokey (1983) study the effects of laborers having tournament-based compensation. Chevalier and Ellison (1999) argue that mutual fund managers are subject to tournament-based compensation and study the implications for their portfolio choice. Cole, Mailath and Postlewaite (1992, 2001) study saving and investment decisions in a matching game between men and women. This is a double-sided tournament in which, after investment returns are realized, men and women with equally ranked wealth are matched. While some of their conclusions are similar to ours,we develop a general equilibrium framework in which prices and relative evaluation arise endogenously.

The endogenous indirect utility function we derive in our model shows that an agent's utility is increasing in own wealth, but decreasing in the wealth of the community. This is related to models, such as Abel's (1990) "catching up with the Jonses" specification, which assume *exogenous* preferences related to relative consumption. In addition to endogenizing these preferences, our model differs in two dimensions. First, our agents care about consumption relative to current, as opposed to lagged, aggregate consumption per capita.[2] Second, and more important, our agents care about their consumption relative to per capita consumption in their local community, not in the whole economy.

Our welfare analysis is related to the classic literature on incomplete markets that started with Hart (1975). This literature has shown that with incomplete markets and multiple goods one should not expect equilibrium to be even constrained efficient (see for example Stiglitz (1982) and Geanakoplos and Polemarchakis (1986)). In this case the welfare implications of removing one friction but not all are indeterminate (see, e.g. Elul (1995)). This is the underlying reason the equilibrium we describe is inefficient; while we assume financial markets are complete and frictionless, there remains a friction in the goods or labor market.

In more recent papers, Chue (2001), Reisman (1999) and Shore and White (2002) study a model that is related to our status interpretation. They examine a partial equilibrium model in which agents have an exogenous preference to mimic other people's consumption. This naturally implies that agents mimic each other's portfolio choice. Because these papers are in a partial equilibrium framework, they do not consider market clearing (which would potentially eliminate the bias) nor the effect on asset prices. We discuss this approach further in section C.

Finally, our paper is related to the theoretical literature on the "home bias" in international markets, in particular, those papers that examine the effect of non-tradable assets (see for example Tesar (1993)). These papers distinguish local versus global goods, but unlike our paper, they represent each country by a single representative agent. This precludes any within community effects and leads to Pareto optimal outcomes. If there are complementarities between non-tradable and tradable goods and the output of the non-tradable good is stochastic then the representative agent biases his portfolio. This bias is a hedge against local productivity shocks. However, Stockman and Tesar (1995) and Lewis (1996) find that these models are rejected empirically, both because the volatility of local productivity is not sufficiently high, and these models predict greater consumption risk sharing than is observed. Our model does not depend on fluctuations in the supply of the local resource – in fact we will assume it is fixed. Also, our model is consistent with a lack of consumption risk sharing across communities.

I. Basic Model

Our model begins with a standard, 2-date stochastic exchange economy: There is a set of investors who live for 2 dates. On the first date, these investors trade securities and choose portfolios. On the second date, the state of nature is realized, which determines investors' endowment income as well as their portfolio payoff. Agents then trade goods in the spot market, and consume. Investors act to maximize their utility over final consumption.

An important feature of our model is the notion of investor "communities," which we define below.

Communities and Consumption Goods: There are two disjoint communities of investors. Community $j \in \{1, 2\}$ has members I_j, so that $I = I_1 \cup I_2$ is the set of all investors.

There are two types of consumption goods: (i) a global good that is consumed by all agents, labelled good 0, and (ii) local goods specific to each community, labelled as goods 1 and 2. Residents in community j consume both the global good 0 and their local good j.

This division into communities with distinct consumption sets is at the heart of our analysis. A natural interpretation is geographical communities with non-tradable goods such as local labor services, real estate, etc. More generally, communities can be thought of as social groups defined by their distinct tastes. For example, the communities of golfers versus skiers are defined by their consumption of country club memberships versus ski resort property and services.

Formally, the local nature of the goods is modelled through agents' preferences, as defined below.

Preferences: All agents maximize expected utility of terminal consumption given a separable CRRA utility function. Agents in community j care only about the global good 0 and the local good j of their community. This leads to the following utility function for $i \in I_j$:

$$u^i(x) = \frac{1}{1-\gamma}\left(x_0^{1-\gamma} + \alpha x_j^{1-\gamma}\right)$$

The parameter $\gamma > 0$ specifies agents' relative risk aversion, while $\alpha > 0$ specifies the relative importance of the local good. Note that our specification of agents' preferences explicitly precludes any complementarity in consumption of the local and global good. This distinguishes our model from others in the literature on home bias and non-tradables.

Assets and Uncertainty: There are $K \leq S$ firms (or "Lucas trees") that produce the global good. The output of each firm is uncertain, and depends upon the state of nature, s, drawn from a finite set $\{1, \ldots, S\}$. We denote by $F^k(s)$ the output of firm k, and we assume they are linearly independent (so that the assets are not redundant).At this point, there is no formal association of firms with communities. However, we will later interpret firms as being located in communities, and that this may be manifested as a bias in agents' initial endowments.

There are also $S - K$ additional zero-net supply securities, with payoffs $A^k(s)$ of the global good for $k = \{K+1, \ldots, S\}$. The role of these securities is to provide complete markets, which we shall assume throughout.

We denote by $Y(s) \equiv (Y^1(s), \ldots, Y^S(s)) = (F^1(s), \ldots, F^k(s), A^{k+1}, \ldots, A^S(s))$ the vector of all security payoffs.

Endowments: Each agent i is initially endowed with a portfolio $\bar{\theta}^i$, so that $\bar{\theta}^i_k$ is agent i's initial endowment of shares of security k. We let $\bar{\theta} = \sum_i \bar{\theta}^i$ be the aggregate endowment of shares, which we normalize so that $\bar{\theta}_k = 1$ for $k \leq K$ and $\bar{\theta}_k = 0$ for $k > K$. As implied by our discussion above the payoff of these portfolios is given in the global good. We assume that there is no other endowment of the global good; this is without loss of generality given complete markets.

Agents may also be endowed with the local good of their community. For agent i in community j, this endowment in state s is denoted $\bar{x}^i_j(s)$. For notational ease, we will extend this notation to all goods, so that $\bar{x}^i_l(s)$ is agent i's endowment of good l in state s, but note that $\bar{x}^i_l(s) = 0$ unless $l = j$.

To summarize, agents are endowed with the global good through shares of securities, and endowed with the local good of their own community. Agents are not endowed with the local good of the other community. We denote the aggregate endowment in the economy by $\bar{X}(s)$, which is given by,

$$\bar{X}_j(s) = \begin{cases} Y(s)\,\bar{\theta} & \text{for} \quad j = 0, \\ \sum_i \bar{x}^i_j(s) & \text{for} \quad j \in \{1, 2\}. \end{cases}$$

We assume that $\bar{X} > 0$, there is a positive supply of all goods in each state.

Timing and Trade: Agents trade shares of the firms on date 1. We let q denote the vector of prices for shares, so that agent i's budget constraint on date 1 is given by

$$q(\theta^i - \bar{\theta}^i) = 0. \tag{1}$$

After forming portfolios at the initial date, the state of nature s is realized and agents trade goods on date 2. We let $p(s) \in \Re^3_+$ denote the vector of spot prices for goods. Without loss of generality, we let the global good be the numeraire so that $p_0(s) = 1$ for all s. Agent i's budget constraint on date 2 can therefore be written

$$p(s) \cdot (x^i(s) - \bar{x}^i(s)) \leq Y(s)\theta^i, \tag{2}$$

in each state s. That is, the agent's net expenditures cannot exceed his portfolio payoff.

Equilibrium: The standard notion of equilibrium in this setting is given by prices, portfolios and allocations $(q, p, (\theta^i), (x^i))$ such that

1. for each agent $i \in I$, (θ^i, x^i) maximizes $E[u^i(x^i)]$ subject to (1) and (2);
2. financial and spot markets clear: $\sum_i \theta^i = \bar{\theta}$, and for $s \in S$, $\sum_i x^i(s) = \bar{X}(s)$.

A. Aggregation and Diversification: The Benchmark Case

In this section we develop some standard results on aggregation and diversification for this economy. We show that equilibrium can be modelled as though there is a representative investor in each economy, and that these investors will hold the global market portfolio. These results will serve as a useful benchmark for our later analysis.

We begin by considering the spot market equilibrium on date 2. In this final date, the economy is a standard exchange economy, and we solve explicitly for equilibrium prices in terms of initial (start of date 2) endowments.

First, recall that at the start of date 2, agent i in community j has endowment $\bar{x}^i_j$ of the local good, and

$$z^i \equiv Y\,\theta^i$$

of the global good. Importantly, note that i has no endowment of the local good of the other community. This implies immediately that there is no trade between members of community 1 and 2 at this stage. Recall that the global good is numeraire, $p_0 = 1$, so that p_j represents the relative price of the local good in community j. A standard derivation yields

$$p_j = \alpha \left(Z^j / \bar{X}_j\right)^\gamma \tag{3}$$

where $Z^j \equiv \sum_{i \in I_j} z^i$ denotes the aggregate date 2 endowment of the global good in community j. That is, the relative price of the local good in community j is inversely related to its relative supply, $\bar{X}_j / Z^j$.

Given the equilibrium prices on date 2, we can now derive agents' indirect utility functions for numeraire wealth on date 2. This utility function can then be used to determine portfolio preferences on date 1.[3]

Lemma 1 *The indirect utility of agent $i \in I_j$ with numeraire wealth w is given by*

$$v^i(w) = \frac{1}{1-\gamma}\, w^{1-\gamma}\, (W^j/Z^j)^\gamma,$$

where $W^j = Z^j + p_j \bar{X}_j$, the aggregate wealth of community j. The ratio of aggregate wealth to tradable wealth can also be written, $(W^j/Z^j) = \phi(p_j) \equiv 1 + \alpha^{1/\gamma} p_j^{1-1/\gamma}$, or alternatively as

$$(W^j/Z^j) = h(Z^j/\bar{X}_j), \tag{4}$$

where $h(z) \equiv 1 + \alpha z^{\gamma - 1}$.

Note that this indirect utility function looks like a standard CRRA utility function over wealth, but multiplied by a community-specific state variable that varies with local prices. In particular, indirect utility is decreasing in the local price, p_j.

Given the indirect utility function, we can restate the initial investment problem for each agent $i \in I_j$ as follows:

$$\begin{array}{ll} \max\limits_{\theta^i} & E\left[v^i(Y\theta^i + p_j \bar{x}^i_j)\right] \\ \text{s.t.} & q(\theta^i - \bar{\theta}^i) \leq 0. \end{array} \tag{5}$$

Thus, the problem at the initial date looks like a standard, one-period, one-good investment problem with CRRA investors, with the critical difference that the indirect utility function v^i depends upon the state variable $\phi(p_j) = W^j/Z^j = h(Z^j/\bar{X}_j)$. This "price dependence" of the utility function will play a key role in our analysis.

