In this appendix, we formally state and prove the results stated in the text.

0.1 Model

The model is composed of three agents: the Shareholders, the Acquirer, and the Insiders.

The value of the target firm is v, which is distributed according to the cumulative distribution function F with a continuous, smooth partial distribution function f on $[v_{\min}, v_{\max}]$. Let $\bar{v}$ be the mean of the distribution. The true value of the firm v is known by the Insiders but is not known by the Shareholders or the Acquirer.

If the target is bought at a price p, then the Acquirer receives $v + s - p$, where $s \geq 0$ is the synergy that the Acquirer has with the target firm; otherwise the Acquirer receives 0. If the firm is bought, the Shareholders obtain p, and receive v otherwise. Finally, the Insiders obtain $p - b$ when the firm is sold and v when the firm is not sold. We assume that $b > 0$: that is, the Insiders strictly prefer to not sell the firm when the price offered is equal to the value of the firm.

We consider three games in the subsections below. The solution concept is Nash Equilibrium. However, occasionally the equilibrium will be unique except for a measure 0 set of realizations of the random variable v, and in that case we will say that we have a generically unique subgame perfect Nash equilibrium. We also make several statements that describe how a set changes in response to a parameter. In this case, a set $A(t)$ is weakly increasing in t if

$$\min A(t) \leq \min A(t') \text{ and } \max A(t) \leq \max A(t')$$

for all $t < t'$.

0.2 The No Poison Pill Case

In this case, the game proceeds as follows:

1. The Acquirer makes an offer p.
2. The Shareholders decide to accept or reject.
3. Payoffs are realized.

It is clear that in the second stage that Shareholders will strictly prefer to sell for any $p - b > v$ and will strictly prefer not to sell for any $p - b < v$, and are indifferent at $p - b = v$. Hence we have the following theorem:

Proposition 1 *The unique subgame perfect equilibrium of the no poison pill game is for the Acquirer to offer $\bar{v}$ to the Shareholders and for Shareholders to accept.*

0.3 The Poison Pill and Effective Staggered Board Case

In this case, the game proceeds as follows:

1. The Acquirer makes an offer p.
2. The Insiders decide to accept or reject.
3. Payoffs are realized.

It is clear that in the second stage the Insiders will strictly prefer to sell for any $p - b > v$ and will strictly prefer not to sell for any $p - b < v$. Hence, the problem of the Acquirer in the first stage is to solve

$$P^* \equiv \arg \max_{p \geq v_{\min}} \int_{v_{\min}}^{p-b} (v + s - p) f(v) \, dv \tag{1}$$

Proposition 2 *In any subgame perfect Nash equilibria of the poison pill and effective staggered board game the Acquirer offers $p^* \in P^*$ and the Insiders accept if $p^* > v + b$ and reject if $p^* < v + b$.*

Corollary 3 *If f is a weakly decreasing function on its domain, there is a generically unique subgame perfect Nash equilibrium of the poison pill as $|P^*| = 1$. If $s > v_{\min} + b$, then $p^* \in P^*$ is interior, and it is given by the solution to*

$$(s-b)\, f(p^* - b) - F(p^* - b) = 0.$$

The corollary follows as the second-order condition of the Acquirer's problem is $(s-b)\, f'(p-b) - f(p) < 0$ for all p if f is a weakly decreasing function. Furthermore, if $s > b$, then $b + v_{\min}$ is clearly nonoptimal, as $b + v_{\min} + \frac{s}{2}$ provides strictly positive utility in expectation, while $b + v_{\min}$ provides exactly 0 utility in expectation.

For a uniform distribution, for example, the optimal offer is $s + v_{\min}$ if $s \geq b$. For an exponential distribution $f(v) = \lambda e^{-\lambda v}$ on $[0, \infty]$, the optimal offer by the Acquirer is given by

$$\begin{aligned}
(s-b)\, \lambda e^{-\lambda(p^*-b)} - \left(1 - e^{-\lambda(p^*-b)}\right) &= 0 \\
-\lambda (p^* - b) &= \ln\left(\frac{1}{\lambda(s-b)+1}\right) \\
p^* &= \frac{1}{\lambda} \ln(\lambda(s-b)+1) + b
\end{aligned}$$

if $s \geq b$.

