Revenue Decentralization, the Local Income Tax Deduction, and the Provision of Public Goods

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Abstract

We consider a model where local and national governments invest in both productive and consumptive public goods using income tax revenue. Local governments will overprovide the consumptive public good if the local income tax is (fully or partially) deductible. However, without full deductibility, local governments will underprovide local productive public goods. Hence, to reduce the distortions in the local governments' decisions, a welfare-maximizing national government will underinvest in both types of public goods. We also consider an alternative fiscal structure where the national government sets one national tax rate and provides transfers to the local governments: This results in lower welfare than one where local governments raise revenue independently.

JEL classification codes: H21, H23, H71, H72, H77

1 Introduction

In the United States, the federal income tax deduction for state and local income taxes costs the federal government approximately fifty billion dollars annually in revenue.[1] Indeed, the recent President's Advisory Panel on Federal Tax Reform (2005) has suggested that this deduction should be eliminated, arguing that the deduction is expensive for the federal government and inappropriately provides incentives for state and local expenditures that in many cases reflect personal consumption expenditures. This paper shows that the local income tax deduction is not necessarily inefficient: its efficiency or inefficiency depends on the nature of the local public goods. Local consumptive public goods will be overprovided relative to the efficient level with the local income tax deduction, while local productive public goods will be underprovided without it, and so national policymakers face a trade-off when choosing the level of deductibility of local income taxes. However, the misalignment in local governments' incentives can not be corrected by having the national government set one national tax rate and provide lump sum transfers to local governments; this actually exacerbates the misalignment.

The local income tax deduction induces local governments to spend more on public goods, as the cost to local taxpayers is reduced; for every $1 spent on public goods, residents who itemize deductions on their federal tax returns will only pay a fraction of that, since they will not have to pay federal taxes on the income lost to local taxation. Hence, in the absence of important national external benefits, spending on local consumptive public goods will be inefficiently high, since the local residents do not bear the full burden of the local spending. This intuition is the basis of many articles decrying this "tax break."[2] This intuition is also, in some respects, similar to that for soft budget constraints[3]: with a soft budget constraint, or with a local

[1] See Joint Committee on Taxation (2008).
[2] See Mitchell (2005).
[3] See Qian and Roland (1998) and references contained therein.

income tax deduction, the national government will pay for some investment by local governments, and that leads local governments to overinvest in consumptive public goods.

However, for productive public goods, this intuition is reversed. Suppose there is no local income tax deduction, and consider the investment choice by local governments in a local productive public good that raises local incomes (and hence national income tax revenues). Without a local income tax deduction, local governments will underinvest in these public goods, since some of the gains from such investment will be taken by the national government through the income tax, while citizens of the local government must pay all of the costs. However, if the national government enacts a local income tax deduction, the local government also no longer pays all of the costs of investment in productive public goods. While the local government still does not capture all of the gains of investment, these forces exactly balance out so that it is optimal for the local government to invest an efficient amount in local productive public goods. This is somewhat analogous to the difference between an income tax and a consumption tax: implementing a local income tax deduction, like switching from an income tax to a consumption tax, no longer taxes agents (i.e. local governments) on income used for investment. Many of the most important responsibilities of local governments are productive public goods: e.g., roads, fire protection, and police and the courts.[4]

For any level of deductibility of local income taxes the local government's spending decisions will not be first-best. A welfare-maximizing national government must balance the incentives it gives local governments through the tax code to under- or overinvest in the two categories of public goods. Underinvestment in local productive public goods may be particularly costly as it reduces the income of the citizenry

[4]For instance, the vast majority of spending on police protection is done by local governments: Local governments spent approximately $207bn on police protection, judicial costs, and corrections in 2008 (US Census, 2011), while the Department of Justice only spent $21bn (DoJ, 2009).

directly and indirectly, as the national government must now impose a higher tax rate (and hence use a more distortionary tax system) to finance a particular level of national government expenditures. The key issue is the inability of the national government to distinguish between local productive and consumptive goods. If the national government is able to distinguish between the two types of goods, a local income tax deduction would be wasteful: the national government could instead implement an appropriate matching grant for local productive public goods to obtain the first-best.

We also consider the effect of this distortion on the incentives facing a welfare-maximizing national government. The size of the inefficiency will depend on the national income tax rate: the higher the national income tax rate, the larger the distortion in the incentives of the local governments. Hence, a welfare-maximizing national government will take these effects into account when setting its own fiscal policy, and we show that it will underinvest in both productive and consumptive public goods.[5]

The local income tax deduction can be thought of as a simple subsidy for local spending by the national government. While for a purely consumptive local public good, such a subsidy would induce local governments to spend too much on public goods, here it can be thought of as helping to correct the distortion in local governments' incentives to invest in productive public goods from a national income tax. Hence, the local income tax deduction may be a second-best solution to the national government's problem.

If there is only one type of local public good that is both productive and consumptive, we show that an appropriately chosen local income tax deduction can achieve the first-best. Without any deduction, the local government will spend too little, and

[5]This last result holds so long as national spending has no direct effect on the incentives of local governments. If, for instance, local and national consumptive public goods act as substitutes, the national government may overinvest in the national consumptive public good to reduce the incentives of a local government to invest in the local consumptive public good.

with a full deduction, the local government will spend too much—hence the national government can find a partial deduction which induces the local government to spend at an efficient level, but this result would disappear with multiple local public goods. We also show that if the national government can provide a matching grant for productive public goods, it can also achieve the first-best, since the national government can independently change the local government's decision regarding productive and consumptive public goods.

We also consider an alternative fiscal form, where all tax revenues are collected by the national government, and transfers are provided to local governments to pay for local public goods. Hence, local governments do not decide how much to spend, but do decide the distribution of spending on the two types of lcoal public goods. Versions of this system are prevalent in much of the world, but we find that a system where the national government has sole taxing authority is worse than one with local revenue generation.[6] The national government will choose a higher national tax rate when it must also fund expenditures by local governments, and since the size of the distortion in local govenments' incentives depends on the size of the national income tax rate, the local governments will choose inferior policies in this case.

Indeed, a stronger result can be shown: It is never optimal for a national government to provide pure transfers to the local governments when local governments have the ability to tax. Local governments will simply use these transfers to reduce their own local tax rates and refund the money to their citizens. Instead, the national government can simply reduce its tax rate, which refunds the money to the citizens directly, and at the same time reduces the distortion in the incentives for local governments.[7]

Surprisingly, for such an important provision of the tax code, the local income tax

[6] Versions of this fiscal form are used in many Latin American countries, including Argentina and Peru.

[7] Boadway and Keen (1996) also provide a model where negative transfers are optimal, though for different reasons than in our model.

deduction has received scant attention in the public finance literature. It is discussed by Feldstein and Metcalf (1987), but there they focus on the effects on the choice of local tax instruments, not on the effects on the efficiency of investment in local public goods. These effects have been discussed more comprehensively, although less formally, in the law literature: see Billman and Cunningham (1985). Gramlich (1985) mentions the possible efficiency gains by eliminating the deduction (as he considers only consumptive public goods) but his argument concentrates on the redistributive consequences of eliminating the deduction. Gordon (1983) mentions that the local income tax deduction may be a solution to distortions in local governments' incentives to underinvest in public goods from spillovers; in our case, the distortion in local governments' incentives to invest in the productive public good is the presence of the national income tax itself.

However, there is a large literature on vertical fiscal externalities, which should be seen as complementary to our work here. Beginning with Cassing and Hillman (1982), and recently summarized by Keen (1998), this literature considers the problem of overlapping jurisdictions that have the power to use the same distortive tax instrument. This "fiscal externality" leads to suboptimal taxation policy. There are two central issues to be considered when determining the effects on the national government of actions by subnational units. The first issue regards the choice and level of taxation instruments, and the fact that these taxes may distort individual agents' economic decisions. This issue, first analyzed by Flowers (1988) and subsequently by Keen and Kotsogiannis (2002), shows that local governments will not consider the effects on federal revenues when setting their own tax rates, and so the combined tax rate of national and subnational governments is higher than it would be under a unitary government. Hoyt (2001) shows that these problems can be mitigated via matching grants. However, our work is concerned with the second issue, i.e. how the taxing decisions of the national government distort the local governments' spending decisions, rather than how giving the local and national governments access to the same distortive tax leads to suboptimal outcomes. Hence, we analyze how the national government's taxation and spending decisions affect the level and composition of spending by local governments; many of these effects are discussed in Dahlby (1996).[8]

The paper most closely related to ours is Dahlby and Wilson (2003). They consider a model similar to ours (but without a local income tax deduction) and show that, for a large set of parameters, localities will overinvest in consumptive public goods, and underinvest in productive public goods. We show that the local income tax deduction can mitigate the inefficiencies identified by Dahlby and Wilson, and that a full local income tax deduction induces efficient investment in local productive public goods. More generally, we show that the local income tax deduction should be chosen to balance the distortive effects of national taxation on local investment in productive and consumptive public goods. We also consider the general equilibrium effects of the fiscal externality between the two levels of government, showing that the national government will underinvest in both productive and consumptive public goods. The national government does this to reduce the fiscal externality on local governments and therefore induce more efficient local spending patterns. Finally, we consider when the government should use a local income tax deduction, and whether other fiscal forms may help solve this problem.