With CRRA utility, it is usually possible to aggregate investors into a single, "representative," investor. This is not the case here, due to the presence of the state variable. However, because the state variable is the same within each community, we can aggregate into a single representative agent *for each community*:

Lemma 2 *If we replace a subset of investors of community j, $\hat{I}_j \subset I_j$, with a single aggregate investor with endowment $\sum_{i \in \hat{I}_j} \bar{x}^i$ and initial shareholdings $\sum_{i \in \hat{I}_j} \bar{\theta}^i$, then equilibrium prices and allocations (for the remaining agents) are unchanged.*

Thus, we can without loss of generality think of there being a single investor in each community. We will let investor $j = 1$ (2) be the representative investor for community 1 (2). Note that in equilibrium, representative investor j has terminal wealth

$$W^j = Z^j + p_j \bar{X}_j.$$

Given the indirect utility function from Lemma 1, the marginal utility of income for j is

$$v^{j\prime}(W^j) = (W^j)^{-\gamma}(W^j/Z^j)^{\gamma} = (Z^j)^{-\gamma}. \tag{6}$$

That is, in equilibrium the representative investor behaves as though he has CRRA utility directly over consumption of the global good. This leads immediately to the following, familiar benchmark result.

Proposition 1 *In equilibrium, the consumption of the global good is perfectly correlated in the two economies ($Z^1 = \lambda Z^2$). Each representative investor holds the market portfolio ($\theta^j \propto \bar{\theta}$).*

The preceding result is not surprising, and confirms that absent market imperfections, we should not observe a "bias" in communities' investment portfolios.[4] What is critical for this result is that the aggregate wealth of each representative investor fluctuates in a way that offsets the price dependence of the indirect utility function. When this is the case, we have a standard representative agent framework, and the equilibrium is Pareto optimal. In the next section, we introduce frictions that break the link between fluctuations in aggregate wealth and the price dependence of the indirect utility function, and show that this leads to the possibility of suboptimal equilibria in which communities herd into undiversified portfolios.

II. Local Labor and Borrowing Constraints

A natural interpretation for the local good is local services, real estate and other local resources. To simplify the presentation, in this section we focus on local labor services, but discuss other possibilities in Section IV. A key feature of local labor services is that endowments consist of human capital. Due to moral hazard constraints, it is reasonable to assume that these agents cannot use this endowment as collateral for trading assets at the initial date. In this section, we incorporate this feature into the

model and explore its consequences.

A. Asymmetric Participation

Formally, we assume that the community is composed of two distinct types of agents, $I_j = I_j^I \cup I_j^L$. Agents in I_j^I are the **investors** in community j; these agents are endowed with shares of firms which they trade to construct portfolios on date 1. Specifically, for $i \in I_j^I$, the goods endowment is zero: $\bar{x}^i = 0$. They are endowed with shares $\bar{\theta}^i$, and we assume that $Y\bar{\theta}^i > 0$.

The second group of agents in community j, I_j^L, we refer to as **laborers**. These agents are only endowed with the human capital that produces units of the local good at date 2.

That is, for $i \in I_j^L$, the share endowment is zero, $\bar{\theta}^i = 0$. Only the local good endowment is positive, that is $\bar{x}_j^i > 0$.

At the initial date, in addition to the budget constraint, $q(\theta^i - \bar{\theta}^i) \leq 0$, we impose the "collateral constraint" that

$$Y\theta^i \geq 0. \tag{7}$$

That is, agents cannot borrow in the securities markets.

This collateral or borrowing constraint affects the two types of agents differently. Since investors have no endowment of goods, the constraint (7) is necessary in order to have positive consumption in all states. Thus, (7) does not bind for the investors, but is a natural consequence of their utility maximization.

On the other hand, (7) prevents laborers from using their endowment income on date 2 as collateral to trade securities on date 1. Since they also have no shares to trade, any non-trivial portfolio that satisfies the budget constraint and (7) represents an arbitrage opportunity, which cannot occur in equilibrium.[5]

We summarize this below:

Lemma 3 *In equilibrium, the constraint (7) does not bind for $i \in I_j^I$. For $i \in I_j^L$, constraint (7) implies that $\theta^i = 0$.*

Thus, each community is composed of investors, who trade on date 1, and laborers, who are constrained from trading on date 1. Applying Lemma 2, we represent the set of investors I_j^I as a single aggregate investor on date 1. This aggregate investor has terminal (date 2) wealth,

$$Y\theta^j = Z^j.$$

This differs from the community wealth, $W^j = Z^j + p_j \bar{X}_j$, which includes the endowment of the laborers, and so contrasts with the standard case consider previously in Section A. There, investor wealth and community wealth coincided. As a result, aggregate wealth of each representative investor fluctuated in a way that offset the price dependence of the indirect utility function. Here, since the two differ ($Z^j \neq W^j$), we obtain from Lemma 1 below a more complicated expression for marginal utility. This in turn will lead to the possibility of suboptimal equilibria in which communities heard into undiversified portfolios.

Proposition 2 *In equilibrium, the marginal utility of income for the representative investor of community j is given by*

$$v^{j\prime}(Z^j) = (Z^j)^{-\gamma}(W^j/Z^j)^{\gamma} = (Z^j)^{-\gamma}\phi(p_j)^{\gamma} = (Z^j)^{-\gamma}h(Z^j/\bar{X}_j)^{\gamma},$$

where $\phi(p_j) \equiv 1 + \alpha^{1/\gamma} p_j^{1-1/\gamma}$, and $h(z) \equiv 1 + \alpha z^{\gamma-1}$.

Relative to the standard case considered in Section A,

Proposition 2 reveals that when laborers are constrained from participating in the asset market, the marginal utility of community j investors is altered. Comparing (6) with (2), we see that the nature of the effect depends critically on the magnitude of the risk aversion parameter γ. When $\gamma > 1$, the functions ϕ and h are increasing. Thus, the marginal utility of income is higher when the price p_j of the local good is higher, or equivalently when the global good is in relatively greater supply in the community. In this case, the agent has a desire to hedge and hold assets that payoff more when local prices are high. The effect is reversed if $\gamma < 1$. In that case, the agent exploits the price variability by holding assets that payoff when local prices are low. Finally, in the special case $\gamma = 1$, the effect disappears, and we have the following:

Corollary 1 *If $\gamma = 1$ (log utility), then the equilibrium coincides with that in Proposition 1.*

When $\gamma \neq 1$, equilibria will no longer coincide with that of Proposition 1 in general. We characterize equilibria for general γ next, and show that investors may hold undiversified portfolios.

B. Characterizing Equilibria with Local Labor

In this section we analyze equilibrium portfolio choices in the presence of the frictions introduced in the previous section. Of interest is whether agents may choose to hold under-diversified portfolios. To simplify the analysis we make the following additional assumptions:

1. There is no aggregate risk, $\bar{X}_1 = \bar{X}_2 = 1$ and $\bar{X}_0 = 2$.

2. There are two equally likely states, $s \in \{1, 2\}$.

3. There are two firms. Each firm j pays

$$Y^j(s) \equiv F^j(s) = \begin{cases} 1+d & \text{if} \quad s = j \\ 1-d & \text{if} \quad s \neq j \end{cases}$$

   for some $d \in (0, 1]$.

4. Communities are symmetrically endowed, $\bar{\theta}_1^1 = \bar{\theta}_2^2$.

Note that for item (1), we normalize the aggregate supply of each good to one per community.[6] Thus, there is no aggregate risk in the economy and the Pareto optimal allocation is obvious – each investor should hold a fully diversified, riskless portfolio. This is the unique equilibrium that corresponds to the conclusion of Proposition 1. This setting is therefore ideal for identifying any biases due to local labor effects.

Since communities are symmetrically endowed, it is natural to consider a symmetric equilibrium in which the two securities have equal prices, $q_1 = q_2$. We restrict attention to this case, and note that the budget constraint for each community is then simply

$$E[Z^j] = 1.$$

Thus, we can represent the consumption of community 1 by its "volatility" σ. That is, if community 1 consumes $1 + \sigma$ in state 1, it must consume $1 - \sigma$ in state 2, where $\sigma \in [-1, 1]$. By market clearing, community 2 therefore consumes $1 - \sigma$ and $1 + \sigma$ in states 1 and 2, respectively.

Consider the portfolio choice for an investor i in community j. The payoff of i's portfolio can be decomposed as

$$z^i = \begin{cases} \bar{z}^i(1 + \sigma^i) & \text{if } s = 1 \\ \bar{z}^i(1 - \sigma^i) & \text{if } s = 2, \end{cases} \tag{8}$$

where $\bar{z}^i$ is the mean and σ^i is the volatility of i's portfolio choice. How does the optimal volatility choice for agent i relate to the choice of his community?

Taking Z^j, or equivalently the community volatility σ, as given,[7] investor i chooses a portfolio to equate his marginal utility of income across states. Using Lemma 1, this implies that

$$\frac{h(1+\sigma)}{1+\sigma^i} = \frac{h(1-\sigma)}{1-\sigma^i}.$$

Solving for σ^i, we find that investor i's best response volatility choice is given by

$$\sigma^i = m(\sigma) \equiv \frac{h(1+\sigma) - h(1-\sigma)}{h(1+\sigma) + h(1-\sigma)}. \tag{9}$$

The best response function is illustrated for $\alpha = 1$ and $\gamma \in \{1/2, 1, 2, 3, 4\}$ in Figure 1. Since community volatility is the aggregate volatility of investors' portfolios, an equilibrium is a fixed point $m(\sigma) = \sigma$; in the figure, this is where m crosses the 45^o line. Below we establish a number of properties of m which are evident from the figure.

*** Insert Figure 1 ***

Lemma 4 *The best response function m satisfies*

1. *m is continuous in σ,*
2. *$m(0) = 0$,*
3. *$m(\sigma) = -m(-\sigma)$,*
4. *m is increasing (decreasing) in σ for $\gamma > (<) 1$,*
5. *m is increasing in γ for $\sigma > 0$,*
6. *m is increasing in α for $m > 0$,*
7. *$m(1) < 1$,*
8. *$m'(0) = \frac{\alpha(\gamma-1)}{1+\alpha} > 1$ if and only if $\gamma > 2 + 1/\alpha$.*

Property 2 above verifies that full diversification ($\sigma = 0$) is always an equilibrium. That is, if the community portfolio is unbiased, it is optimal for each individual investor to fully diversify as well. Property 3 implies that the reaction function is symmetric, as should be expected given the symmetry of the model.