Given the first-order condition in the above proposition, it is easy to show two properties of the solution. First, since

$$\frac{\partial^2}{\partial s \partial p} \int_{v_{\min}}^{p-b} (v + s - p)\, f(v)\, dv = f(p-b) \geq 0$$

we have by Topkis's theorem the following:

Proposition 4 *In the poison pill and effective staggered board game, the set of optimal offers P^* by the Acquirer is weakly increasing in the synergy s.*

Further, if $s < b$, then the first-order condition is always negative, and so we have the following:

Proposition 5 *In the poison pill and effective staggered board game, if $s < b$, it is optimal for the Acquirer to make an offer that will never be accepted.*

Finally, we can take the cross-partial with respect to b and p to obtain

$$\frac{\partial^2}{\partial b \partial p} \int_{v_{\min}}^{p-b} (v + s - p)\, f(v)\, dv = -(s-b)\, f'(p-b)$$

which is positive if f is decreasing and $s > b$, and so (again by Topkis's theorem) in this case the optimal offer will be increasing with b.

Proposition 6 *In the poison pill and effective staggered board game, if f is a weakly decreasing function and $s > b$, then the unique optimal offer p^* by the firm is weakly increasing in the private benefits b.*

We also have that the optimal choice of b for the Shareholders may be nonzero. This is because b has two effects. First, for a fixed price offering, a larger b increases the range where the Insiders choose to not sell the firm even though the Shareholders would wish them to do so. However, as shown above, a larger b may also increase the offered price.

Consider the case when $f(v) = e^{-v}$ on $[0, \infty]$. Then the optimal offer by the Acquirer is given by $p^* = \ln(1 + s - b) + b$ from the calculations above. Hence, for a given synergy s, the payoff to the Shareholder is given by

$$\int_0^{\ln(1+s-b)} p^* f(v)\, dv + \int_{\ln(1+s-b)}^{\infty} v f(v)\, dv$$
$$\ln(1 + s - b) + \frac{1 + b(s-b)}{1 + s - b}$$

At $b = 0$, the derivative of this expression is positive with respect to b, showing that the optimal private benefits are higher than zero from the perspective of the Shareholders. Indeed one can solve for the optimal benefits from the perspective of the Shareholder, and obtain

$$b^* = \frac{1 + 2s - \sqrt{1 + 4s}}{2} > 0$$

0.4 The Poison Pill without an Effective Staggered Board Case

In this case, the game proceeds as follows:

1. The Acquirer makes an offer p.
2. The Insiders choose to accept or reject.
3. If the Insiders reject, the Shareholders choose whether or not to overrule the Insiders and accept the offer.
4. Payoffs are realized.

Let us first consider the subgame after the Acquirer has offered p. Note that for the Insiders, it is a weakly dominant strategy to accept any offer $p > v + b$. It is also weakly dominant for the Insiders to reject any offer $p < v + b$. Given this, it is strictly optimal for the Shareholders to reject whenever the Insiders reject if

$$\tilde{v}(p) \equiv \frac{\int_{p-b}^{v_{\max}} v f(v)\, dv}{\int_{p-b}^{v_{\max}} f(v)\, dv} > p.$$

$\tilde{v}(p)$ is the expected value of the firm, conditional upon the Insiders rejecting playing the weakly undominated strategy outlined above. Hence, if this condition holds, the generically unique weakly undominated Nash equilibrium of the subgame, given an offer p, is for the Insiders to reject the offer if $p < v + b$, and to accept it otherwise, and for the Shareholders to support their decision.

Hence, consider $\hat{p}$ defined by

$$\hat{p} \equiv \min \{p \in \mathbb{R} : \tilde{v}(p) \leq p\}$$

At this price, even if the Insiders suggest rejection, the expected value of the firm is weakly less than the offered price. Hence, in any weakly undominated subgame perfect Nash equilibrium the Shareholders will overrule the Insiders and accept the offer.

The existence of a price $\hat{p}$ at which the Shareholders will always accept the offer means that the Acquirer can now offer the price $\hat{p}$ and obtain the firm for sure. Hence, the Acquirer will either offer a price $p^* \in P^*$ and obtain the firm only if the Insiders agree, or the higher price $\hat{p}$ and obtain the firm for sure.

Proposition 7 *In any weakly undominated subgame perfect Nash equilibrium of the poison pill with no effective staggered board game, the Acquirer either*

- *Offers the price $\hat{p}$, which is then accepted by the Shareholders if rejected by the Insiders, or*
- *Offers $p^* \in P^*$ and for the Insiders to accept if $p^* > v + b$ and reject if $p^* < v + b$ with the Shareholders supporting the decision of the Insiders. In this case, $p^* < \hat{p}$.*

We will call the strategy elucidated in the first bullet point the Shareholder-oriented strategy, and the strategy elucidated in the second bullet point the Insider-oriented strategy.