The paper is structured as follows. The next section introduces the model, and provides conditions for optimality and equilibrium. Section 3 characterizes the equilibrium policy under decentralized revenue collection; Section 4 characterizes policy under centralized revenue collection. Section 5 shows that the national government can always do better, if allowed to choose the level of deductibility of local taxes, under decentralized revenue collection than centralized revenue collection; as a corol-

[8] Similarly, in Johnson (1988), states engage in excessive redistribution in order to reduce the tax burden of their citizens.

lary, with decentralized revenue collection the national government should not make positive transfers to local governments. The final section concludes. All proofs may be found in the Appendix.

2 Model

We consider an economy with two levels of government, national and local. The national government in this model is beneficent: it chooses policy so as to maximize the welfare of the representative agent. The local government is also beneficent: it chooses policy to maximize the welfare of the representative agent of its district.[9] For simplicity, we assume that all districts are identical and normalize the population within each district to 1.[10]

The national government decides on a national income tax rate $\tau_n \in [0, 1]$, a local income tax deduction rate $\alpha \in [0, 1]$, and per capita levels of investment in a productive public good $p_n \geq 0$ and consumptive public good, $g_n \geq 0$. The national government must also balance its budget, taking into account the subsequent decisions of the local government.

A local government chooses policy for its district, taking the national level of income taxation, the local income tax deduction rate, and national investment in public goods as fixed. In other words, a local government acts as a "policy-taker": A district is small compared to the national economy, so a local government does not consider the effect on national finances of its decisions. Hence we can model the actions of local governments by considering one representative local government.

[9]The local governments in this model could as state, county or municipal governments in the U.S. context, or as provinces or subprovincial units in other nations. We shall refer to the jurisdictions of local governments throughout as districts for brevity.

[10]We model the governments as beneficent for simplicity. A more realistic model could be built which explicitly models the political process. However, so long as this political process produces a Condorcet winner when one exists, the results will be unchanged, as all agents are identical. We could also allow for agents to be differentiated according to a parameter that determines their productivity: using the methodology of Lindbeck and Weibull (1987) to model political compeititon would also produce the same results.

This local government chooses a local income tax $\tau_l \in [0,1]$ and per capita levels of investment in a productive public good $p_l \geq 0$ and consumptive public good $g_l \geq 0$. Finally, after all policy choices have been made, agents produce and then consume their private consumption and the consumptive public good.

The marginal productivity of labor within each district is given by $F(p_l, p_n)$, which is strictly increasing, strictly concave, twice continuously differentiable and satisfies the Inada conditions in each variable.[11] We say that p_l and p_n are weak substitutes if the cross-partial of $F(p_l, p_n)$ is not positive. Finally, to simplify the analysis, we assume that the labor supply of the representative agent within each district is fixed at 1, in order to concentrate on intergovernmental inefficiencies.

We can now write the budget constraints faced by the national and local governments. The local government faces a budget constraint

$$F(p_l, p_n)\tau_l = p_l + g_l.$$

However, the national government, given the income tax rates τ_n and τ_l, remits back to the citizens an amount equal to $\alpha \in [0,1]$ times the tax paid on the income those citizens lost to local taxation; if $\alpha = 0$, there is no local income tax deduction, and if $\alpha = 1$, local income taxes are fully deductible. The budget constraint of the national government is

$$F(p_l, p_n)\tau_n(1 - \alpha\tau_l) = p_n + g_n.$$

An agent's consumption is given by

$$c = F(p_l, p_n)\left(1 - \tau_n - \tau_l + \alpha\tau_n\tau_l\right)$$

and each agent has a utility function given by

$$u(c) + H(g_l, g_n)$$

[11]This last condition is not necessary, but greatly simplifies the analysis in that it eliminates the need to consider boundary conditions.

where H denotes the level of utility obtained from investment in local and national consumptive public goods, while $u(c)$ is the level of utility the agent obtains from private consumption. These functions are strictly increasing, concave, twice continuously differentiable and satisfy the Inada conditions. We shall say that g_l and g_n are weak complements if the cross-partial of $H(g_l, g_n)$ is not negative.

2.1 Welfare Optimality

We first characterize the optimal choice of policy from the point of view of the representative agent. Consider the problem of a social planner who can decide on both τ_n and τ_l, as well as the investment levels in both national and local public goods, subject only to the budget constraints of the governments. The social planner will solve

$$\max_{\tau_l, p_l, g_l, \alpha, \tau_n, p_n, g_n} \left\{ u\left(F(p_l, p_n)\left(1 - \tau_n - \tau_l + \alpha\tau_n\tau_l\right)\right) + H(g_l, g_n) \right\}$$

subject to

$$\begin{aligned} F(p_l, p_n)\tau_l &= p_l + g_l \\ F(p_l, p_n)\tau_n &= p_n + g_n + \alpha\tau_l\tau_n F(p_l, p_n). \end{aligned}$$

By substituting the budget constraints into the maximization problem, we have that the social planner solves:

$$\max_{p_l, p_n, g_l, g_n} \left\{ u\left(F(p_l, p_n) - (p_l + p_n + g_l + g_n)\right) + H(g_l, g_n) \right\}$$

Hence, the social planner simply chooses the optimal level of investment in each of the public goods, taking into account the resulting decrease in consumption.

The first-order conditions of the social planner's problem are[12]:

$$\begin{aligned} F_l\left(p_l^*, p_n^*\right) &= 1 \\ F_n\left(p_l^*, p_n^*\right) &= 1 \\ \frac{H_l\left(g_l^*, g_n^*\right)}{u'(c)} &= 1 \\ \frac{H_n\left(g_l^*, g_n^*\right)}{u'(c)} &= 1 \end{aligned}$$

The first two equations state that at the optimal policy, the marginal increase in production by increasing public investment is exactly offset by the cost of that investment. The third and fourth equations are essentially Samuelson conditions for investment in the consumptive public good. We shall refer to the investment levels defined above as first-best.

2.2 Equilibrium

The national government chooses a level of investment in both productive and consumptive public goods, as well a national income tax rate and local income tax deduction. In so doing, the national government must ensure that its choices satisfy the national budget constraint, taking into account the decision local governments will make. The local governments choose their levels of investment in productive and consumptive public goods, as well as their tax rate, taking into account their own budget constraints, and the investment levels and tax policies of the national government. We assume that a district is small—that is, the local government does not take into account the effect of its own policies on the national government's spending decisions and taxation rate, but rather takes these as given when making its decision.

Hence, an equilibrium is a set of investment levels $\{p_l, p_n, g_l, g_n\}$ and tax policies $\{\tau_l, \tau_n, \alpha\}$ such that:

[12]We denote the derivatives of $F\left(p_l, p_n\right)$ with respect to p_l and p_n by $F_l\left(p_l, p_n\right)$ and $F_n\left(p_l, p_n\right)$, respectively. $H_l\left(g_l, g_n\right)$ and $H_n\left(g_l, g_n\right)$ are similarly defined.

1. The local government maximizes the welfare of the representative agent within its district taking the national income tax rate and national levels of investment in the national public goods as fixed. The local government solves:

$$\max_{\tau_l, p_l, g_l} \{u(F(p_l, p_n)(1 - \tau_n - \tau_l + \alpha\tau_n\tau_l)) + H(g_l, g_n)\} \tag{1}$$

   subject to the budget constraint that

$$F(p_l, p_n)\tau_l = p_l + g_l \tag{2}$$

   taking the tax rate τ_n and investment decisions p_n and g_n of the national government as given.

2. The national government maximizes the welfare of the representative agent taking the districts' response to the national government's policy choices as given. The national government solves

$$\max_{\alpha, \tau_n, p_n, g_n} \{u(F(p_l, p_n)(1 - \tau_n - \tau_l + \alpha\tau_n\tau_l)) + H(g_l, g_n)\} \tag{3}$$

   subject to the budget constraint that

$$F(p_l, p_n)\tau_n(1 - \alpha\tau_l) = p_n + g_n \tag{4}$$

   taking the tax rate τ_l and investment decisions p_l and g_l of the local government as functions of the national government's decisions, as calculated in Condition 1 of the equilibrium definition.

3 Policy under the Decentralized Revenue Mechanism

We first consider the policy decisions made by local governments, who take both the national tax rate and the provision of national public goods as given.

3.1 The Problem of the Local Government

Substituting the local budget constraint (2) into the local maximization problem (1), the problem for the local government is

$$\max_{p_l, g_l} \left\{ u\left(F(p_l, p_n)\left(1 - \tau_n\right) - \left(1 - \alpha\tau_n\right)\left(p_l + g_l\right)\right) + H(g_l, g_n)\right\}. \tag{5}$$

The first order condition for this problem with respect to investment in the local productive public good is

$$F_l(p_l, p_n) = \frac{1 - \alpha\tau_n}{1 - \tau_n}. \tag{6}$$

If $\alpha < 1$, the local productive public good is underprovided, relative to the optimal level of provision $p_l^*(p_n)$. The local government discounts part of the return from investing in the local productive public good as some of the benefits of that investment go to the national government. The national income tax has created a fiscal "tragedy of the commons"—a local government will not invest in up to the point where the total marginal benefit is equal to the total marginal cost, as some of the returns from such investment are captured by other districts in the form of the national provision of public goods.