Property 4 demonstrates the key theme of this paper: that investors have a tendency to "herd" and choose portfolios close to their community when they are more risk averse than log-utility.[8] Moreover, this tendency is increasing with risk aversion, and with the importance of the local good, from properties 5 and 6.

The case $\gamma = 2$ in Figure 1 illustrates that this tendency towards herding need not be sufficient to lead to an undiversified equilibrium. In section III, we will show that if some investors in the community are constrained or biased in their portfolio choice, however, this herding tendency will induce rational investors to magnify the bias.

Moreover, when investors are sufficiently risk averse so that $\gamma > 2 + 1/\alpha$, properties 1, 7 and 8 establish the existence of undiversified equilibria, *even absent any constrained or biased investors*. That is, since $m'(0) > 1$, for σ sufficiently close to zero, $m(\sigma) > \sigma$; investors initially "overreact" to a bias in the community. Because $m(1) < 1$, continuity implies that m must cross the 45^o line for some $\sigma > 0$. Thus, with sufficiently risk-averse investors, the community effects may be so strong as to be self-sustaining. We summarize the key results below:

Theorem 1 *For $\gamma > 1$, investors have a tendency to herd due to community effects. For $\gamma \leq 2$, however, full diversification ($\sigma = 0$) remains the only equilibrium. If $\gamma > 2 + 1/\alpha$, then there exist self-sustaining undiversified equilibria. That is, there exists a unique $\sigma^* \in (0, 1)$ such that $m(\sigma^*) = \sigma^*$, and the set of equilibria is given by $\{-\sigma^*, 0, \sigma^*\}$. The volatility σ^* of the undiversified equilibrium is increasing in γ and α.*

This result highlights the fact that our model generates *portfolio externalities*. That is, the optimal portfolio choice of an investor depends upon the portfolio choices of his neighbors. When investors are sufficiently risk averse, this effect is self-sustaining: in equilibrium, agents do not diversify because the rest of their community is not diversified.

Theorem 1 provides conditions on γ and α that guarantee the existence of an undiversified equilibrium. Alternatively, one can solve the equilibrium condition $m(\sigma) = \sigma$ for the required importance of the local good α. In this case it can be shown that for $\gamma > 2$, there exists an equilibrium with income volatility $\sigma > 0$ if the importance α of the local good (status) satisfies

$$\alpha = \frac{2\sigma}{(1 - \sigma^2)[(1 + \sigma)^{\gamma - 2} - (1 - \sigma)^{\gamma - 2}]}. \tag{10}$$

Thus, for sufficiently risk averse agents, *any* level of income volatility can be supported as an equilibrium given appropriate importance of the local good.

C. Equilibrium Stability and Welfare

Before considering the impact of constrained or biased investors in the model, we explore further the possibility of self-sustaining undiversified equilibria. In this section, we address first how reasonable such equilibria are (relative to full diversification) and then consider their welfare properties.

As Theorem 1 and Figure 1 make clear, when investors are sufficiently risk averse multiple equilibria exist. How plausible are the undiversified equilibria in this case?

One refinement criteria that has been used in the literature is that of dynamic stability. The definition of stability relies on an iterative procedure in which agents react to the last period's outcome.Alternatively, one can view this iteration as taking place as "fictitious play" in the minds of the agents. A stable equilibrium can be thought as a limiting outcome of such a process. Hence this refinement has the view that an equilibrium is an outcome of a gradual process in which agents converge to an equilibrium strategy. Formally,

Definition 1 *An equilibrium σ is* ***locally stable*** *if for every σ' in a neighborhood of σ, the sequence $\{\sigma_n\}_{n=0}^{\infty}$ defined by $\sigma_0 = \sigma'$ and $\sigma_{i+1} = m(\sigma_i)$ converges to σ. An equilibrium σ is* ***globally unstable*** *if any sequence $\{\sigma_n\}_{n=0}^{\infty}$ for which $\sigma_{i+1} = m(\sigma_i)$ and $\sigma_0 \neq \sigma$ does not converge to σ.*

As Figure 1 makes clear, multiple equilibria exist when individual investors overreact to small deviations from full diversification by their community. This overreaction makes the fully-diversified equilibrium inherently unstable, as we show below:

Theorem 2 *If $\gamma > 2 + \frac{1}{\alpha}$ then:*

- *the full diversification equilibrium , $\sigma = 0$, is globally unstable,*
- *the undiversified equilibrium $\sigma^* > 0$ is a locally stable in the neighborhood $(0, 1]$ (as is $-\sigma^*$ in $[-1, 0)$).*

Proof. We first observe the fact that $m'(\sigma) > 1$ implies that σ is unstable. Hence, (i) follows from property 8 of m. From Theorem 1, σ^* is the unique point in $(0, 1)$ such that $m(\sigma) = \sigma$. Then given properties 1, 7, 8, and 4, it must be that $m(\sigma) \in (\sigma^*, \sigma)$ for all $\sigma \in (\sigma^*, 1]$, and $m(\sigma) \in (\sigma, \sigma^*)$ for all $\sigma \in (0, \sigma^*)$. Thus, starting from any point in $(0, 1]$, the sequence converges monotonically to σ^*. The case of $-\sigma^*$ is symmetric. ■

The result of the theorem can be seen in Figure 1. For $\gamma = 4$, starting from σ arbitrarily close to but not equal to zero, investors choose progressively less diversified portfolios until the undiversified equilibrium is reached.

Now that we have seen that the undiversified equilibria are stable, what can we say about their efficiency properties? In particular, how does the welfare of both investors and of laborers compare with the fully-diversified equilibrium?

Given that there is no aggregate risk in the economy, it is immediate that the fully diversified equilibrium is Pareto optimal, whereas an undiversified equilibrium cannot be (giving each agent the average consumption bundle that he consumes would make him better off). What is less clear is the welfare comparison of the two types of equilibria. While some agents must be worse off in an undiversified equilibrium relative to the fully diversified one, other agents might be better off (that is, the equilibria may be Pareto incomparable). The following result shows that this is not the case, and that in fact the full diversification equilibrium Pareto dominates the undiversified equilibria.

Theorem 3 *Every investor is worse off in the undiversified equilibrium than in the full diversification equilibrium. The same is true for every laborer as long as their endowments of the local good are uncorrelated with the payoffs of the firms.*[9]

The above theorem has important policy implications. First, it demonstrates that in this setting, restricting investors to invest in diversified portfolios can solve the coordination problem and make all agents strictly better off. In this sense, diversification has "public good" attributes. Alternatively, financial innovation that allows investors to hold riskier portfolios can reduce welfare by leading to less diversified equilibria.

Similar results apply to financial integration, which generally enhances welfare by allowing agents to better hedge against local risk factors. In our framework, however, financial integration creates new opportunities to trade risk with outsiders, and therefore creates the opportunity for agents to move towards a less diversified (rather than more diversified) equilibrium. For example, suppose $d < \sigma^*$ and compare the case in which the communities are separate autarkies to the case in which they are integrated. With autarky, each community will face the risk d of the production of the local firm. This equilibrium is obviously unique and stable, even if financial markets are complete. In contrast, if we integrate the communities, the only stable equilibria is the undiversified equilibrium, σ^*. Thus, financial integration will lead to increased risk in community consumption. We formalize this with the corollary,

Corollary 2 *There exists a $\hat{d} > 0$ such that for $d \in [0, \hat{d}]$, every agent is better off under autarky than in the stable equilibrium if financial markets are integrated.*

This conclusion is similar to Newberry and Stiglitz (1984). They examine trade in a production economy and conclude that with incomplete markets an autarky may Pareto dominate free trade.

III. Biased or Constrained Traders

Thus far we have assumed that all investors are rational and unconstrained. In this section, we introduce a subset of investors who face portfolio constraints. For example, these investors might be subject to a behavioral bias, such as "familiarity," that leads to a preference for investment in local stocks. Alternatively, some agents may receive compensation that is directly tied to the performance of the local firms (e.g., option and bonus compensation, etc.). In addition, agents may hold skills or human capital whose value is positively correlated with the productivity of a certain sector. If these agents are unable to trade against this income, they will be constrained in their portfolio choice, affecting the equilibrium outcome.

We will show that the presence of such constrained investors can lead to a unique equilibrium outcome. In this equilibrium, the entire community is "pulled" in the direction of the constrained traders. Moreover, we will show that in equilibrium, the bias of the constrained traders can be rationalized so that their constraints no longer bind in equilibrium.

Suppose, for example, that a subset of the population with wealth ω is subject to a behavioral bias towards investing in "local" firms. As a result, they hold a portfolio with payoffs

$$\omega \begin{bmatrix} 1+\hat{\sigma} \\ 1-\hat{\sigma} \end{bmatrix}.$$

independent of behavior of other traders. That is, they hold portfolios with bias given by $\hat{\sigma}$. What effect does the presence of these behavioral investors have on the equilibrium portfolios of rational investors?

To preserve symmetry and simplify the analysis, we assume that the same decomposition applies to community 2, with $\hat{\sigma}$ replaced by $-\hat{\sigma}$, and look at equilibria in which securities are equally priced. In this case, given an aggregate (rational and behavioral) community volatility of σ, the reaction function $m(\sigma)$ gives the optimal portfolio volatility for rational investors. Since with equally priced securities $EZ^j = 1$, the rational investors have aggregate expected wealth $1-\omega$. This yields the equilibrium condition:

$$(1-\omega)m(\sigma) + \omega\,\hat{\sigma} = \sigma,$$

which can be rewritten as

$$m(\sigma) = \hat{\sigma} + \frac{\sigma - \hat{\sigma}}{1-\omega} \equiv f(\sigma|\omega, \hat{\sigma}).$$

This equilibrium condition is illustrated in Figure 2 with $\alpha = 1$ and $\gamma = 4$. Rather than an equilibrium being defined as the intersection of m with the 45^o line, it is now the intersection of m with the line defined by f. The line f can be thought of as a rotation of the 45^o line around the point $\sigma = \hat{\sigma}$ until it has slope $1/(1-\omega)$. This is illustrated with $\hat{\sigma} = 50\%$ and $\omega = 20\%$.