This proposition then allows us to show that the synergy increases the offer by the firm. Clearly, as synergy increases, if the firm uses the same strategy, the price offered will either stay the same ($\hat{p}$) or increase (as in Proposition 3). Since $p^* < \hat{p}$, if the firm goes from an Insider-centric strategy to a Shareholder-centric strategy, the price will also increase. Finally, it is clear that the firm will never switch from a Shareholder-centric strategy to an insider-centric strategy as s increases. This is because the value the Acquirer obtains from a Shareholder-centric strategy increase one-to-one with the synergy, while only increasing at the rate

$F(p^* - b)$ with respect to the insider-centric strategy. To see this, note that the insider-centric strategy has an expected value of

$$\int_{v_{\min}}^{p^*(s)-b} (v + s - p^*(s)) f(v)\, dv$$

and so, by the envelope theorem, the derivative of this with respect to s is simply

$$\int_{v_{\min}}^{p^*(s)-b} f(v)\, dv = F(p^*(s) - b) < 1$$

So we have:

Proposition 8 *The set of optimal offers by the firm is weakly increasing in the synergy s. In particular, if the firm chooses the Shareholder-oriented strategy for a synergy level s, it will also choose the Shareholder-oriented strategy for any synergy greater than s.*

0.5 Optimal Level of Takeover Defenses

Shareholder preferences for defense levels depends on parameters. For instance, consider the case where the value of the firm is uniformly distributed between 250 and 750, and the private benefits are 100. Then if the synergy is low, e.g. 300, the Shareholders prefer no effecitve staggered board, as they obtain an expected utility of 650 without the effective staggered board and 580 with the effective staggered board, while if the synergy is 500, then the Shareholders still obtain an expected utility of 650 without the effective staggered board (as the Acquirer still always buys the firm), but now obtains 740 with the effective staggered board.

However, we can characterize the kind of firms where Shareholders prefer an effective staggered board. In particular, firms that expect a large s are exactly those for which the enhanced bargaining power of the effective staggered board is helpful for Shareholders.

Proposition 9 *There exists an $\bar{s}$ such that for all $s > \bar{s}$, the Shareholders prefer to have an effective staggered board.*

The logic of the proposition is straightforward. Suppose that Shareholders do prefer an effective staggered board for a given s, and consider a given $\dot{s} > s$. Then, since the Shareholders strictly prefer the effective staggered board for the synergy s, the firm would make a Shareholder-oriented offer to the target without the effective staggered board, and hence would also do so for synergy $\dot{s}$. Then, with the effective staggered board, we know that the Acquirer will make a strictly larger offer for synergy $\dot{s}$ to the firm by Proposition 3, and so we just need to show that shareholders prefer larger offers in the poison pill with an effective staggered board game.

However, it is straightforward that the shareholders always prefer a higher price be offered. Let the price the Acquirer offers for synergy s ($\dot{s}$) be denoted p ($\dot{p}$). If the true value of the firm is such that the Insiders will not sell even at $\dot{p}$, the increased synergy had no effect on the outcome for the shareholders. If the true value of the firm was such that the Insiders would sell for either p or $\dot{p}$, then the Shareholders are better off as they receive a higher price. Finally, consider the interval of values for the firm where at the price p, the Insiders would not sell, but do sell at price $\dot{p}$. For these values, for synergy s the Shareholders receive only v, while for $\dot{s}$, they receive $\dot{p} > v + b$, and so the Shareholders are better off in this final scenario as well.

More generally, we can calculate the optimal choice of defenses from the perspective of the Shareholders. If parameter values are such that the Acquirer will make an Insider-oriented offer when there is no effective staggered board, then the existence of an effective staggered board has no effect on Shareholders' utility However, if the Acquirer makes a Shareholder-oriented offer when there is no effective staggered board, then there is a difference in welfare for the Shareholders between the two regimes. In that case, the Shareholders' prefer an effective staggered board if and only if

$$p^* F(p^* - b) + \int_{p^*-b}^{v_{\max}} v f(v)\, dv > \hat{p}$$

where p^* is the offer made by the firm when there is an effective staggered board.

Furthermore, it is clear that if shareholders prefer an effective staggered board, then the Insiders do as well, as Insiders prefer an effective staggered if and only if

$$(p^* - b)\, F\,(p^* - b) + \int_{p^*-b}^{v_{\max}} v f\,(v)\, dv > \hat{p} - b$$

and so we have the following:

Proposition 10 *If shareholders prefer an effective staggered board, then Insiders prefer an effective staggered board.*

However, it is not the case that Insiders always prefer an effective staggered board, as shown in an example in the text in part III.E.1.