If $\alpha = 1$, then the local productive public good is provided at efficient levels. An increase in taxes of ε to increase p_l costs district residents $\approx \left(1 - \alpha\tau_n\right)\varepsilon$ and results in an increase in local income of $\approx F_l(p_l, p_n)\left(1 - \tau_n\right)\varepsilon$. Hence, local governments will choose to have $F_l(p_l, p_n) = 1$ and will invest efficiently in the local productive public good if and only if $\alpha = 1$.

We can also calculate how a change in the tax rate τ_n (holding fixed α, p_n, and g_n) affects local investment in productive public goods. As τ_n increases, $\frac{1-\alpha\tau_n}{1-\tau_n}$ increases if $\alpha < 1$. Hence, from (6) we have that $F_l(p_l, p_n)$ is increasing in τ_n, and since $F(\cdot)$ is concave in p_l, the investment by the local government in productive public goods is decreasing in τ_n. Intuitively, as the national tax rate increases, the fraction of returns from local public investment that go to the national government also increase, and so the incentive for the local government to invest in productive public goods is smaller.

Taking the first order condition for the problem of the local government with respect to local investment in the consumptive public good, we obtain

$$\frac{H_l(g_l, g_n)}{u'(c)} = 1 - \alpha\tau_n. \tag{7}$$

If $\alpha > 0$, the local consumptive public good is overprovided, relative to the provision $g_l^*(g_n)$, where $g_l^*(g_n)$ is the Samuelsonian choice of investment in g_l given g_n. When a district spends \$1 on local consumptive public goods, the cost to its citizens is not \$1, but instead \$1 $- \alpha\tau_n$, as the district citizens receive a rebate from the national government for the spending done by their local government. Hence we have a different fiscal tragedy of the commons—each local government overinvests in consumptive public goods since some of the costs of this investment are borne by other districts through the mechanism of the local income tax deduction.

If $\alpha = 0$, there is no local income tax deduction, and local governments will invest efficiently in the local consumptive public good, since spending \$1 on the consumptive public good costs the residents of the district \$1, and so the local government will spend up to the point where $u'(c) = H_l(g_l, g_n)$.

The preceding results are summarized in the following proposition.

Proposition 1 *For a given local income tax deduction level $\alpha \in [0, 1]$, national tax rate $\tau_n > 0$, and investments of p_n and g_n,*

1. *the level of local investment in productive public goods $p_l(p_n)$ is less than first-best, i.e. $p_l(p_n) \leq p_l^*(p_n)$, with equality if and only if $\alpha = 1$, and*

2. *the level of local investment in consumptive public goods $g_l(g_n)$ is more than first-best, i.e. $g_l(g_n) \geq g_l^*(g_n)$, with equality if and only if $\alpha = 0$.*

Note that, in both cases, the level of distortion in the decisions by the local governments is increasing in the level of the national income tax. We can define a level of distortion in local decisions, independent of the level of α, as the ratio of the two first order conditions:

$$\frac{H_l(g_l, g_n)}{F_l(p_l, p_n)u'(c)} = 1 - \tau_n \tag{8}$$

As the national income tax rate increases, this "tax wedge" between spending on local productive and consumptive public goods increases. By changing α, we can change which decision gets distorted, but the level of distortion, as measured by the tax wedge, remains the same.

Furthermore, the local income tax deduction acts as a subsidy for public spending by local governments. It subsidizes local spending by effectively giving the local government $\frac{\alpha}{1-\tau_n}$ for every dollar of local spending. In general, such a subsidy would distort the spending of the local government away from efficient levels, and in the case of consumptive public goods, it does exactly that. However, for productive public goods, this subsidy exactly offsets the distortion imposed by the existence of the national income tax itself.

We can also calculate how local government policy will change as a function of the taxation parameters chosen by the national government.

Proposition 2 *Given investments p_n and g_n by the national government,*

1. *Holding the national income tax rate τ_n fixed, local investment in both types of public goods is increasing in the level of the local income tax deduction α, i.e. $\frac{\partial p_l}{\partial \alpha}, \frac{\partial g_l}{\partial \alpha} \geq 0$, and*

2. *Holding the local income tax deduction α fixed, local investment in productive public goods is decreasing with the national income tax rate τ_n, i.e. $\frac{\partial p_l}{\partial \tau_n} \leq 0$.*

The first result states that both types of local public goods are increasing in the tax deduction. Investment in the local productive public good increases as more of the income generated from this public investment is retained by the local citizenry.

Investment in the local consumptive public good, however, increases for two reasons. First, there is a price effect: the price of the local consumptive public good (from the perspective of the local government) decreases, as an increase in α means that the national government bears a larger burden of the costs. Second, there is an income effect: The after-tax income of the citizens increases with α, and so the local government will now wish to spend more on the local consumptive public good to equalize the marginal utility from consumption and the consumptive public good.

The second result states that an increase in the national income tax reduces local investment in the productive public good. When the national tax rate is increased, local governments now invest less in local productive public goods, since less of the income generated from this public investment is retained by the local citizenry. However, we can not sign the change in the local consumptive public good as the income and price effects are offsetting. The price of the local consumptive public good is decreasing in τ_n and so the local government should purchase more of it (the price effect). On the other hand, the after-tax income of the citizen is also decreasing in τ_n, and so the local government should purchase less of the local consumptive public good (the income effect).

3.2 The Problem of the National Government

We now turn to the problem of the national government. The national government wishes to maximize the welfare of all citizens, and it takes the response functions of the local governments as given. By plugging in the identity $F(p_l, p_n)\tau_l = p_l + g_l$ for the local income tax, we obtain that the problem of the national government is:

$$\max_{\alpha,\tau_n,p_n,g_n} \{u\left(F(p_l,p_n)\left(1-\tau_n\right) - \left(p_l+g_l\right)\left(1-\alpha\tau_n\right)\right) + H(g_l,g_n)\} \tag{9}$$

subject to

$$F(p_l,p_n)\tau_n = p_n + g_n + \alpha\tau_n\left(p_l+g_l\right)$$

where p_l and g_l are functions of the national government's decisions.

Taking the first order condition with respect to the tax rate τ_n, we obtain

$$\lambda = u'(c) \frac{F(p_l, p_n) - \alpha (p_l + g_l)}{F(p_l, p_n) - \alpha (p_l + g_l) + \tau_n \frac{\partial I}{\partial \tau_n}} \tag{10}$$

where λ represents the Lagrange multiplier with respect to the budget constraint of the national government, and where $I \equiv F(p_l, p_n) - \alpha (p_l + g_l)$ represents income taxable by the national government. The "shadow price" of the budget constraint is larger than $u'(c)$, so long as taxable income is decreasing in τ_n. As in the analysis of how g_l changes with τ_n, there is both an income and a price effect of changing τ_n. We can write

$$\frac{\partial I}{\partial \tau_n} = (F_l(p_l, p_n) - \alpha) \frac{\partial p_l}{\partial \tau_n} - \alpha \frac{\partial g_l}{\partial \tau_n}$$

From Proposition 2, we have that $\frac{\partial p_l}{\partial \tau_n} \leq 0$; since $F_l(p_l, p_n)$ is larger than 1, the first term must be negative. The second term is the effect on local spending on consumptive public goods from a change in the national tax rate. This effect is composed of both an income effect which lowers spending—raising the national tax rate increases the marginal utility of private consumption—and a price effect, which raises spending—raising the national tax rate lowers the cost of providing a given amount of the local consumptive public good. As long as the income effect does not dominate both the price effect and the effect of taxes on local productive public goods, it will be the case that $\lambda > u'(c)$.[13] When $\lambda > u'(c)$, spending is costly for the national government as it both reduces the consumption of the representative agent directly and it further distorts decisions by local governments.

Taking the first order condition with respect to the local income tax deduction rate α, we obtain:

$$\lambda = u'(c) \frac{\tau (p_l + g_l)}{\tau (p_l + g_l) - \tau_n \left((F_l(p_l, p_n) - \alpha) \frac{\partial p_l}{\partial \alpha} - \alpha \frac{\partial g_l}{\partial \alpha} \right)}.$$

[13]For instance, if utility is quasilinear in consumption, this condition will always hold, as there will be no income effect.

If we assume that $\lambda > u'(c)$, we have that

$$(F_l(p_l, p_n) - \alpha)\frac{\partial p_l}{\partial \alpha} - \alpha \frac{\partial g_l}{\partial \alpha} > 0.$$

The above expression is the indirect effect on national revenues from a change in the local income tax deduction rate, which is positive so long as $\lambda > u'(c)$. That is, the increase in taxable income from a greater investment in productive public goods will more than offset the decrease in taxable income from the increase in local spending on local public goods. In particular, estimates of the change in national revenues due to eliminating the local income tax deduction (such as Mitchell, 2005) will likely overstate the gains to national revenue if they do not consider the effect that eliminating this deduction will have on local spending decisions.