*** Insert Figure 2 ***

The following results can be seen easily from the figure:

Theorem 4 *For any $\gamma > 1$, $\omega > 0$, and $\hat{\sigma} > 0$, full diversification is no longer an equilibrium. There exists a largest equilibrium, σ^*, with the following properties:*

1. *$\sigma^* > 0$ and is stable.*

2. *For any $\hat{\sigma} > 0$, there exists large enough ω such that σ^* is the unique equilibrium.*

3. *Since $m(\sigma^*) > 0$, we have that $\sigma^* > \omega\hat{\sigma}$; that is, rational investors accentuate the bias.*

4. *If $m(\hat{\sigma}) > \hat{\sigma}$, then $m(\sigma^*) > \sigma^* > \hat{\sigma}$; that is, rational investors are even more biased than the behavioral investors, and amplify the behavioral bias.*

These properties are illustrated in the example in the figure. Note that property 4 is likely to occur when $\gamma > 2 + 1/\alpha$ and $\hat{\sigma}$ is small, since we know from our earlier results that in that case investors "overreact" to small biases in the community.[10] The figure also makes clear the following comparative statics properties of σ^* when $\gamma > 1$, which both follow from the fact that m is increasing:

- As the wealth of the behavioral investors increases so does their influence on rational investors. That is, $|m(\sigma^*) - \hat{\sigma}|$ is decreasing in ω, and σ^* converges monotonically to $\hat{\sigma}$ as $\omega \to 1$.

- The more volatile the bias is, the more volatile the portfolio of the rational investors. That is, σ^* and $m(\sigma^*)$ are increasing in $\hat{\sigma}$.

Thus, the existence of biased behavioral investors breaks the symmetry of the model and biases the equilibrium portfolio choice of the community.

In the above analysis we have assumed that behavioral investors *must* hold portfolios with bias $\hat{\sigma}$. Alternatively, we could consider a setting in which $\hat{\sigma}$ represents instead the *minimal* bias these investors can hold, but they are free to hold more biased portfolios. In this case, the equilibrium would be described by the intersection of m with the function

$$\hat{f}(\sigma) = \min(f(\sigma), \sigma).$$

That is, $\hat{f}$ coincides with f below $\hat{\sigma}$, and with the 45^o line above $\hat{\sigma}$. As before, if there are enough such investors the unique equilibrium is one with a local bias. However, while the constraint has an effect on the equilibrium set, it need not bind in equilibrium. For instance, in the example of Figure 2, investors who are forced to hold a bias of at least $\hat{\sigma}$ would choose an even greater bias in equilibrium. Moreover, the behavioral bias is rational in the sense that if we "cure" a behavioral investor he would not change his portfolio. In other words, while the behavioral bias selects this equilibrium, in the resulting equilibrium *all* investors are behaving rationally!

Finally, we note that the analysis of this section may also be important from a public policy standpoint. For example, if the government constrains some traders to hold diversified portfolios, we can interpret this as the case $\hat{\sigma} = 0$. If enough traders are so constrained, this will guarantee $\sigma = 0$ as the unique equilibrium, enhancing welfare.

IV. Robustness – Alternative Model Specifications

Throughout the analysis we have made a number of simplifying assumptions to facilitate tractability and allow for a clearer exposition of the key features of our model. We have assumed that agents endowed with the local good are constrained from participating in financial markets. We also assumed that agents are unable to migrate from one community to another. Finally, we have used a separable constant relative risk aversion (CRRA) specification of the utility function. One may wonder how our results change if we alter these assumptions.

In this section we explore these assumptions and discuss how relaxing them impacts the community effects we have described. We demonstrate that while the magnitude of the bias and price volatility may decline, the main qualitative results of our model are robust.

One important feature of our model is the fact that preferences regarding relative consumption arise endogenously. We conclude this section by discussing an alternative approach in which one assumes that agents care explicitly about the aggregate wealth in their community – that is, they have a concern for their relative "status." Specifically, we point out that while some status specifications may yield similar implications as our model, many standard models of status do not.

A. Participation Constraints and Labor Mobility

Thus far we have assumed that agents endowed with the local good are completely constrained from participating in the financial market. Given that participation rates in the stock market in the U.S. are still below 50% (and as recently as 1989 were close to 30%), and that participation is strongly correlated with wealth, it seems reasonable to assume that participation rates are relatively low for individuals in the local labor market. In addition, even if individuals endowed with local goods do participate in the financial market, it is likely that they are unable to fully collateralize the value of their future endowment – brokers typically do not accept future labor income, real estate, etc., as collateral for margin accounts.

That said, these constraints are unlikely to be as extreme as those imposed in the model. Here we show that we can relax these constraints somewhat without undermining the main results. Suppose an agent endowed with 1 unit of the local good is permitted to participate in the financial market. Then, given community risk σ, his optimal trade will be to adjust his portfolio risk to the optimal risk given by the reaction function $m(\sigma)$. That is, he will choose a portfolio that pays $\{-b, b\}$, where b satisfies:[11]

$$\frac{P(1+\sigma)-b}{P(1-\sigma)+b} = \frac{1+m(\sigma)}{1-m(\sigma)},$$

where $P(z)$ is the price of the local good (and therefore the value of his endowment) given global community income z. Using (3) to compute P with $\bar{X}_j = 1$, solving for b yields:

$$b(\sigma) = .5\alpha[(1-m(\sigma))(1+\sigma)^{\gamma} - (1+m(\sigma))(1-\sigma)^{\gamma}].$$

Thus, if a fraction l of the endowment of local good is held by agents who are unconstrained, we get the aggregate reaction function

$$m_l(\sigma) = m(\sigma) - lb(\sigma).$$

Figure 3 illustrates this reaction function for the case $\gamma = 4$, $\alpha = 2$ and $l \in \{0, 10\%, 25\%\}$.

*** Insert Figure 3 ***

Increasing l diminishes the equilibrium bias since the tendency of investors to herd is offset somewhat by the hedging of the holders of the local good.[12] However, in the example above as long as $l < 25\%$, undiversified equilibria still persist. Indeed, we have the following general result, which shows that our results do not depend on the extreme assumption that $l = 0$.

Proposition 3 *When a fraction l of the laborers are unconstrained,*

$$m_l'(0) = m'(0) - l\,\frac{\alpha(\alpha+\gamma)}{1+\alpha}$$

Thus, there is a herding effect if $l < \frac{\gamma-1}{\gamma+\alpha}$, and there exists an undiversified equilibrium if in addition $l < \frac{\gamma-(2+1/\alpha)}{\gamma+\alpha}$.

Proof. Using (15) and the fact that $b'(0) = \alpha(\gamma - m'(0))$, we can solve for $m_l'(0)$. The inequalities for l are equivalent to $m_l'(0) > 0$ and $m_l'(0) > 1$. ■

Another issue that may affect our results is the elasticity of the supply of the local good. We have assumed a fixed supply in our model. However, in the case of local labor, if there is some labor mobility then we would expect laborers to migrate from the poor community to the rich community on date 2. This will reduce the volatility of the price of labor p_j. Alternatively, if the local good is a commodity that can be produced (e.g., build new housing), this will also mitigate the price volatility.

Specifically, consider the case where local laborers can migrate between the two communities at some cost c. So far we implicitly assumed that $c = \infty$, consider instead the case where $c < \infty$. Because migration implies an upper bound on the variability of local prices, it also implies an upper bound on investors' hedging demand and hence on the reaction function.

Let σ_c denote the bias that implies a price difference in the two communities of exactly c. It then follows that the reaction function, m_c, is capped. That is, for positive σ we have

$$m_c(\sigma) = m(\min(\sigma, \sigma_c)).$$

An immediate conclusion is that the conditions for community effects and the existence of an undiversified equilibrium are unchanged.[13] Investors will still herd in their portfolio choice if they are more risk averse than log-utility. The effect of migration is to put an upper bound on the magnitude of the bias, but not eliminate it.The same is true if we allow investors to migrate at date 2 to lower cost of living communities. As long as their is a positive cost of moving, our effects remain, though the magnitude is capped.

B. Alternative Utility Specifications

A CRRA specification of the utility function, while convenient, is admittedly special. In this section we discuss alternatives and their implications.

For example, rather than assume constant relative risk aversion, it is straightforward to extend the model to HARA class utility

$$u^i(x) = \frac{1}{1-\gamma} \sum_j \alpha_j^i (\frac{\gamma}{\gamma_1} - 1 + x_j)^{1-\gamma},$$

which is parameterized by γ_1, the coefficient of relative risk aversion at a consumption level of 1, and γ, the coefficient of relative risk aversion in the limit for high levels of consumption. By varying γ_1 above or below γ, we can vary agents' relative risk aversion, and allow it to be increasing ($\gamma_1 < \gamma$) or decreasing ($\gamma_1 > \gamma$). In this case, it can be shown that the initial slope of the reaction function, $m'(0)$, is increasing in γ_1 and decreasing in γ. In other words, the tendency toward herding is strongest when agents are very risk averse in the neighborhood of full diversification, but this relative risk aversion declines with wealth.

A second limitation of the utility specification we have chosen is that the parameter

γ controls both the relative risk aversion *and* the degree of substitutability between the goods. However, there is no a priori reason to expect these to be related to each other. A more general, non-separable utility function would allow us to separate these.

One obvious specification is to allow a constant elasticity of substitution (CES) between the local and the global good. The combination of CES/CRRA leads to a utility function for agent i of community j of the form,

$$u^i(x) = \frac{1}{1-\gamma_r}\left(x_0^{1-\gamma_c} + \alpha x_j^{1-\gamma_c}\right)^{\frac{1-\gamma_r}{1-\gamma_c}}$$

Here, γ_r is the coefficient of relative risk aversion, while γ_c is the inverse of the elasticity of substitution. Thus, γ_c measures the degree of complementarity between the goods.

In this case, the complementarity between the goods, γ_c, will determine the sensitivity of the price of the local good to fluctuations in community wealth. That is, (3) becomes

$$p_j = \alpha\left(Z^j/\bar{X}_j\right)^{\gamma_c}$$

Solving the model in this case leads to the following formula for the initial slope of the reaction function,

$$m'(0) = \gamma_c\left(\frac{\gamma_r - 1}{\gamma_r}\right)\left(\frac{\alpha}{1+\alpha}\right) \tag{11}$$

This coincides with our results in Section B when $\gamma_c = \gamma_r$. More generally, equation (11) implies that there is a tendency to herd ($m'(0) > 0$) if and only if agents are more risk averse than log (i.e. $\gamma_r > 1$). The magnitude of the herding effect is increasing in

- the degree of risk aversion, γ_r,
- the degree of complementarity between the global and local goods, γ_c, and
- the importance of the local good, α.

C. Relative Consumption and Community Status

In this subsection we discuss an alternative approach in which one assumes that agents care explicitly about their community's wealth. As we shall see, while this approach may yield similar effects it depends crucially on the exact functional form. Relative utility functions that are commonly used such as Abel's (1990, 1999) notion of "catching up with Joneses" cannot generate herding behavior.

