

# Polarization and the Power of Lobbyists

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## Abstract

We consider how changes in the polarization of a legislature affect the power of lobbyists. If there is only one lobbyist, and he cares about policy along the dimension of polarization, an increase in polarization causes the cost of implementing any given proposal to fall and for the equilibrium policy to be farther from the median legislator's ideal point. However, even if the lobbyist's policy goals are orthogonal to the dimension of polarization, i.e. all legislators have the same ideal point on the lobbyist's policy dimension, an increase in polarization along the general policy dimension will decrease the cost to the lobbyist of implementing any given policy in the lobbyist's policy dimension, and move the equilibrium policy in this dimension closer to the lobbyist's ideal point. Furthermore, the policy choice in the dimension of legislature polarization will likely move away from the median voter's ideal point, even though the lobbyist has no interest in the general policy dimension *per se*. With competing lobbyists, the effects on implementation cost and on equilibrium policy of an increase in polarization are ambiguous; however, the size of the optimal supermajority will fall as polarization increases.

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# 1 Introduction

Over the past few decades, American politics has become increasingly polarized. McCarty et al. (2006) argue that the center is becoming largely deserted, as liberal Republicans from the Northeast and conservative Democrats from the South dwindle in number and are replaced by more polarizing figures. However, the effects on public policy of this increase in polarization are not well understood and, in particular, the relationship between the power of lobbyists and the degree of polarization has been unexamined.<sup>1</sup> The existence of lobbying, i.e. the influence of small, organized groups to use political means to gain at the expense of the population of the whole, has long been recognized as an institutional failure of democratic government. Grossman and Helpman (1994), Becker (1983), Snyder (1991), and others have argued that lobbying distorts policy choice away from the median voter's ideal point and towards the preferences of these organized interests. In this paper, we ask how the increase in polarization that McCarty et al. (2006) and others have recognized will change the power of lobbyists.<sup>2</sup>

The question of how polarization affects lobbying outcomes contains two separate cases. The first case is when the lobbyist's preferences fall along the axis of polarization. For instance, unions arguing for greater bargaining rights for labor falls along the axis of polarization: the farther to the left the legislator, the more likely he is to support such an initiative. Environmental policy, redistributive policy, and worker safety regulation all fall into this category. The second case is exemplified in issues such as infrastructure spending: while there is no clear difference in preferences on infrastructure spending at the national level, the firms who construct highways, bridges, etc. are likely to lobby Congress for more spending in this area. Indeed, many issues that are beneath the notice of most voters have this feature: e.g., policy regarding digital rights management, the extension of copyright laws, or the differences in tax treatment between stock and mutual life insurance companies.<sup>3</sup>

We find that a more polarized legislature will result in decreasing costs to lobbyists and equilibrium policies farther from the ideal point of the median legislator in both cases. While this result is perhaps not that surprising when the lobbyist's preferences are oriented along the same dimension as the polarization of the legislature, it is also true even if all legislators agree about the best policy along the dimension that concerns the lobbyist:

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<sup>1</sup>See, however, Alesina (1988), Gilmour (1995), Groseclose and McCarty (2001), and Krehbeil (1998) among others for a discussion of general policy implications of the increase in polarization.

<sup>2</sup>Fiorina (2005) argues that the American electorate has not become significantly more polarized over time. However, we are concerned with how an increase in the polarization of a legislative body will affect policy outcomes.

<sup>3</sup>For an excellent account of the lobbying battles over this last issue, see Birnbaum (1993).

polarization along a separate policy dimension will, in equilibrium, produce a policy more to the liking of the lobbyist. In addition, increased polarization is quite likely to move policy farther from the median legislator along the other policy dimension as well.

When the policy dimension that the lobbyist is concerned with is the same as that along which preferences of legislators are polarized, the key observation is that a lobbyist needs only half the votes in order to win passage of his bill, as in Snyder (1991), and those votes will come from legislators who naturally support moving policy in the lobbyist's favor. A larger degree of polarization means that the supporting legislators will be even more amenable to moving the policy in the direction the lobbyist favors. Consider the case with three legislators: if they all have the same preferences, the lobbyist must pay an amount twice the loss in utility to the median legislator when policy moves from a status quo of the median legislator's ideal point. On the other hand, if the first and third legislators have preferences that are far from (and on opposite sides of) the median, it will cost the lobbyist only the loss in utility to the median legislator: the first or third legislator will not need to be bribed to move policy away from the median legislator's ideal point. Of course, one legislator will be vehemently against the bill, but since the lobbyist does not need his vote, the vehemence of his preferences does not matter.