Taking the first order condition of the problem of the national government with respect to g_n, and simplifying, we obtain

$$H_n(g_l, g_n) = \lambda\left(1 + \alpha\tau_n \frac{\partial g_l}{\partial g_n}\right).$$

The national government will likely underinvest in national consumptive public goods so long as $\lambda > u'(c)$; that is, it will choose a $g_n < g_n^*$, the solution to the social planner's problem. By underinvesting in national consumptive public goods, the national government can reduce the tax wedge, and hence reduce the distortion from optimality in choices by local governments.

However, if investments in consumptive public goods are substitutes, then $\frac{\partial g_l}{\partial g_n}$ is negative.[14] In that case, the national government may wish to overinvest in the national consumptive public good so as to reduce investment in the local consumptive public good by local governments. However, if g_l and g_n are complements, then the national government has even more reason to underinvest in the consumptive public good, as such investment would increase the already inefficiently high investment in consumptive public goods by local governements.

[14] For a proof of this statement, see the proof of Proposition 3 in the appendix.

Taking the first order condition of the problem of the national government with respect to p_n, and simplifying, we obtain

$$F_n(p_l, p_n) = \frac{\lambda\left(1 + \alpha\tau_n \frac{\partial g_l}{\partial p_n} - \frac{\partial p_l}{\partial p_n}\tau_n\left(F_l(p_l, p_n) - \alpha\right)\right)}{\lambda\tau_n + u'(c)\left(1 - \tau_n\right)}.$$

The national government will underinvest in national consumptive public goods so long as $\lambda > u'(c)$; that is, they will choose a $p_n < p_n^*$, the solution to the social planner's problem. Note that $\frac{\partial g_l}{\partial p_n} > 0$, as the more the national government invests in productive public goods, the more income is available to residents within each district, and so the local government will choose to invest even more in the local consumptive public good. This gives the national government yet another reason to be cautious investing in the national productive public good, as local governments overinvest in the local consumptive public good. However, the national government will not wish to invest more in the national productive public good to make up for the fact that the local governments underinvest in productive public goods; it would be more effective to lower the national tax rate instead.

However, if investments in productive public goods are complements, then $\frac{\partial p_l}{\partial p_n}$ may be positive.[15] In that case, the national government may wish to overinvest in the national productive public good so as to induce more investment in the local productive public good by local governments, by making such investment more effective. However, if p_l and p_n are substitutes, then the national government has even more reason to underinvest in the productive public good, as investment in the national productive public good will reduce the already inefficiently low investment in productive public goods by local governments.

We summarize these results in the following proposition:

Proposition 3 *So long as taxable national income is decreasing in the national tax rate, the national government will choose:*

[15] For a proof of this statement, see the proof of Proposition 3 in the appendix.

1. *a level of national investment in productive public goods* $\hat{p}_n$ *that is less than first-best, i.e.* $F_n\left(p_l(p_n), \hat{p}_n\right) > 1$*, as long as* p_l *and* p_n *are weak substitutes, and*

2. *a level of national investment in consumptive public goods* $\hat{g}_n$ *that is less than first-best, i.e.* $H_n\left(g_l\left(\hat{g}_n\right), \hat{g}_n\right) > u'(c)$*, as long as* g_l *and* g_n *are weak complements.*

If local governments are given only one responsibility, and the national government can choose α, then it is possible to attain the first-best. For example, assume that $H(g_l, g_n) = H(g_n)$, i.e., local consumptive public goods do not exist. Then by choosing $\alpha = 1$, the national government can attain first-best. Proposition 1 assures us that the local productive public good will be chosen efficiently; since this happens regardless of the national income tax τ_n, the national government can now choose to implement the first-best national policy.

3.3 Comparative Statics of Optimal Policy

There is no straightforward comparative static of the optimal national policy with respect to the local income tax deduction. In particular, the optimal choice of α is not necessarily monotonic in the importance of local consumptive public goods. Consider the case where the utility of the agent is given by

$$c + H\left(g_l\right) + J(g_n)$$

so that utility is quasilinear. Further assume that there is no national productive public good. If $H\left(g_l\right) = \theta \min\left\{1, g_l\right\}$, then for $\theta = 0$ or $\theta \geq 1$, the optimal policy choice by the national government is to choose $\alpha = 1$. If $\theta = 0$, there is no local consumptive public good, and so the national government can obtain the first-best by choosing $\alpha = 1$, which eliminates the distortion in the choice of p_l by the local government. If $\theta \geq 1$, then the local government will choose $g_l = 1$ for any nonnegative local income tax deduction rate α, and so the national government can again obtain the first-best by choosing $\alpha = 1$. However, if $\theta = \frac{1}{2}$, then it may be optimal for the national government to choose an $\alpha = \theta = \frac{1}{2} < 1$ so that the local government does not wish to invest in the local consumptive public good.

3.4 A Unitary Local Public Good

In this section, we show that the national government can achieve the first-best when local government spending is unitary. By unitary, we mean that the local government spending g_l finances a general public good used for production and consumption, so that the production function is now $F(g_l, p_n)$. Let (g_l^*, p_n^*, g_n^*) be the first-best levels of public good provision.

The problem of the local government becomes

$$\max_{\tau_l, g_l} \{u(F(g_l, p_n)(1 - \tau_n - \tau_l + \alpha\tau_n\tau_l)) + H(g_l, g_n)\}$$

subject to the budget constraint that

$$F(g_l, p_n)\,\tau_l = g_l$$

taking the tax rate τ_n and investment decisions p_n and g_n of the national government as given.

Taking the first order condition for the problem of the local government with respect to investment in the local public good, we obtain

$$F_l(g_l, p_n) + \frac{H_l(g_l, g_n)}{u'(c)} = 1 + \tau_n\,(F_l(g_l, p_n) - \alpha)\,.$$

It then follows that the national government can obtain the first-best choice of g_l by choosing

$$F_l(g_l^*, p_n) = \alpha$$

regardless of the choice of τ_n, p_n, or g_n.

Intuitively, the national government directly controls the level of spending by the local governments by choosing the level of the local income tax deduction. Hence, the national government can choose its taxation and spending levels to implement the first-best national policies, and then use the local income tax deduction to induce the local governments to choose the first-best level of local spending. Since the national government maximizes the welfare of the represenative agent, it will choose policies to implement the first-best.

Proposition 4 *The national government will choose* $p_n = p_n^*, g_n = g_n^*$*, and* $\alpha = F_l\left(g_l^*, p_n^*\right)$*. The equilibrium outcome is first-best.*

This result highlights the fact that the national income tax distorts the mix, not the level, of spending by the states in our basic model. When states have only one spending decision, the national government can implement the first-best by careful choice of the national tax rate and local income tax deduction. However, if states have discretion over spending on productive public goods versus consumptive public goods, the national government can no longer implement the first-best.

3.5 Matching Grants

In this section, we show that a national government can use a targeted matching grant in order to achieve the first-best outcome. Suppose that the national government can choose a matching grant level γ, where the national government agrees to pay for a proportion γ of local spending on the local productive public good.

The local budget constraint then becomes

$$F(p_l, p_n)\tau_l = p_l\left(1-\gamma\right) + g_l.$$

Using this new budget constraint, we can solve the local government's problem (1) and obtain:

$$\begin{aligned} F_l(p_l, p_n) &= \frac{\left(1-\gamma\right)\left(1-\alpha\tau_n\right)}{1-\tau_n} \\ \frac{H_l(g_l, g_n)}{u'(c)} &= 1-\alpha\tau_n \end{aligned}$$

Setting $\gamma = \tau_n$ and $\alpha = 0$ incentivizes the local government to choose optimal values of investment in both types of public goods, regardless of the national income tax rate. The budget constraint for the national government is given by

$$F(p_l, p_n)\tau_n (1 - \alpha\tau_l) = p_n + g_n + \gamma p_l$$

Since the national government can ensure that local governments invest optimally in both types of public goods by choosing $\gamma = \tau_n$ and $\alpha = 0$, it can then obtain the first-best by choosing the national income tax rate to cover the cost of not only the matching grant, but also first-best investment in the national productive and consumptive public goods.

Proposition 5 *The national government will choose $p_n = p_n^*$, $g_n = g_n^*$, $\alpha = 0$, and $\gamma = \tau_n$. The local government will choose $p_l = p_l^*$ and $g_l = g_l^*$*

The national income tax induces the local governments to invest too little in productive public goods; the matching grant then allows the national government to adjust this investment back to first-best levels. The key assumption is that the matching grant can be targeted at a particular type of local public good. A grant to all forms of local government spending would encounter the same problems that the local income tax deduction does: It would reduce the underinvestment in productive public goods by local governments, but increase the spending on local consumptive public goods beyond efficient levels.