In the relative consumption framework there is only a single good. Individuals have utility directly over the consumption of this good, as before. In addition, however, agents also care about how their individual consumption compares to the aggregate level of consumption in the community as a whole. We can interpret this concern for relative consumption as a concern for community "status." Each agent cares about his or her status, and so aggregate consumption appears directly in the utility function. To generate similar effects, assume that the utility of agent $i \in I_j$ is increasing in direct consumption x_0^i of the global good, and decreasing in $Z^j = \sum_{i \in I_j} x_0^i$, so that,

$$U^i(x_0^i, Z^j) = \frac{1}{1-\gamma}(x_0^i)^{1-\gamma} H(Z^j), \tag{12}$$

where H is a positive function and $H/(1-\gamma)$ is decreasing. That is, utility is increasing in own consumption, but decreasing in community consumption. If $\gamma > 1$ then H is increasing, so that the marginal utility of consumption is increasing with the level of community consumption – individuals value income more if their community is rich.

This functional form captures the idea of agents concern for their community status, yet preserves aggregation and allows us to model the community as a single aggregate investor. Exogenously assuming such preferences will therefore yield similar implications as our model; indeed, our model of local labor is equivalent to the case of $H = h^\gamma$. That is, when individuals must compete for scarce local resources, relative wealth matters. We view our model as way to "endogenize" individuals preferences regarding relative consumption. While the equilibrium outcomes are the same for either setting, the welfare implications and interpretation of the equilibrium will differ.

That said one should use caution when using a utility function that has a relative component. In many cases such utility functions do *not* yield herding. Consider for example a "catching up with the Joneses" utility function in which utility depends purely on an individual's share of aggregate community wealth. That is, suppose the utility function takes the form:

$$u(x_0^i / Z^j).$$

This specification does not support an undiversified equilibrium in the absence of exogenously constrained investors. Intuitively, if all agents choose the same undiversified portfolio, then there is no "relative" risk. However, each agent will find it profitable to deviate slightly towards a more diversified portfolio: in terms of relative wealth, it is profitable to give up a dollar when the community is rich and gain one when it is poor. This destroys an undiversified equilibrium.[14]

In general, to support an undiversified equilibrium it is necessary that in some instances individuals prefer to hold portfolios that are more extreme than the rest of the community (so that the reaction function has $m(\sigma) > \sigma > 0$). Many standard models of status do not produce this. In our setting, it occurs endogenously through the effect on relative prices.

On the other hand, it is possible that both status and community effects are important. In this case, the two effects will reinforce each other, increasing the tendency for investor herding.

V. Asymmetric Economies and Asset Price Effects

Thus far, we have considered the case of two symmetric communities. This symmetry simplifies the analysis by allowing us to ignore price effects in the first period securities market, since a bias towards one security in one community is offset by a bias towards the other security in the other community. However, all of our analysis generalizes to an asymmetric setting. When the economy is asymmetric, then the biases we have identified will in addition have consequences for equilibrium asset prices. In fact, we will show below that even absent aggregate risk, securities may exhibit high equilibrium risk premia.

For the general model, let there be N distinct communities. Each community is composed of laborers and investors, as in Section II. Each community $j \in \{1, \ldots, N\}$ has preferences given by risk aversion parameter γ_j, and taste parameter α_j, which may differ across communities.

As before, there are two states and two securities, with aggregate endowment $Y(s)\bar{\theta} = 1$ in each state s; that is, there is no aggregate risk. The investors in each community j have aggregate share endowment of $\bar{\theta}^j = w_j\bar{\theta}$, with $\sum_j w_j = 1$. Similarly, the laborers in each community have aggregate endowment of $\bar{X}_j = w_j$ units of the local good of their community.[15] Thus w_j measures the relative size of community j in terms of its wealth. The wealth distribution across communities is arbitrary.

Thus, we have described a general N-community economy, with communities of arbitrary size, and where each has its own risk aversion and relative importance for local goods. Since there is no aggregate risk, the natural, Pareto-optimal equilibrium for this economy is again full diversification. Our next result provides a general sufficient condition for the existence of an undiversified equilibrium.

Theorem 5 *There exists an equilibrium in which community j is undiversified if*

$$\left(\frac{w_j}{\phi_j}\right)\left(\sum\nolimits_{\hat{j}\neq j}\frac{w_{\hat{j}}}{\phi_{\hat{j}}}\right)\left(\sum\nolimits_{\hat{j}}\frac{w_{\hat{j}}}{\phi_{\hat{j}}}\right) > 0, \tag{13}$$

where $\phi_j = \frac{\gamma_j\alpha_j}{1+\alpha_j}\left[\gamma_j - \left(2 + \frac{1}{\alpha_j}\right)\right]$.

Note that (13) is always satisfied if $\phi_j > 0$ for all j. This leads to the following immediate corollary, which generalizes our result from Theorem 1:

Corollary 3 *If* $\gamma_{\hat{j}} > 2 + \frac{1}{\alpha_{\hat{j}}}$ *for every community* $\hat{j}$*, then for any initial wealth distribution and any community* j*, there exists an equilibrium in which community* j *is undiversified.*

Though $\phi_j > 0$ for all j is sufficient, it is not necessary for an undiversified equilibrium. It is enough that $\phi_j > 0$ for a single community, as long as that community is sufficiently small.

Corollary 4 *If* $\gamma_j > 2 + \frac{1}{\alpha_j}$*, then there exists an equilibrium in which community* j *is undiversified if* w_j *is sufficiently small.*

This follows from the theorem since for w_j small, the sign of the second and third terms in (13) are identical. The economic intuition for this result is that for w_j sufficiently small, community j investors will have little impact on equilibrium asset prices. But if asset prices are unaffected, from the perspective of community j this is equivalent to the symmetric setting of Section B.

Of course, the above conditions are sufficient but not necessary for an undiversified equilibrium for a community. For example, consider the case in which the conditions of Corollary (4) are satisfied for the first of two communities. By market clearing, in equilibrium both communities must be undiversified, even though the second community may be large and have $\gamma_2 < 2 + 1/\alpha_2$. The reason the second community is undiversified, in this case, is due to the price impact of the herding within community 1.

To see this, and to better understand the potential magnitude of the asset price effects, we consider next a simple example.[16] Suppose that $N = 2$ and for one of the communities, there are no scarce local resources; i.e., let $\alpha_2 = 0$. In this case, $\phi_2 = -\gamma_2 < 0$, and (13) simplifies to

$$\frac{w_1}{1 - w_1} < \frac{\gamma_1}{\gamma_2} \frac{\alpha_1}{1 + \alpha_1} \left[\gamma_1 - \left(2 + \frac{1}{\alpha_1} \right) \right]$$

Thus, if $\gamma_1 > 2 + \frac{1}{\alpha_1}$ there exists a range of w_1 sufficiently small supporting an undiversified equilibrium. But note that this range is diminishing in γ_2. This is because the price impact of community 1's trades increases with the risk aversion of community 2, as well as community 1's relative size. If this price impact is large enough, it will offset the community effects and prevent an undiversified equilibrium.

*** Insert Figure 4 ***

We illustrate this numerically in Figure 4. Here we solve for the undiversified equilibrium in the case $\gamma_1 = \gamma_2 = 4$, $\alpha_1 = 1$ and $\alpha_2 = 0$. In the figure, σ_1^* denotes the bias in community 1's portfolio. Note that σ_1^* declines with the size w_1, and thus the price impact, of community 1. But as long as $w_1 < 1/3$, this price impact is not so large as to prevent an undiversified equilibrium due to community effects in community 1.

Of course, since $\sigma_1^* > 0$, market clearing implies $\sigma_2^* < 0$. Since there are no community effects in community 2 ($\alpha_2 = 0$), investors in community 2 will demand a risk premium to hold a biased portfolio. That is, the relative price π of consumption in state 1 versus state 2 must be larger than 1. We can express this by computing the maximal Sharpe ratio that is available in the asset market:[17]

$$\rho = \max_R \left| \frac{E[R] - r_f}{\sigma(R)} \right| = \left| \frac{\pi - 1}{\pi + 1} \right|.$$

Figure 4 illustrates the maximal Sharpe ratio ρ for varying sizes w_1 of the community 1. Note that the effect on the equilibrium Sharpe ratio ρ is non-monotonic. When community 1 is small, so is their price impact, and so returns are hardly affected. On the other hand, when community 1 is large enough, only the fully diversified equilibrium remains, and the Sharpe ratio is again zero.

For intermediate sizes, however, the Sharpe ratio can be high even though aggregate consumption is *riskless*. Thus, our model produces an "equity premium puzzle." The resolution of the puzzle in the context of our model is that while aggregate consumption is smooth, individual consumption is very volatile due to the "herding" of community 1 investors. That is, community 1 investors are willing to accepting lower returns in order to hedge their community risk. On the other hand, community 2 investors take advantage of the high Sharpe ratio, biasing their trades in favor of higher returns.

VI. Empirical Relevance

In this section we evaluate the empirical relevance of our model. First we discuss empirical evidence related to the importance of local goods. We then describe a number of testable predictions that follow from our analysis.

One key parameter of our model determining the magnitude of the herding effect is the importance of local goods, represented by α. Higher α implies a greater magnitude of the hedging effect in our model. For our purposes, α can be measured empirically by looking at the value of expenditures on local goods and services to those on global goods in a given community.[18]

Evidence regarding this expenditure ratio is primarily available in an international context. Traditional estimates indicate that $\alpha = 1$(see for example Stockman and Tesar (1995) and Kravis, Heston and Summers (1982)). These estimates are based on a rough categorization of tradable versus non-tradable goods. More recent work such as Burstein, Neves and Rebelo (2001) implies that $\alpha = 3$. The reason for this difference is that they find that roughly half of the price of tradable goods is due to a non-tradable components such as labor costs.

For another example, we can consider "generational" communities. That is, younger investors represent a distinct community from older investors since they will consume in periods that the old will not. Here, α is the discount factor young investors place on future consumption. Indeed, we are currently in the process of extending our model to an overlapping generations model of this sort. Note that in this setting, "local goods" correspond to future consumption. Thus, local price volatility corresponds to fluctuations in interest rates and asset prices.

An important empirical implication of our model is that local goods prices should be sensitive to community wealth. One might be concerned whether we see local goods prices move in this way. We should first note that even modest price variability can lead to a significant portfolio bias using an appropriate parameterization of our model; e.g., with $\gamma = 4$, $\alpha = 3$, and $d = 15\%$, a price volatility of 5% leads to 20% bias in portfolio choice. Moreover, the low empirical correlation between good prices and wealth is true only for short horizons and national price indices. The importance of the horizon is shown in Boudoukh and Richardson (1993). They look at the relationship between stock market returns and inflation,[19] and find that while there is no positive correlation at high frequencies (e.g., monthly or quarterly), there is a strong positive correlation over longer horizons.[20] This is consistent with the fact that both consumption and good prices tend to be sticky. Prices adjust only gradually in response to more permanent shocks in community wealth. These price effects are also more likely to be observed in local, rather than national, markets. See, for example, Green (2002) for evidence from the real estate market.