When the policy dimension that the lobbyist is concerned with is orthogonal to that on which preferences of legislators are polarized, the lobbyist can still use increasing polarization to further his aims. In particular, he can propose a policy to the right (or left) of the median legislator as part of his "payment" to the winning coalition. By doing this, he may reduce his overall costs from moving policy along the dimension he cares about away from the legislature's preferred point. In this case, polarization is likely not only to move the equilibrium policy in the lobbyist's favor, but also as an unintended consequence to move policy away from the median legislator on the general policy dimension as well. Indeed, this may make it difficult to identify issues that are, in fact, orthogonal to the axis of polarization: lobbyists will intentionally polarize their issue in order to decrease costs to implement the policy they actually care about.

The case when there are competing lobbyists is considerably more complex, since if the lobbyist wishes to bribe a supermajority, as in Groseclose and Snyder (1996), polarization could reduce or increase his costs for a given policy proposal. However, the size of winning coalitions will fall, as an increase in polarization will make the marginal member of a supermajority more expensive, and so make it more likely the marginal member will be dropped and the winning coalition will shrink. This result is similar to the result in Banks (2000), where the winning coalition gets smaller as the legislators' preferences move farther from that of the lobbyist; as long as the most costly member of any supermajority

coalition is becoming more expensive, the size of the optimal supermajority coalition falls. Furthermore, if the policy dimension that the lobbyist is concerned with is orthogonal to that on which preferences of legislators are polarized, the minimal cost to pass a given policy is maximized under no polarization. Hence, the Groseclose and Snyder claim that, in (large) legislatures where legislators are indifferent over policy outcomes, the first lobbyist must spend in bribes twice the amount the second lobbyist is willing to pay in order ensure passage is inexact; by playing off of polarization in other dimensions, the lobbyist will pay at most twice the other lobbyist's willingness to pay.

The next section presents the model used throughout. The third section considers the case of one lobbyist, whose policy preferences lie along the axis of polarization, while the fourth considers the case where the one lobbyist's preferences are orthogonal to the axis of polarization. The fifth section considers the complications of a second, competing lobby. The last section concludes. All proofs may be found in the appendix.

## 2 Model

We shall use a model that is similar to the Snyder (1991) and Groseclose and Snyder (1996) models of vote-buying in legislatures. We shall consider a compact two dimensional policy space  $X \times Y \subseteq \mathbb{R}^2$  where there is a status quo policy  $(s, q) \in X \times Y$ . The first dimension we will call the dimension of polarization: the preferences of legislators along this dimension vary. The second dimension we shall call the lobbyist policy dimension: all legislators have the same preferences in this dimension.

### 2.1 Legislatures

There are a set of legislators  $\mathbb{N} = \{1, \dots, N\}$ , with  $N$  odd. Each legislator  $n \in \mathbb{N}$  has an ideal point  $x_n \in X$  and has a utility function over outcomes and money of

$$u_n(x, y, T_n) = f(x - x_n) + g(y) + T_n$$

We require that  $f(\cdot)$  is a smooth, concave function with a global maximum of 0; hence, holding transfers and  $y$  constant, agent  $n$ 's utility is maximized at his ideal point  $x_n$ . We require that  $g(\cdot)$  is a smooth, concave function with a global maximum of 0; since all the legislators have the same preferences over policy in the  $y$  dimension, we simply normalize the ideal point of each legislator to 0.  $T_n$  is any transfer payment made to the legislator by a lobbying group.

A legislature is defined by the ideal points of the legislators within it; we shall refer to  $L \equiv \{x_1, \dots, x_n\}$  as a legislature. For notational simplicity, we will always order agents by

their ideal points: if  $n < n'$ , then  $x_n \leq x_{n'}$ . Also, let  $m$  be the median legislator, that is,  $m \equiv \frac{N+1}{2}$ .

We shall say that a legislature  $L'$  is *more polarized* than a legislature  $L$  if

1.  $x'_n \leq x_n$  for all  $n < m$ ,
2.  $x'_m = x_m$ , and
3.  $x'_n \geq x_n$  for all  $n > m$ .

Essentially, an increase in polarization is a median-preserving spread of the distribution of ideal points. To isolate the effect of polarization, we shall hold the median voter constant, and consider how outcomes change as a function of how far the other legislators are from the median voter.

It is helpful to break down increases in polarization into two types, increases among high types and increases among low types. A legislature  $L'$  is *more polarized among low types* if

1.  $x'_n \leq x_n$  for all  $n < m$  and
2.  $x'_m = x_m$  for all  $n \geq m$ .

The definition for high types is analogous.