4 Policy under the Centralized Revenue Mechanism

We now consider a different model of federalism. Local governments retain their power to make spending decisions, but their budget is now set by the national government, who provides all of their funding via a transfer T. The national budget constraint is now

$$F(p_l, p_n)\tau_n = p_n + g_n + T$$

and the local budget constraint is completely defined by the funding of the national government:

$$p_l + g_l = T.$$

4.1 The Problem of the Local Government

Under a centralized revenue mechanism, a local government faces the following problem:

$$\max_{p_l, g_l} \{u\left(F(p_l, p_n)\left(1 - \tau_n\right)\right) + H(g_l, g_n)\} \tag{11}$$

subject to

$$p_l + g_l = T.$$

The first order conditions of the local government's problem are

$$\begin{aligned} F_l(p_l, p_n) &= \frac{\kappa}{u'(c)\left(1 - \tau_n\right)} \\ H_l(g_l, g_n) &= \kappa \end{aligned} \tag{12}$$

where κ is the Lagrange multiplier with respect to the budget constraint of the local government. Combining these two expressions, we find that the tax wedge is

$$\frac{H_l(g_l, g_n)}{F_l(p_l, p_n)u'(c)} = 1 - \tau_n. \tag{13}$$

The choices of the local government are distorted in a similar way to the decentralized revenue mechanism. However, τ_n is likely to be larger under the centralized revenue mechanism, as all government revenues come from the national income tax.

4.2 The Problem of the National Government

The national government maximizes the welfare of the represenative agent, taking the response functions of local governments as given. Hence, we model the national government as choosing p_l, p_n, g_l, g_n, and T subject to the national budget constraint, the local budget constraint and the constraint (13). The problem of the national government is

$$\max_{p_l, p_n, g_l, g_n, \tau_n} \left\{ u\left(F(p_l, p_n)\left(1 - \tau_n\right)\right) + H(g_l, g_n) \right\} \tag{14}$$

subject to

$$\begin{aligned} F(p_l, p_n)\tau_n - (p_n + g_n) - T &= 0 \\ p_l + g_l &= T \\ (1 - \tau_n) - \frac{H_l(g_l, g_n)}{F_l(p_l, p_n)u'(c)} &= 0. \end{aligned}$$

This problem simplifies to

$$\max_{p_l, p_n, g_l, g_n, \tau_n} \left\{ u\left(F(p_l, p_n)\left(1 - \tau_n\right)\right) + H(g_l, g_n) \right\} \tag{15}$$

subject to

$$\begin{aligned} F(p_l, p_n)\tau_n - (p_l + p_n + g_l + g_n) &= 0 \\ (1 - \tau_n) - \frac{H_l(g_l, g_n)}{F_l(p_l, p_n)u'(c)} &= 0. \end{aligned}$$

Let μ denote the Lagrange multiplier for the first constraint, and ν the Lagrange multiplier for the second constraint. Both Lagrange multipliers are positive.

The first order condition with respect to the tax rate τ_n is

$$\mu = u'(c) + \nu \left(\frac{1}{F(p_l, p_n)} + \frac{u''(c)}{u'(c)} \left(1 - \tau_n\right) \right). \tag{16}$$

Unfortunately, as (16) shows, the Lagrange multiplier on the government budget constraint may be greater or smaller than the marginal utility of consumption. This is because an increase in the tax rate has two indirect effects: First, it widens the tax wedge (13), which further distorts decisions by local governments. Second, it increases the marginal utility of consumption, and so the local government may actually increase investment in the local productive public good and decrease investment in the local consumptive public good as the tax rate increases.

Taking the first order condition with respect to the local productive public good, and simplifying using (13) and (16), we obtain:

$$F_l(p_l, p_n) = \frac{\left[\mu - \nu \frac{u''(c)}{u'(c)} (1 - \tau_n) \tau_n\right] - \nu \frac{(1-\tau_n) F_{l,l}(p_l, p_n)}{F_l(p_l, p_n)}}{\left[\mu - \nu \frac{u''(c)}{u'(c)} (1 - \tau_n) \tau_n\right] - \frac{1-\tau_n}{F(p_l, p_n)}}.$$

We have $F_l(p_l, p_n) > 1$, so the local productive public good will be underprovided. The national government would have to set taxes very high to fund the local productive public good at efficient levels, as it would have to provide a large enough transfer T to the local governments that they would fund p_l at the efficient level, even given the tax wedge. The tax wedge at that point would be quite large, since the national government must raise all the revenue to fund p_l. Instead, the national government provides a transfer of the size that the local productive public good will be underprovided, as providing a large enough transfer to fund p_l at efficient levels requires a large overinvestment in the local consumptive public good.

Taking the first order condition with respect to the local consumptive public good, and simplifying using (13), we obtain:

$$H_l(g_l, g_n) = \mu + \nu \frac{H_{l,l}(g_l, g_n)}{H_l(g_l, g_n)} (1 - \tau_n)$$

We can not determine whether the local consumptive public good is under- or overprovided. From (16), μ may be greater or less than the marginal utility of consumption $u'(c)$. The national government may not provide a large enough transfer so that g_l will be overprovided, as by keeping the transfer small, the national government can reduce the distortion in the decisions by local governments.

Taking the first order condition with respect to the national productive public good, and simplifying using (13), we obtain:

$$F_n(p_l, p_n) = \frac{\left[\mu - \nu \frac{u''(c)}{u'(c)} (1 - \tau_n) \tau_n\right] - \nu \frac{(1-\tau_n) F_{l,n}(p_l, p_n)}{F_l(p_l, p_n)}}{\left[\mu - \nu \frac{u''(c)}{u'(c)} (1 - \tau_n) \tau_n\right] - \frac{1-\tau_n}{F(p_l, p_n)}}$$

and so $F_n(p_l, p_n) > 1$ as long as $F_{l,n}(p_l, p_n)$ is not too positive, implying the national productive public good will be underprovided. As long as p_l and p_n are weak substitutes, the national government will underfund the national productive public good. By doing so, it is able to reduce taxes, and hence lower the tax wedge which distorts the choices of local governments.

The first order condition with respect to the national consumptive public good is

$$H_n(g_l, g_n) = \mu + \nu \frac{H_{l,n}(g_l, g_n)}{H_l(g_l, g_n)} (1 - \tau_n) .$$

As in the case of the local consumptive public good, we can not determine whether it is under- or over-provided, as μ may be greater than or less than the the marginal utility of consumption $u'(c)$.

We summarize these results in the following proposition:

Proposition 6 *The optimal policy of the national government with centralized revenue generation will result in:*

1. *the level of local investment in productive public goods is less than first-best, i.e.* $F_l(p_l, p_n) > 1$,
2. *the level of national investment in productive public goods is less than first-best, i.e.* $F_n(p_l, p_n) > 1$, *as long as* p_l *and* p_n *are weak substitutes,*
3. *the level of investment in both local and national consumptive public goods* g_l *and* g_n *may be less than, equal to, or more than first-best.*

5 Regime Comparison

In this section, we show that the representative agent has a higher welfare under decentralized revenue generation than under centralized revenue generation. This result may be surprising in light of the fact that the national government, who wishes to maximize the welfare of the representative agent, has more control under the centralized revenue mechanism: The national government can determine exactly how much is spent by the local government. However, the fundamental problem for the national government is not how much the local government spends, but the misallocation of the money it does spend.

Recall that the tax wedge is given by:

$$\frac{H_l(g_l, g_n)}{F_l(p_l, p_n)u'(c)} = 1 - \tau_n. \tag{17}$$

Under both systems, the national income tax distorts the allocation of resources: The local government spends too much on local consumptive public goods, and too little on local productive public goods. Under the decentralized revenue mechanism, which we denote by a superscript d, we have that

$$\tau_n^d = \frac{p_n^d + g_n^d + \alpha\tau_l^d\tau_n^d F(p_l^d, p_n^d)}{F(p_l^d, p_n^d)} = \frac{p_n^d + g_n^d + \alpha\tau_n^d(p_l^d + g_l^d)}{F(p_l^d, p_n^d)} < \frac{p_n^d + g_n^d + p_l^d + g_l^d}{F(p_l^d, p_n^d)} \tag{18}$$

as $\alpha \leq 1$ and $\tau_n^d < 1$. However, under a centralized revenue mechanism, which we denote by a superscript c, we have that

$$\tau_n^c = \frac{p_n^c + g_n^c + T}{F(p_l^c, p_n^c)} = \frac{p_n^c + g_n^c + p_l^c + g_l^c}{F(p_l^c, p_n^c)}. \tag{19}$$

Hence, for the same level of funding of the public goods, the tax wedge will be smaller under a decentralized revenue mechanism. This is the key idea behind the following result.