The international finance literature also supports the view that in the long run, community wealth has an important effect on local prices (see for example the survey paper, Rogoff (1996)). Specifically this is true for the relative price of tradable versus non-tradables. This can be seen most strikingly in the cross section, comparing deviations from PPP across countries with measures of a country's wealth: prices are higher in wealthier countries, primarily due to higher prices of non-tradables. It also can be seen in the time series of PPP deviations. One example is the appreciation of Japan's real exchange rate following World War II and the depreciation of the yen versus the dollar in real terms following the recession in Japan in the 90s. A recent paper by Chen and Rogoff (2002) provides additional evidence for even shorter horizons. They focus on countries where primary commodities constitute a significant share of their exports. Since we can observe commodity prices wealth effects are more easily detected. They find a strong link between the real exchange rate and wealth effects, consistent with our model.

In addition to the link between local prices and local wealth, our model has a number of other implications for portfolio choice which can be directly tested. For example, the key driver of community effects in our model is competition over local resources. This implies that:

1. Communities where local resources are scarce relative to wealth are more subject to herding effects in portfolio choice, and as a result will be less diversified.

Hence, for example, we would expect urban investors to exhibit more correlated portfolios than rural investors. Also, competition for local resources will be more intense in areas where wealth has increased substantially relative to the supply of local resources. Thus, we should see greater herding by investors in "boom town" economies.

In Section III of the paper, we demonstrated that when some investors are constrained in their portfolio choice, the reaction of unconstrained investors is to tilt their portfolios in the same way. One important class of constrained investors is that of agents who work in local firms. Due to moral hazard considerations their income is tied both explicitly and implicitly to the performance of these firms. Hence we expect that:

2. In areas where there is dominant company or sector of companies, unconstrained investors will be less diversified (and biased towards the local firms).

Indeed, unconstrained investors who do not work for the local firms may exhibit the most biased stock portfolios. For example, the neighbors of Silicon Valley engineers may be more heavily invested in the hi-tech industry than the engineers themselves (of course, the *total* exposure of the engineers may be greater once restricted stock, options and labor income are included – we are comparing the unconstrained components of the portfolios).

Another implication of the results in Section III is that

3. These effects will be increasing with the volatility of the local sector.

Thus, we expect to see greater herding in hi-tech towns than in towns dominated by less volatile industries (e.g., auto manufacturing).

The points above suggest that herding effects will be more pronounced in areas such as Manhattan or the San Francisco Bay Area. A recent survey by the California Association of Realtors (Krueger and Cauley, 1999) documents that home buyers in the Bay Area are roughly 3 times as likely to alter their behavior in response to stock market performance as compared to buyers in Southern California. This is consistent with the fact that home buyers in the Bay Area have higher stock holdings. Moreover, the Bay Area housing market is among the most inelastic in the country (Mayer and Somerville, 2000). Not surprisingly, Green (2002) finds that the housing market in Southern California is much less sensitive to stock price fluctuations than the Bay Area market. Consistent with this, our paper predicts that herding effects in portfolio choice should also be much more important in the Bay Area.

Another factor that we discussed is mobility. To the extent that estimates can be obtained for migration costs then we should find:

4. Herding effects will be larger in communities with higher migration costs.

Furthermore, if investors migrate between communities we should observe that

5. Investors who move from one community to another will shift their portfolio so it becomes less (more) correlated with the portfolio of the community they migrated from (to).

Early empirical support for this prediction is provided by Bodnaruk (2002). He shows that investors that move sell shares of companies located in their old residence and buy ones closer to their new home. This is consistent with our model, but also with the hypothesis that physical proximity facilitates information transmission. Evidence on the performance of stocks that movers sell compared with stock that they buy might help separate between the two explanations.

VII. Conclusion

Our paper provides an explanation for herding and lack of risk sharing by investors. Indeed, we demonstrate that individuals may choose undiversified portfolios even in an environment with complete financial markets and no aggregate risk. We begin by showing that competition for local resources (such as local real estate, labor and other services) creates an externality so that individuals care about their relative wealth in the community. This effect has important consequences. If the local resources cannot be fully collateralized, and if investors are sufficiently risk averse, then individual investors will try to correlate their wealth with that of their community.

Absent aggregate risk, there always exists an equilibrium in which all investors are fully diversified. While this equilibrium is Pareto optimal, we show that when agents are sufficiently risk averse, this equilibrium is not stable. In all stable equilibria, investors in a given community tilt their portfolio in the same direction, taking unnecessary risk. As a result of this "herding" effect, agents are worse off than in a fully diversified equilibrium.

We use this to examine the impact of a behavioral bias. If some agents are subject to a behavioral bias then rational agents adopt this bias and amplify its effect. This in turn can rationalize the bias. A similar conclusion follows if some agents' income is tied to the productivity of a certain sector.

We also consider the implications of our model for asset returns. We show that the presence of a small subset of agents in the economy that are subject to community effects is sufficient to significantly impact returns. Specifically, equilibrium Sharpe ratios can be high, even though aggregate consumption is riskless.

Finally, we discuss the empirical evidence regarding the assumptions in our model. We also consider empirical implications regarding cross-sectional differences in the magnitude of herding across communities.

Within our model, diversification is a public good. One implication for this is a history dependence in portfolio choice. Prior to the development of financial markets, communities were likely unable to diversify many "local" risks. As markets have become more complete, one would expect investors to diversify their portfolios away from such risks. Our results make clear, however, that there is a coordination aspect to such diversification. As a result, the community is likely to remain in a stable equilibrium in which the local risk is still held.

This has obvious policy implications. For example, there is a role for social policies which subsidize investor diversification. There can be welfare gains from restricting investor portfolio choice in retirement accounts in a way that prevents them from holding undiversified positions. Indeed, our results imply that much of the policy implications related to public goods may also apply to investor diversification.

VIII. Appendix

A. Proof of Lemma 1

Recall that the agent consumes only the global good and the local good j, and that the we let the global good be numeraire, $p_0 = 1$. The necessary and sufficient first order condition for agent i is that the marginal rate of substitution equals the relative price:

$$p_j = \alpha(x_j^i/x_0^i)^{-\gamma}.$$

Equivalently,

$$x_j^i = (\alpha/p_j)^{1/\gamma} x_0^i, \tag{14}$$

Using the budget constraint and (14), we have

$$y = x_0^i + p_j x_j^i = x_0^i + p_j(\alpha/p_j)^{1/\gamma} x_0^i = x_0^i \phi(p_j).$$

From the definition of u^i and (14),

$$u^i(x^i) = \frac{1}{1-\gamma}\left[{x_0^i}^{1-\gamma} + \alpha {x_j^i}^{1-\gamma}\right] = \frac{1}{1-\gamma}\ {x_0^i}^{1-\gamma}\phi(p_j).$$

Combining these yields the expression for v^i in terms of ϕ. Finally, using (3),

$$\phi(p_j) = (1 + \alpha^{1/\gamma} p_j^{1-1/\gamma}) = (1 + p_j(\alpha/p_j)^{1/\gamma}) = (1 + p_j(\bar{X}_j/Z^j)) = (W^j/Z^j).$$

Similarly, h follows from ϕ by substituting for p_j using (3).

B. Proof of Lemma 2

This is the standard aggregation result for CRRA utility functions. Here we have state dependent utility, but as the multiplicative factor is the same for all agents within community j, it can be treated as a change of measure, and the usual proof of aggregation applies. Note that one condition for aggregation is that endowments are traded. In our setting this is equivalent to

$$\bar{x}_0^i + p_j \bar{x}_j^i \in \text{span}(Y),$$

for all $i \in I_j$, for which complete markets is sufficient (though this can be weakened).

C. Proof of Lemma 3

If $\gamma > 0$, the marginal utility of consumption is infinite at zero. In equilibrium, agents consumption is therefore strictly positive. Thus, the constraint (7) does not bind for $i \in I_j^I$.

For $i \in I_j^L$, the budget constraint implies $q\theta^i \leq 0$. This together with (7) implies an arbitrage opportunity unless $Y\theta^i = 0$. Given the non-degeneracy of the asset payoffs, this implies $\theta^i = 0$.

D. Proof of Proposition 1

With complete markets, the marginal utility of income must be proportional for all investors. Thus, $(Z^1)^{-\gamma} = \hat{\lambda}(Z^2)^{-\gamma}$. The theorem then follows with $\lambda = \hat{\lambda}^{-1/\gamma}$. Since there are no redundant assets and $Z^j = \bar{x}_0^j + Y\theta^j$, if $\bar{x}_0^j = 0$ then $\theta^1 = \lambda\theta^2$. This plus market clearing $(\theta^1 + \theta^2 = \bar{\theta})$ implies the result.

E. Proof of Lemma 4

Property 1 follows since h is continuous and $h(z) > 1$. Properties 2 and 3 follow immediately from the definition of m. Property 4 follows from the monotonicity of h. Property 5 follows since for $\sigma > 0$,

$$\frac{\partial}{\partial\gamma}m(\sigma) = \frac{2\alpha(G(\sigma) - G(-\sigma))}{(2 + \alpha((1-\sigma)^{\gamma-1} + (1+\sigma)^{\gamma-1}))^2}$$

where $G(\sigma) = \ln(1+\sigma)(1+\sigma)^{\gamma-1}h(1-\sigma)$ and thus $G(\sigma) > 0 > G(-\sigma)$. Property 6 follows since for $m > 0$,

$$\frac{\partial}{\partial\alpha}\ln(m(\sigma)) = \left(\alpha + \tfrac{1}{2}\alpha^2((1-\sigma)^{\gamma-1} + (1+\sigma)^{\gamma-1})\right)^{-1} > 0$$

Property 7 follows since $h(0) = 1$. For 8,

$$m'(\sigma) = 2\frac{h'(1+\sigma)h(1-\sigma) + h'(1-\sigma)h(1+\sigma)}{(h(1+\sigma) + h(1-\sigma))^2},$$

and so

$$m'(0) = \frac{2\alpha\,(\gamma-1)\,(2+2\alpha)}{(2+2\alpha)^2} = \frac{\alpha\,(\gamma-1)}{1+\alpha} \tag{15}$$

which implies that $m'(0) > 1$ if and only if $\gamma > 2 + \frac{1}{\alpha}$.