Proposition 7 *The optimal policy for the national government under decentralized revenue collection provides higher welfare than the optimal policy for the national government under centralized revenue collection.*

We prove this result by proving that a national government under the decentralized taxation regime can replicate the outcome under the nationalized taxation regime. Let the national government under the decentralized taxation regime choose the same levels of national public goods. The national government can also replicate the local provision of public goods by choosing τ_n and α.[16] If the level of funding for all the public goods is the same, then the national government under the decentralized revenue mechanism has a budget surplus, as it is collecting $\tau_n^d F(p_l, p_n)$ but only spending $p_n + g_n + \alpha\tau_n(p_l + g_l) < p_n + g_n + p_l + g_l$. Suppose that it returns this budget surplus to the citizens as a lump-sum rebate. Then the national government under decentralized revenue generation will have replicated the outcome of the national government with centralized revenue generation. The national government under the decentralized revenue mechanism can then lower its taxes (and the lump-sum rebate), which will make its citizens better off as the distortion in the local governments' decisions due to the tax wedge will decrease.

5.1 Decentralized Revenue Generation with Transfers to the States

We now consider a combined mechanism: that is, the districts raise their own revenue, but the government may supplement also make a transfer $T \geq 0$ to local governments. The problem for the local government is now

$$\max_{p_l, g_l, \tau_l} \{u\left(F(p_l, p_n)\left(1 - \tau_n - \tau_l + \alpha\tau_n\tau_l\right)\right) + H(g_l, g_n)\} \tag{20}$$

subject to

$$F(p_l, p_n)\tau_l + T = p_l + g_l$$

while the problem for the national government is

$$\max_{p_n, g_n, \alpha, \tau_n, T} \{u\left(F(p_l, p_n)\left(1 - \tau_n\right)\right) + H(g_l, g_n)\} \tag{21}$$

subject to

$$F(p_l, p_n)\tau_n = p_n + g_n + \alpha\tau_l\tau_n F(p_l, p_n) + T.$$

[16] If $g_l^c < g_l^*(g_n)$, then the national government must choose an $\alpha < 0$. While this can not be optimal for the national government under the decentralized revenue mechanism, we are only interested here in showing it can do better than the national government under the centralized revenue mechanism.

However, the optimal transfer to local governments will be 0. The local governments will simply treat the transfer as a lump sum and reduce their own taxation by an equivalent amount. Hence, holding p_n, g_n, and τ_n constant, the decisions by local governments regarding p_l and g_l will be the same regardless of the transfer, so long as the transfer is not so large as to completely crowd out local revenue generation. Note that the first order conditions for a local government are

$$\begin{aligned} F_l(p_l, p_n) &= \frac{1-\alpha\tau_n}{1-\tau_n} \\ \frac{H_l(g_l, g_n)}{u'(c)} &= 1-\alpha\tau_n \end{aligned} \tag{22}$$

which is exactly the same as without any transfer. Hence, it would always be better to reduce the transfer so as to reduce national taxation and the tax wedge distorting the choices of local governments.

Proposition 8 *The optimal policy for the national government under decentralized revenue collection with transfers is to set the transfer to* 0*.*

Indeed, in our model, a national government that could require transfers from the districts to the national government could achieve first-best by choosing the first-best levels of both national public goods, and then simply demand the local governments provide the appropriate amount of money to pay for national expenditures. The tax wedge would then be zero, as national income taxes are zero, and so local governments would have no incentive to underinvest in productive public goods or overinvest in consumptive public goods.

6 Conclusions

We have shown that the local income tax deduction can be welfare-enhancing if the local government must provide both productive and consumptive public goods. While a local income tax deduction will cause overinvestment in local consumptive public goods, it will increase investment in local productive public goods, which will be underprovided without a full local income tax deduction. Furthermore, since there does not exist a local income tax deduction rate that correctly aligns local governments' incentives for both productive and consumptive public goods, national governments will underinvest in public goods to reduce the distortion in the local governments' decisions due to the national income tax.

The key issue here is that some public goods that local governments provide produce taxable benefits, such as increased income, while other public goods produce nontaxable benefits, such as a more beautiful neighborhood. In this paper, productive public goods provide taxable benefits, while consumptive ones provide nontaxable ones. The income tax, then, creates a tax bias favoring the nontaxable consumptive public goods over the taxable productive public goods, and hence creates incentives for local governments to invest more in consumptive than productive public goods. However, if the national government is able to isolate which local public goods are productive, i.e. provide taxable benefits, and which are consumptive, i.e. provide nontaxable benefits, then a local income tax deduction should not be implemented, but rather an appropriate matching grant for local productive public goods should be used. It is only in the case when this differentiation can not be made by the national government that a local income tax deduction may be optimal.

This problem is not solved, but is rather exacerbated, by having the national government be the sole revenue collector. When the national government raises the revenue for local and national spending, the national tax rate is higher, and hence the incentives of local governments are further distorted to underinvest in productive public goods and overinvest in consumptive public goods. Indeed, a national government can always do better by allowing the local governments to raise all the resources they wish to spend and choosing an appropriate level of local income tax deduction.

Using an average marginal tax rate of 21.7% (Barro and Redlick, 2009), the implied subsidy from the local income tax deduction by the federal government to the states is on the order of \$43bn.[17] Property taxes used by local governments are also deductable, and the implied subsidy to local governments for spending is on the order of \$69bn.[18] The numbers estimated here are, of course, only very rough estimates of the size of the effects of the local income tax deduction. More research is needed to properly estimate the quantitative effects of removing this deduction on national tax revenues, as well as on the spending of state and local governments within the United States on both consumptive and productive public goods.

In this paper, we have assumed welfare-maximizing governments, and such governments are not wholly reflected in reality, to say the least. If local governments are (partial) Leviathans, who will spend too much on both types of public goods, then reducing the local income tax deduction would be beneficial, as it would reduce spending on both types of public goods—first-best would still not be achieved, however, as such local governments would still overspend on consumptive public goods relative to productive public goods.

Accounting for the effect of the tax deduction on agents' labor decisions also weakens the case for a local income tax deduction. Increasing the local income tax deduction incentivizes local governments to spend, and thus tax, more. It is well-known that local governments overtax under these assumptions, as they do not consider how the effect of their taxes on labor supply affects national government revenues (Flowers, 1988). A local income tax deduction further incentivizes local governments to raise local income taxes, further exacerbating the effect on national revenues.

On the other hand, if local government investment is subject to positive spillovers

[17]According to the U.S. Census of Governments, states collect approximately \$246bn in income taxes. While not all taxpayers itemize, approximately 80% (Toder and Rosenberg, 2007) of tax liability is paid by itemizing taxpayers (and these taxpayers likely face a higher marginal rate). Furthermore, taxpayers are allowed to itemize state sales taxes when they do not itemize state income taxes, so the true size of the effect on the decisions of state governments is larger than that calculated by just looking at income taxes.

[18]The U.S. Census of Governments report that local governments collected \$396bn in property tax revenues in the 2007 fiscal year.

in either productive or consumptive public goods, this only strengthens the case for a local income tax deduction. Provision of both types of goods are increasing in the level of local income tax deduction, so if both are underprovided with no local income tax deduction, the introduction of such a deduction may bring us closer to welfare-maximizing levels of both types of goods.

This paper has shown that the local income tax deduction may be a useful tool for national governments to mitigate the distortionary effects of national tax policy. The local income tax deduction increases local spending on both productive and consumptive public goods, and so its welfare consequences depend on the mix of public goods available to local governments as well as the underlying incentive structure of local policymakers. Understanding the effects of national tax policy on local tax and spending decisions remains an area ripe for future research.

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7 Appendix

A number of the results in this section rely on ideas of monotone comparative statics, developed in Milgrom and Shannon (1994).[19] The key idea is that for the maximization problem

$$x^*(\theta) = \arg \max_{x \in X \subseteq \mathbb{R}^m} \{f(x, \theta)\}$$

if $\frac{\partial^2 f}{\partial x_i \partial \theta} \geq 0$, and $\frac{\partial^2 f}{\partial x_i \partial x_j} \geq 0$ for all $i \neq j$, then $\frac{\partial x_i^*(\theta)}{\partial \theta} \geq 0$ for all i.

The proof of Propostion 1 follows:

Proof. We have from Equation (6) that

$$F_l(p_l, p_n) = \frac{1 - \alpha\tau_n}{1 - \tau_n} \geq 1$$

Since $F(\cdot)$ is strictly concave in p_l, $p_l(p_n)$ is less than the first-best level $p_l^*(p_n)$, with equality if and only if $\alpha = 1$. Totally differentiating this expression with respect to

[19] An excellent exposition of these techniques is also given in Ashworth and Bueno de Mesquita (2006).

τ_n, we obtain

$$\frac{\partial p_l}{\partial \tau_n} = \frac{1-\alpha}{F_{l,l}(p_l,p_n)\left(1-\tau_n\right)^2} \leq 0$$

Totally differentiating with respect to α, we obtain

$$\frac{\partial p_l}{\partial \alpha} = \frac{-\tau_n}{F_{l,l}(p_l,p_n)\left(1-\tau_n\right)} \geq 0$$

Similarly, we have from Equation (7) that

$$\frac{H_l(g_l,g_n)}{u'(c)} = 1-\alpha\tau_n \leq 1$$

Since $H\left(\cdot\right)$ is strictly concave in g_l, $g_l(g_n)$ is less than the first-best level $g_l^*(g_n)$, with equality if and only if $\alpha = 1$. Totally differentiating this with respect to α, we have

$$\begin{aligned}
H_{l,l}(g_l,g_n)\frac{\partial g_l}{\partial \alpha} &= -u'(c)\tau_n + \left(1-\alpha\tau_n\right)u''(c)\left[\begin{array}{c} \tau_n\left(p_l+g_l\right)+ \\ \left(F'\left(p\right)\left(1-\tau\right)-\left(1-\alpha\tau_n\right)\right)\frac{\partial p_l}{\partial \alpha}+ \\ -\left(1-\alpha\tau_n\right)\frac{\partial g_l}{\partial \alpha}\end{array}\right] \\
\frac{\partial g_l}{\partial \alpha} &= \frac{-u'(c)\tau_n + \left(1-\alpha\tau_n\right)u''(c)}{H_{l,l}(g_l,g_n)+\left(1-\alpha\tau_n\right)^2 u''(c)} \geq 0
\end{aligned}$$

■

The proof of Proposition 2 follows:

Proof. First, note that the cross-partial with respect to p_l and p_n of the maximand of the local government's problem is

$$u''(c)\left(1-\tau_n\right)^2 F_l\left(\cdot\right)F_n\left(\cdot\right) + u'(c)F_{l,n}\left(\cdot\right)$$

and may only be positive $F_{l,n}\left(\cdot\right) > 0$. Hence, by the theory of monotone comparative statics (see Milgrom and Shannon, 1994), $\frac{\partial p_l}{\partial p_n}$ may only be positive if $F_{l,n}\left(\cdot\right) > 0$, i.e. p_l and p_n are complements.

Simliarly, the cross-partial with respect to g_l and g_n of the maximand of the local government's problem is $H_{l,n}\left(\cdot\right)$. Hence, by the theory of monotone comparative statics (see Milgrom and Shannon, 1994), $\frac{\partial g_l}{\partial g_n}$ will be negative if and only if $H_{l,n}\left(\cdot\right) <$ 0, i.e. g_l and g_n are substitutes.

From (6) we have that

$$F_l(p_l, p_n) = \frac{1 - \alpha\tau_n}{1 - \tau_n}$$

Implicitly differentiating with respect to τ_n, we have

$$\begin{aligned} F_{l,l}(p_l, p_n)\frac{\partial p_l}{\partial \tau_n} &= \frac{1 - \alpha}{(1 - \tau_n)^2} \\ \frac{\partial p_l}{\partial \tau_n} &= \frac{1 - \alpha}{F_{l,l}(p_l, p_n)(1 - \tau_n)^2} < 0 \end{aligned}$$

and implicity differentiating with respect to α, we have

$$\begin{aligned} F_{l,l}(p_l, p_n)\frac{\partial p_l}{\partial \alpha} &= \frac{-\tau_n}{(1 - \tau_n)^2} \\ \frac{\partial p_l}{\partial \alpha} &= \frac{-\tau_n}{F_{l,l}(p_l, p_n)(1 - \tau_n)^2} > 0 \end{aligned}$$

For the second part, we have from (7) that

$$u'(c)(1 - \alpha\tau_n) = H_l(g_l, g_n)$$

Implicitly differentiating with respect to α, we have

$$\begin{aligned} u''(c)(1 - \alpha\tau_n)\left(\begin{array}{c} [F_l(p_l, p_n)(1 - \tau_n) - (1 - \alpha\tau_n)]\frac{\partial p_l}{\partial \alpha} - \\ (1 - \alpha\tau_n)\frac{\partial g_l}{\partial \alpha} \end{array}\right) - \tau_n u'(c) &= H_{l,l}(g_l, g_n)\frac{\partial g_l}{\partial \alpha} \\ -u''(c)(1 - \alpha\tau_n)^2\frac{\partial g_l}{\partial \alpha} - \tau_n u'(c) &= H_{l,l}(g_l, g_n)\frac{\partial g_l}{\partial \alpha} \\ \frac{-\tau_n u'(c)}{H_{l,l}(g_l, g_n) + u''(c)(1 - \alpha\tau_n)^2} &= \frac{\partial g_l}{\partial \alpha} \end{aligned}$$

which is positive. (Note that from (6) we have that $F_l(p_l, p_n)(1 - \tau_n) - (1 - \alpha\tau_n) = 0$.) ■

The proof of Proposition 3 follows:

Proof. The problem of the national government is:

$$\max_{\alpha, \tau_n, p_n, g_n} \{u(F(p_l, p_n)(1 - \tau_n) - (p_l + g_l)(1 - \alpha\tau_n)) + H(g_l, g_n)\}$$

subject to

$$F(p_l, p_n)\tau_n = p_n + g_n + \alpha\tau_n(p_l + g_l)$$

Taking the first order condition of the problem of the national government with respect to p_n and g_n, and simplifying, we obtain

$$\begin{aligned} H_n(g_l,g_n)+\frac{\partial g_l}{\partial g_n}\left(H_l(g_l,g_n)-u'(c)\left(1-\alpha\tau_n\right)\right) &= \lambda\left(1+\left(1-\alpha\tau_n\right)\frac{\partial g_l}{\partial g_n}\right) \\ H_n(g_l,g_n) &= \lambda\left(1+\alpha\tau_n\frac{\partial g_l}{\partial g_n}\right) \end{aligned}$$

using (7) and

$$\begin{aligned} u'(c)\left(\begin{array}{c} F_n(p_l,p_n)\left(1-\tau_n\right)+ \\ \frac{\partial p_l}{\partial p_n}\left(F_l(p_l,p_n)\left(1-\tau_n\right)-\left(1-\alpha\tau_n\right)\right) \\ +\frac{\partial g_l}{\partial p_n}\left(H_l(g_l,g_n)-u'(c)\left(1-\alpha\tau_n\right)\right) \end{array}\right) &= \lambda\left(1-\left(\begin{array}{c} F_n(p_l,p_n)\tau_n+ \\ \frac{\partial p_l}{\partial p_n}F_l(p_l,p_n)\tau_n- \\ \alpha\tau_n\left(\frac{\partial p_l}{\partial p_n}+\frac{\partial g_l}{\partial p_n}\right) \end{array}\right)\right) \\ u'(c)F_n(p_l,p_n)\left(1-\tau_n\right) &= \lambda\left(1-\left(\begin{array}{c} F_n(p_l,p_n)\tau_n+ \\ \frac{\partial p_l}{\partial p_n}\tau_n\left(F_l(p_l,p_n)-\alpha\right)- \\ \alpha\tau_n\frac{\partial g_l}{\partial p_n} \end{array}\right)\right) \\ F_n(p_l,p_n) &= \frac{\lambda\left(1-\frac{\partial p_l}{\partial p_n}\tau_n\left(F_l(p_l,p_n)-\alpha\right)+\alpha\tau_n\frac{\partial g_l}{\partial p_n}\right)}{\lambda\tau_n+u'(c)\left(1-\tau_n\right)} \end{aligned}$$

using (6) and (7). Note that $\frac{\partial g_l}{\partial p_n}\geq 0$ by Topkis' theorem as we have that

$$\frac{\partial^2}{\partial p_n\partial g_l}\left[u\left(\begin{array}{c} F(p_l,p_n)\left(1-\tau_n\right)- \\ \left(1-\alpha\tau_n\right)\left(p_l+g_l\right) \end{array}\right)+H(g_l,g_n)\right]=-u''(c)\left(1-\alpha\tau_n\right)F_n(p_l,p_n)\left(1-\tau_n\right)\geq 0$$

Now taking the first order condition with respect to τ_n, we have

$$\begin{aligned} u'(c)\left(\begin{array}{c} F(p_l,p_n)-\alpha\left(p+g\right)+ \\ \left[F_l(p_l,p_n)\left(1-\tau_n\right)-\left(1-\alpha\tau_n\right)\right]\frac{\partial p_l}{\partial \tau_n} \end{array}\right) &= \lambda\left(\begin{array}{c} F(p_l,p_n)-\alpha\left(p_l+g_l\right)+ \\ \left[F_l(p_l,p_n)\tau_n-\alpha\tau\right]\frac{\partial p_l}{\partial \tau}-\alpha\tau\frac{\partial g_l}{\partial \tau} \end{array}\right) \\ u'(c)\left(F(p_l,p_n)-\alpha\left(p+g\right)\right) &= \lambda\left(F(p_l,p_n)-\alpha\left(p_l+g_l\right)+\tau_n\frac{dI}{d\tau_n}\right) \\ \lambda &= u'(c)\frac{F(p_l,p_n)-\alpha\left(p+g\right)}{F(p_l,p_n)-\alpha\left(p_l+g_l\right)+\tau_n\frac{dI}{d\tau_n}} \end{aligned}$$

where the second step follows from (6) and (7). So $\lambda > u'(c)$ so long as $\frac{dI}{d\tau_n}$, and the results stated in the proposition follow from the above equations for the equilibrium choices of p_n and g_n. ■

The proof of Proposition 4 follows from the arguments in the text.