F. Proof of Theorem 1

For $\gamma \leq 2$, note that with complete markets, the equilibrium condition is that agents' marginal utilities of income are proportional. This implies that the marginal utility of income in community 1 is increasing with the marginal utility of income in community 2. Using Proposition 2, the marginal utility of income is monotone in $h(z)/z = 1/z + \alpha z^{\gamma-2}$, which is decreasing for $\gamma \leq 2$. If the marginal utility of income is decreasing in income, this implies that Z^1 is increasing in Z^2. Since $Z^1 + Z^2 = \bar{X}_0$ a constant, this implies that Z^1 and Z^2 are constant as well. Thus, both communities must fully diversify.

For $\gamma > 2+1/\alpha$, existence follows from the argument in the text. To establish uniqueness, note that from Equation (10), the condition for an equilibrium $\sigma^* > 0$ can be written as $\psi(\sigma^*) = \frac{1}{\alpha}$ where $n = \gamma - 2$ and,

$$\psi(\sigma) \equiv \frac{(1-\sigma^2)[(1+\sigma)^n - (1-\sigma)^n]}{2\sigma}.$$

Note that $\lim_{\sigma \downarrow 0} \psi(\sigma) = n$ and $\psi(1) = 0$, so that for $n = \gamma - 2 > 1/\alpha$, the continuity of ψ ensures that such σ^* exists. Rewriting the expression for ψ and simple differentiation yield:

1. $\psi(\sigma) = \nu(\sigma) + \nu(-\sigma)$ where $\nu(\sigma) = \frac{1-\sigma^2}{2\sigma}(1+\sigma)^n$,
2. $\psi'(\sigma) = -\kappa(\sigma)\nu(\sigma) + \kappa(-\sigma)\nu(-\sigma)$ where $\kappa(\sigma) = \frac{(n+1)\sigma^2 - n\sigma + 1}{\sigma(1-\sigma^2)}$,
3. $\psi''(\sigma) = -\left[\lambda(\sigma)\kappa(\sigma)\nu(\sigma) + \lambda(-\sigma)\kappa(-\sigma)\nu(-\sigma)\right]$ where $\lambda(\sigma) = \frac{2(n+1)\sigma - n}{(n+1)\sigma^2 - n\sigma + 1} - \frac{(n-2)\sigma^2 - n\sigma + 2}{\sigma(1-\sigma^2)}$.

We now argue that such σ^* is indeed unique. We focus on "critical points" $\hat{\sigma}$ which satisfy $\psi'(\hat{\sigma}) = 0$. We note that at a critical point, ψ'' reduces to:

$$\psi''(\hat{\sigma}) = -\left[\lambda(\hat{\sigma}) + \lambda(-\hat{\sigma})\right]\kappa(\hat{\sigma})\nu(\hat{\sigma}) = \frac{(n+1)n(1+\hat{\sigma})^n}{(1-\hat{\sigma}^2)((n+1)\hat{\sigma}^2 + n\hat{\sigma} + 1)}\left[n(1-\hat{\sigma}^2) - 4\right].$$

Consider the cases:

- Case 1: $n < 4$: the claim follows since $\psi'(\hat{\sigma}) = 0$ implies that $\psi''(\hat{\sigma}) < 0$. Combining with $\psi'(0) = 0, \psi''(0) < 0$ we conclude that ψ is strictly decreasing on $[0,1]$.

- Case 2: $n > 4$: $\psi'(0) = 0$ and $\psi''(0) > 0$ imply that ψ is increasing at zero. For it to satisfy $\psi(1) = 0$ it cannot be increasing on the whole $[0,1]$ interval. Hence, it hits first a local maxima $\hat{\sigma}'$ at which $\psi'(\hat{\sigma}') = 0$ and $\psi''(\hat{\sigma}') \leq 0$; it implies that $n(1-\hat{\sigma}'^2) - 4 \leq 0$. We conclude that for any other $\hat{\sigma} > \hat{\sigma}'$ for which $\psi'(\hat{\sigma}) = 0$ we have that $n(1-\hat{\sigma}^2) - 4 < 0$ and $\psi''(\hat{\sigma}) < 0$. Hence, the claim follows from ψ being strictly decreasing on $[\hat{\sigma}', 1]$.

- Case 3: $n = 4$: This case, $\gamma = 6$, is resolved by using (10) to obtain the explicit solution $\sigma = \sqrt[4]{1 - 1/(4\alpha)}$.

Finally, the comparative statics of σ^* with respect to γ and α follow from properties 5 and 6 of the reaction function.

G. Proof of Theorem 3

From Lemma 1, the indirect utility for any agent in community j is given by

$$v(w) = \frac{1}{1-\gamma} w^{1-\gamma} h(Z^j)^\gamma$$

where w is the agent's numeraire wealth. First consider the investors. From (8) and the budget constraint, in equilibrium the wealth of investor i is given by

$$w^i = \bar{z}^i Z^j,$$

where $\bar{z}^i = \bar{\theta}_1^i + \bar{\theta}_2^i$. In the fully diversified equilibrium, $Z^j = 1$. Thus, since the existence of an undiversified equilibrium implies $\gamma > 2$ (by Theorem 1) and hence that utility is negative, investor i is worse off in the undiversified equilibrium if and only if

$$E\left[(Z^j)^{1-\gamma} h(Z^j)^\gamma\right] > h(1)^\gamma. \tag{16}$$

Now, the equilibrium condition for the undiversified equilibrium is that the investor's marginal utility of income is equated across states, or equivalently, $h(Z^j)/Z^j = c$, for some constant c. Using this plus the fact that $E[Z^j] = 1$, (16) is equivalent to

$$h(Z^j)/Z^j = c > h(1).$$

Suppose $c < h(1)$. Then multiplying by Z^j and taking expectations yields

$$E[h(Z^j)] < h(1),$$

which contradicts the convexity of h for $\gamma > 2$. Thus, $c > 1$, and every investor is worse off in the undiversified equilibrium.

Next consider the laborers. For $i \in I_j^L$, using (3),

$$w^i = p_j \bar{x}_j^i = \alpha \bar{x}_j^i (Z^j)^\gamma.$$

Thus, given $\bar{x}_j^i$ and Z^j are uncorrelated, laborer i is worse off in the undiversified equilibrium if and only if

$$E\left[(Z^j)^{\gamma(1-\gamma)} h(Z^j)^\gamma\right] > h(1)^\gamma.$$

Because $\gamma > 1$, a sufficient condition is

$$E\left[(Z^j)^{(1-\gamma)} h(Z^j)\right] > h(1),$$

which follows immediately since $z^{1-\gamma}h(z) = z^{1-\gamma} + \alpha$ is convex in z for $\gamma > 1$.

H. Proof of Theorem 4

Full diversification is not an equilibrium since $m(0) = 0$ and $f(0|\omega, \hat{\sigma}) < 0$. Stability of σ^* follows by the same logic as in Theorem 2. For uniqueness, we first observe that there exists a finite M such that $m'(\sigma) < M$. This follows from the definition of m plus the fact that $h(z) > 1$ and $h'(z)$ bounded for $z \in [0, 2]$. Since $m(0) = 0 > f(0|\omega, \hat{\sigma})$ and $m(1) < 1 \leq f(1|\omega, \sigma)$, there is at least one equilibrium with $\sigma > 0$. If $1/(1-\omega) > M$, then this equilibrium must be unique. The results for $\gamma > 1$ follow immediately from the fact that m is increasing. When $\gamma > 2 + 1/\alpha$, if $m(\hat{\sigma}) > \hat{\sigma}$ then $m(\sigma^*) > \hat{\sigma}$.

I. Proof of Theorem 5

The general N-community setting is described by the parameters $(\gamma_j, \alpha_j, w_j)$ with $\sum_j w_j = 1$. An equilibrium is described by the relative price π of state 1 consumption relative to state 2 consumption, together with the expected consumption c_j and bias σ_j of each community j. That

is, given (c_j, σ_j), the global consumption of commmunity j is $c_j(1 + \sigma_j)$ in state 1, and $c_j(1 - \sigma_j)$ in state 2.

Since there is no aggregate risk, and the aggregate endowment is 1, the market clearing constraint is given by

$$\sum_j c_j = 1, \qquad \sum_j c_j \sigma_j = 0. \tag{17}$$

Given the relative price π, the budget constraint for each community is given by

$$\pi c_j(1 + \sigma_j) + c_j(1 - \sigma_j) = \pi w_j + w_j \tag{18}$$

Finally, the optimality condition for investors in community j is given by

$$\left(\frac{h^j(c_j(1+\sigma_j)/w_j)}{h^j(c_j(1-\sigma_j)/w_j)} \times \frac{1-\sigma_j}{1+\sigma_j} \right)^{\gamma_j} = \pi \tag{19}$$

where $h^j(z) = 1 + \alpha_j z^{\gamma_j - 1}$.

Given the above equilibrium conditions, one equilibrium is full diversification: $\pi = 1$ and for all j, $\sigma_j = 0$, and $c_j = w_j$. We now consider the reaction function for community 1 around this equilibrium. That is, we consider perturbing σ_1, take into account the effect of this perturbation on other equilibrium variables including prices, and then determine the optimal reaction for an individual investor in community 1. As before, an undiversified equilibrium exists as long as the slope of this reaction function is greater than 1.

Thus, we consider the endogenous variables as implicit functions of σ_1. Taking the derivative of (18) for each j implies

$$c_j'(\pi(1+\sigma_j)+(1-\sigma_j))+\pi'(c_j(1+\sigma_j)-w_j)+\sigma_j'(\pi c_j-c_j)=0.$$

At the full diversification equilibrium this reduces to

$$2c_j'=0.$$

Differentiating (17) then implies, (using $\sigma_1'=1$ and $c_j'=0$),

$$\sum_j w_j\sigma_j' = w_1+\sum_{j>1} w_j\sigma_j'=0. \tag{20}$$

Differentiating the log of (19) for $j>1$ yields (using full diversification and $c_j'=0$),

$$\pi' = 2\gamma_j\left[\frac{h^{j\prime}}{h^j}-1\right]\sigma_j' = 2\phi_j\sigma_j'.$$

Thus, we can substitute $\sigma_j'=\pi'/2\phi_j$ into (20), and solve for π' to yield

$$\pi' = \frac{-2w_1}{\sum_{j>1}\frac{w_j}{\phi_j}}. \tag{21}$$

Equation (21) gives the equilibrium price reaction to a small bias by community 1. To determine the slope of the reaction function for an individual investor in community 1, we need to solve for m_1' based on the optimality condition,

$$\left(\frac{h^1(c_1(1+\sigma_1)/w_1)}{h^1(c_1(1-\sigma_1)/w_1)}\times\frac{1-m_1}{1+m_1}\right)^{\gamma_1}=\pi \tag{22}$$

where (22) reflects the fact that an individual investor in community 1 takes the bias (and thus the state variable h^1) as given. Differentiating (22) yields,

$$m_1'(0)=\frac{h_1'}{h_1}-\frac{\pi'}{2\gamma_1}.$$

Thus, $m_1'-1>0$ if and only if $\gamma_1(m_1'-1)>0$ or

$$\gamma_1\left(\frac{h_1'}{h_1}-1\right)-\pi'/2=\phi_1-\pi'/2>0.$$

Using our expression (21) for π' then gives the condition:

$$\phi_1 + \frac{w_1}{\sum_{j>1} \frac{w_j}{\phi_j}} > 0.$$

Finally, dividing by w_1 and multiplying by $\left(\frac{w_1}{\phi_1}\right)^2 \left(\sum_{j>1} \frac{w_j}{\phi_j}\right)^2$ gives the general condition for an undiversified equilibrium for community 1:

$$\left(\frac{w_1}{\phi_1}\right) \left(\sum_{j>1} \frac{w_j}{\phi_j}\right) \left(\sum_{j} \frac{w_j}{\phi_j}\right) > 0.$$

Since community 1 was arbitrary, the same calculation can be used for any other community.