The proof of the Propostion 5 is as follows:

Proof. Taking the first-order condition with respect to p_l, we obtain

$$0 = \begin{array}{c} u'(c)F_l(p_l,p_n)\left(1-\tau_n\right)+\mu\left(F_l(p_l,p_n)\tau_n-1\right)+ \\ \nu\left[\frac{H_l(g_l,g_n)F_{l,l}(p_l,p_n)}{[F_l(p_l,p_n)]^2u'(c)}+\frac{H_l(g_l,g_n)u''(c)F_l(p_l,p_n)(1-\tau_n)}{F_l(p_l,p_n)[u'(c)]^2}\right]\end{array}$$

Substituting in (16) and (13), and simplifying we obtain

$$\begin{aligned}
0 &= \begin{array}{c} u'(c)F_l(p_l,p_n)\left(1-\tau_n\right)+ \\ \left[u'(c)+\nu\left(\frac{1}{F(p_l,p_n)}+\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\right)\right]\left(F_l(p_l,p_n)\tau_n-1\right)+ \\ \nu\left[\frac{(1-\tau_n)F_{l,l}(p_l,p_n)}{F_l(p_l,p_n)}+\frac{u''(c)F_l(p_l,p_n)(1-\tau_n)^2}{u'(c)}\right]\end{array} \\
0 &= \begin{array}{c} u'(c)\left(F_l(p_l,p_n)-1\right)+ \\ \nu\left(\frac{1}{F(p_l,p_n)}+\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\right)\left(F_l(p_l,p_n)\tau_n-1\right)+ \\ \nu\left[\frac{(1-\tau_n)F_{l,l}(p_l,p_n)}{F_l(p_l,p_n)}+\frac{u''(c)F_l(p_l,p_n)(1-\tau_n)^2}{u'(c)}\right]\end{array} \\
0 &= \begin{array}{c} u'(c)\left(F_l(p_l,p_n)-1\right)+ \\ \nu\frac{F_l(p_l,p_n)\tau_n-1}{F(p_l,p_n)}+\nu\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\left(F_l(p_l,p_n)\tau_n-1+\left(1-\tau_n\right)\right)+\nu\frac{(1-\tau_n)F_{l,l}(p_l,p_n)}{F_l(p_l,p_n)}\end{array} \\
0 &= \begin{array}{c} u'(c)\left(F_l(p_l,p_n)-1\right)+ \\ \nu\frac{F_l(p_l,p_n)\tau_n-1}{F(p_l,p_n)}+\nu\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)^2\left(F_l(p_l,p_n)-1\right)+\nu\frac{(1-\tau_n)F_{l,l}(p_l,p_n)}{F_l(p_l,p_n)}\end{array} \\
F_l(p_l,p_n) &= \frac{u'(c)+\nu\left(\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)^2+\frac{1}{F(p_l,p_n)}\right)-\nu\frac{(1-\tau_n)F_{l,l}(p_l,p_n)}{F_l(p_l,p_n)}}{\left[u'(c)+\nu\left(\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)^2+\frac{\tau_n}{F(p_l,p_n)}\right)\right]} \\
F_l(p_l,p_n) &= \frac{\mu-\nu\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\tau_n-\nu\frac{(1-\tau_n)F_{l,l}(p_l,p_n)}{F_l(p_l,p_n)}}{\mu-\nu\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\tau_n-\frac{1-\tau_n}{F(p_l,p_n)}}
\end{aligned}$$

We then have that $F_l(p_l,p_n)>1$ as $\nu>0$, $F_{l,l}(p_l,p_n)\leq 0$, $\tau_n\leq 1$, and $u''(c)\leq 0$. (Note that the numerator is positive, as each term is positive: since $F_l(\cdot,\cdot)>0$ for all inputs, this implies the denominator is positive as well.)

Taking the first-order condition with respect to p_n, we obtain

$$0 = \begin{array}{c} u'(c)F_n(p_l,p_n)\left(1-\tau_n\right)+\mu\left(F_n(p_l,p_n)\tau_n-1\right)+ \\ \nu\left[\frac{H_l(g_l,g_n)F_{l,n}(p_l,p_n)}{[F_l(p_l,p_n)]^2u'(c)}+\frac{H_l(g_l,g_n)u''(c)F_n(p_l,p_n)(1-\tau_n)}{F_l(p_l,p_n)[u'(c)]^2}\right]\end{array}$$

Substituting in (16) and (13), and simplifying as we did before

$$F_n(p_l,p_n)=\frac{\mu-\nu\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\tau_n-\nu\frac{(1-\tau_n)F_{l,n}(p_l,p_n)}{F_l(p_l,p_n)}}{\mu-\nu\frac{u''(c)}{u'(c)}\left(1-\tau_n\right)\tau_n-\frac{1-\tau_n}{F(p_l,p_n)}}$$

We then have that $F_n(p_l,p_n)>1$ if $F_{l,n}(p_l,p_n)\leq 0$ as $\mu>0$, $\nu>0$, $F_{l,l}(p_l,p_n)\leq 0$, $\tau_n\leq 1$, and $u''(c)\leq 0$. (Note that the denominator is positive, as it is the same as in the expression for p_l.) ■

The proof of the Propostion 6 is as follows:

Proof. Consider the allocation of the government with centralized revenue collection. Now, under decentralized revenue collection, let the level of investment in national public goods be the same and choose α and τ_n^d so that the level of investment in each of the local public goods is the same. Note that if $g_l^c \leq g_l^*$, we will need $\alpha \leq 0$. However, by Proposition 4, we know that the level of provision of p_l^c is less than optimal, and so the necessary $\alpha < 1$.

Since the level of provision of both local goods is the same, we have that

$$\begin{aligned} 1 - \tau_n^c &= \frac{H_l(g_l, g_n)}{F_l(p_l, p_n)u'(c)} = 1 - \tau_n \\ \tau_n^c &= \tau_n^d \end{aligned}$$

Now note that under centralized revenue collection, we have that

$$F(p_l, p_n)\tau_n^c = (p_n) + (p_l^c + g_l^c)$$

and under decentralized revenue collection

$$\begin{aligned} F(p_l, p_n)\tau_n^d \left(1 - \alpha \tau_l^d\right) &= p_n^d + g_n^d \\ F(p_l, p_n)\tau_n^d &= p_n^d + g_n^d + \alpha \tau_l^d \tau_n^d F(p_l, p_n) \end{aligned}$$

but from the solution to the local government's problem, we know that

$$\tau_l^d = \frac{p_l^d + g_l^d}{F(p_l, p_n)}$$

so

$$F(p_l, p_n)\tau_n^d = \left(p_n^d + g_n^d\right) + \alpha \tau_n^d \left(p_n^c + g_n^c\right) < \left(p_n^c + g_n^c\right) + \left(p_l^c + g_l^c\right)$$

as $\alpha < 1$. Hence, if the national government, under the decentralized revenue mechanism, chooses τ_n^d and α in order to incentivize the local government to choose the same investment in local public goods as under the centralized revenue mechanism, and invests the same amount in the national public goods, it will have a revenue surplus. Suppose that it returns this surplus as a lump-sum to the citizens. Then the decentralized government will have replicated the outcome under the centralized regime.

Hence, holding investment in the national public goods fixed at the levels chosen under the centralized revenue mechanism, first increase α so that it is nonnegative, if necessary. Then lower τ_n^d until the national government's budget is balanced, i.e. the lump-sum rebate is 0. Since

$$F_l\left(p_l^d, p_n^d\right) = \frac{1-\alpha\tau_n^d}{1-\tau_n^d} \text{ and } \frac{H_l\left(g_l^d, g_n^d\right)}{u'(c)} = 1-\alpha\tau_n^d$$

this will cause both local investments to move closer to the optimal level.[20] Hence, the welfare of the represenative agent is higher and we have found a policy for the national government, under decentralized revenue generation, that does strictly better than the outcome under the optimal policy choice with centralized revenue generation. Hence, the optimal policy under decentralized revenue generation will result in higher welfare than the optimal policy choice under centralized revenue generation. ■

The proof of the Proposition 7 is as follows:

Proof. Note that the first-order conditions for the local government are the same as in the text. First, consider any policy such that the local government still collects revenue after recieving the transfer. If we plug in the national government's budget constraint into its maximization problem, we obtain:

$$\max_{p_n, g_n, \tau_n, T} \left\{u\left(F(p_l, p_n) - p_n + g_n + \alpha\left(p_l + g_l\right) + T\right) + H(g_l, g_n)\right\}$$

subject to the constraint that $T \geq 0$. Taking a derivative with respect to T, we obtain

$$-u'(c) + \beta = 0$$

where β is the Lagrange multiplier on the constraint that $T = 0$. Hence, β must be positive and hence the constraint is binding: $T = 0$, and so the optimal policy must involve either a transfer of 0 or the local government choosing to raise no revenue.

[20]Unless $\alpha = 0$, in which case only p_l^c will move closer to the optimal level.

However, if the local government is not collecting revenue, then this policy outcome is worse than that under the centralized revenue mechanism, which we know in turn is (by the previous proposition) worse than the optimal policy under the decentralized revenue mechanism. ∎