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Notes

[1]For example, in the case of local labor, due to legal restrictions and frictions such as moral hazard, one cannot sell future labor services in advance.

[2]Abel (1999) and Gali (1994) generalize the structure in Abel (1990) to allow for dependence both on current and on lagged aggregate consumption.

[3]See appendix for proofs not provided in the text.

[4]It is not necessarily the case, however, that each individual agent will hold the market portfolio, since individual endowments of the local good will differ.

[5]The assumption that laborers do not participate at all in financial markets is obviously extreme and made for simplicity. We relax this assumption in section A.

[6]The fact that the supply of local and global goods is equal is without loss of generality – differences in aggregate supplies can be equivalently accommodated through different choices of α.

[7]Since Z^j maps to p_j, this is equivalent to taking the distribution of the local good price as given, as is standard in general equilibrium.

[8]The fact that log-utility is the cutoff for herding effects is not surprising. It is analogous to the result that in inter-temporal investment problems, investors who are less risk averse than log want to have more money when investment opportunities are greater (i.e. prices are lower).

[9]Note that while there is no aggregate risk in the endowment of the local good, there may be risk in the endowments of individual laborers.

[10]When ω is small, our earlier analysis also implies that when $\gamma > 2 + \frac{1}{\alpha}$, there exists a unique equilibrium with $\sigma > 0$ (as is evident from the figure, in addition there exist multiple equilibria with $\sigma < 0$). However, there are cases with $1 < \gamma \leq 2 + \frac{1}{\alpha}$ when multiple equilibria with $\sigma > 0$ exist, some of these being unstable. In this case we focus on the largest equilibrium, which is guaranteed to be stable.

[11]Note that the trade $\{-b, b\}$ has cost zero (with symmetric prices) and so satisfies the budget constraint.

[12]In this framework we assumed that a fraction l is unconstrained and the remainder are fully constrained. However, one can also allow for partially constrained agents. A natural constraint, for example, is that the position is "capped" by some amount $\bar{b}$ which may depend on the equilibrium σ. In this case, $m_l(\sigma) = m(\sigma) - \min(b(\sigma), \bar{b}(\sigma))$. It is easy to show that the undiversified equilibrium persists in this alternative specification as well, as long as $\bar{b}(\sigma)$ is not too large. One possible choice is that $\bar{b} = P(1-\sigma)$, the amount of riskless borrowing a laborer can conduct.

[13]Formally, this follows since with $c > 0$, $m_c'(0) = m'(0)$.

[14]Formally, marginal utility of income for the representative agent is $u'(1)/Z^j$, which is strictly decreasing in Z^j (and is identical to log utility). Thus, full diversification is the unique equilibrium.

[15]The fact that w_j specifies community j's share of *both* goods is without loss of generality,

since we allow α_j, the importance of the local good, to vary across communities. A relative scarcity of the local good in community j is equivalent to a higher choice of α_j.

[16]In related work, we examine the consequences of community effects on asset prices in a dynamic context. We present here a simplified version of this model that demonstrates the potential implications.

[17]The second equality follows immediately from the Hansen-Jagannathan bound, interpreting π as the stochastic discount factor.

[18]To see why, note that in our model, the expenditure ratio

$$q = p_j \bar{X}_j / (p_0 Z^j) = \alpha (Z^j / \bar{X}_j)^{\gamma - 1}.$$

Taking full diversification as the benchmark and without loss of generality normalizing $Z = 1$ and $\bar{X}_j = 1$ (this is nothing more than an arbitrary choice of units for each good), $q = \alpha$.

[19]Note that pure nominal inflation is not relevant in our model; what matters is the change in the *relative price* of the local goods, for which inflation might be a proxy if purchasing power parity holds more closely for traded goods.

[20]Not surprisingly papers such as Cooper and Kaplanis (1994) that use *monthly* international data do not find support to the idea that local stocks provide a better hedge against inflation.

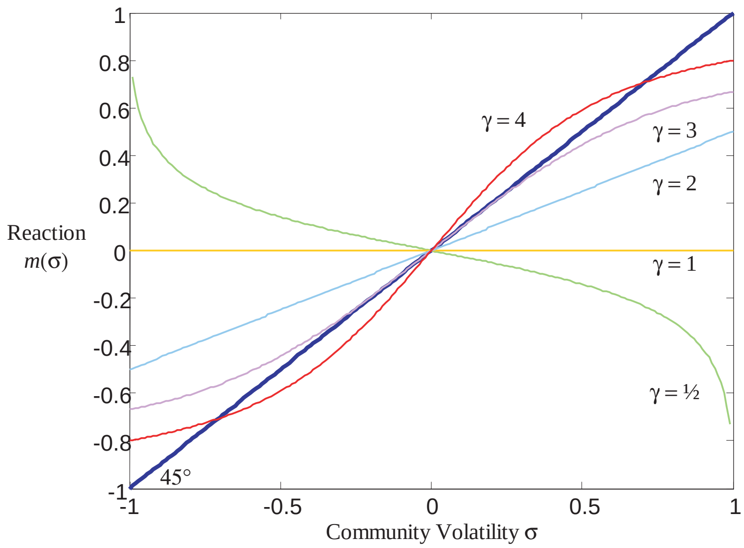

Figure 1. Optimal Portfolio Choice σ^i given Community Volatility σ ($\alpha = 1$)

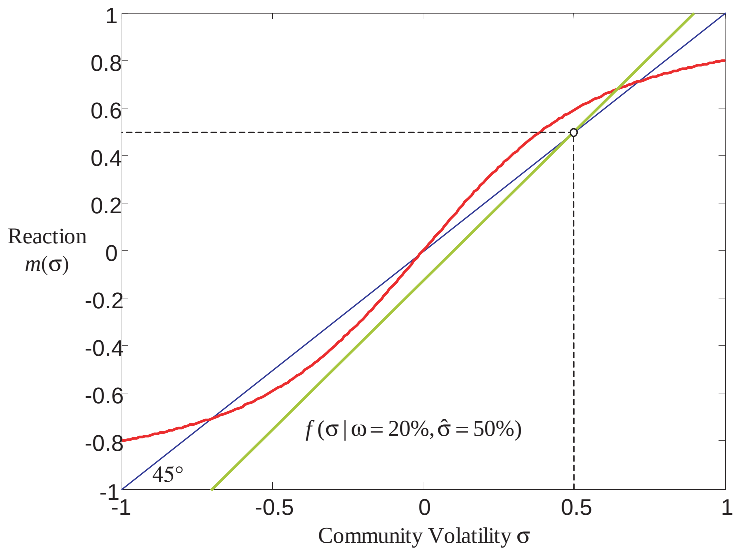

Figure 2. Equilibrium with Constrained Investors

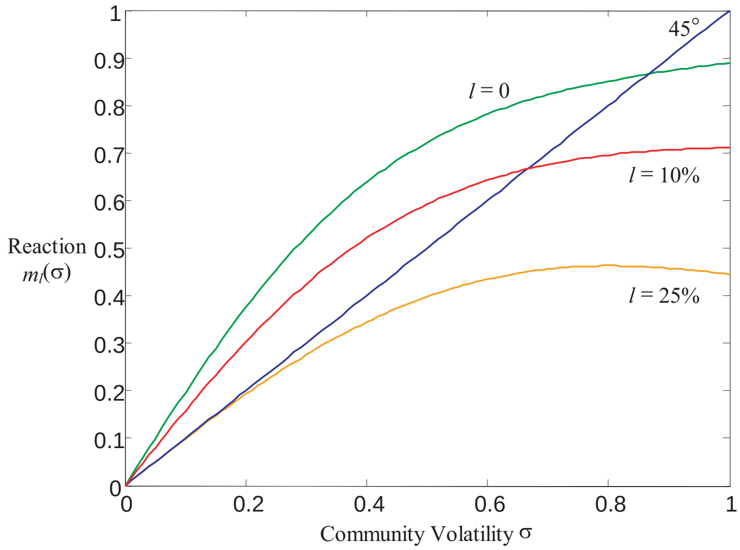

Figure 3. Equilibrium with Unconstrained Holders of the Local Good

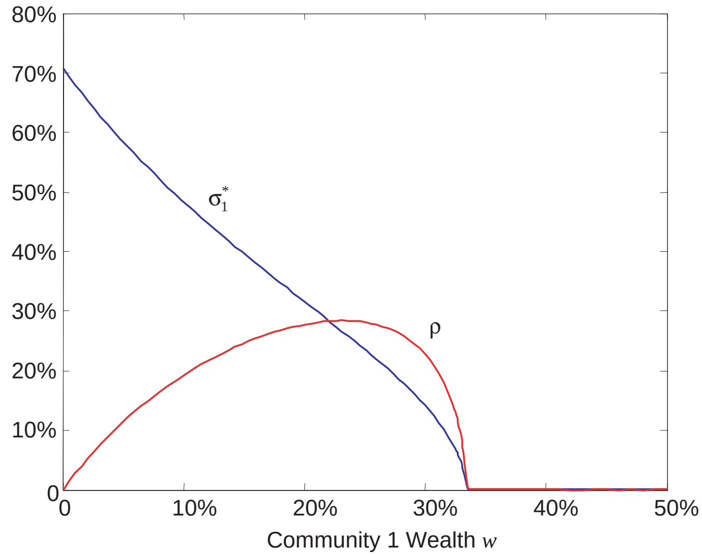

Figure 4. Equilibrium Volatility and Sharpe Ratio vs. Community Size

Captions

Figure 1

Optimal Portfolio Choice σ^i given Community Volatility σ ($\alpha = 1$)

Figure 2

Equilibrium with Constrained Investors

Figure 3

Equilibrium with Unconstrained Holders of the Local Good

Figure 4

Equilibrium Volatility and Sharpe Ratio vs. Community Size