## 2.2 Lobbyist Preferences

The game includes up to two lobbyists. The first lobbyist makes the initial proposal, and then can offer bribes to each legislator that are contingent on his voting for the bill.<sup>4</sup> The lobbyist making the proposal can be thought of either as an agenda setting legislator or as an outside lobbyist who is working with an agenda setting legislator. Note that the bribes can be legislator specific, and that the lobbyist knows the type, i.e. the ideal point  $x_n$ , of each legislator. The second lobbyist may then offer bribes contingent on voting against the bill. The second lobbyist also can offer legislator specific bribes and knows the type of all  $N$  legislators, as well as the promises made by the first lobbyist.

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<sup>4</sup>We assume that lobbyists can not make bribes contingent on pivotality or other aspects of the voting profile. Dal Bó (2007) shows that a more general contracting environment may lead to costless capture: however, if legislators have even relatively weak position-taking preferences, then vote-contingent bribes are optimal. See also Mayhew (1974).

### 2.2.1 Parallel Preferences

We shall call the case where the lobbyist's utility is determined by the policy decision along the dimension of polarization of the legislature *parallel preferences*. In this case, the first lobbyist's utility function is given by

$$v(x, T) = p(x) - \sum_{n=1}^N T_n$$

where  $p(\cdot)$  is an increasing function of the policy outcome  $x$ .

The second lobbyist has similar preferences, but does not want policy to increase. Hence, his utility is given by

$$\hat{v}(x, \hat{T}) = \hat{p}(x) - \sum_{n=1}^N \hat{T}_n$$

where  $\hat{p}(\cdot)$  is a decreasing function.

### 2.2.2 Orthogonal Preferences

We shall call the case where the lobbyist's utility is determined by the policy decision in a dimension orthogonal to the dimension of polarization *orthogonal preferences*. In this case, the first lobbyist's utility function is given by

$$v(y, T) = h(y) - \sum_{n=1}^N T_n$$

where  $h(\cdot)$  is an increasing function of the policy outcome  $y$ .

The second lobbyist has similar preferences, but does not want policy to increase. Hence, his utility is given by

$$\hat{v}(y, \hat{T}) = \hat{h}(y) - \sum_{n=1}^N \hat{T}_n$$

where  $\hat{h}(\cdot)$  is a decreasing function.

Note that in this case we assume that the lobbyists are completely indifferent over outcomes along the axis of polarization.

## 2.3 Game Structure and Timing

The game is structured as follows:

1. The first lobbyist makes a proposal  $(x, y) \in X \times Y$ .
2. The first lobbyist makes legislator-specific bribes conditional on voting for the proposal.

3. The second lobbyist makes legislator-specific bribes conditional on voting for the status quo. The second lobbyist has full information regarding the strategy of the first lobbyist.
4. Each legislator either votes for or against the proposal; the proposal passes with  $\frac{N+1}{2}$  or more votes.
5. Bribes are paid and payoffs realized.

Our equilibrium concept will be subgame perfect Nash equilibrium.

In our game, the lobbyist gets to make the proposal and there is a closed amendment rule. We believe these assumptions can be justified by considering a lobbyist who may include as conditions of his bribe that a particular member propose his bill and that all members vote against any amendment to the bill. As long as the lobbyist has sufficient commitment power, amending the above legislative game to allow for an open rule will not change the results below.

### 3 Equilibrium with One Lobbyist and Parallel Preferences

We first consider the case where the second lobbyist does not make any counterbribes after the first lobbyist; that is, step 3 of the game form does not happen. We shall call this the one-lobbyist game. Furthermore, in the case of parallel preferences, there is no second policy dimension: hence, we shall assume that  $g(y) = 0$  for all  $y \in Y$ .

#### 3.1 Characterization of Equilibrium

As in Snyder (1991), it is optimal for the lobbyist to bribe only a minimal winning coalition. Given any proposal  $x$ , the lobbyist will wish to choose the lowest cost way to obtain majority support for the proposal. Hence, he will choose to bribe the  $\frac{N+1}{2}$  legislators with the highest ideal points; that is, legislators  $m, m+1, \dots, N$ . Furthermore, he will bribe each of them the minimal amount to induce them to vote for the bill, which is  $\max\{f(s - x_n) - f(x - x_n), 0\}$  for legislator  $n$ .

**Proposition 1** *With only one lobbyist and parallel preferences, the lowest cost way to pass a given proposal  $x \geq s$  is to bribe each legislator in  $\{m, m+1, \dots, N\}$  an amount  $\max\{f(s - x_n) - f(x - x_n), 0\}$ .*

#### 3.2 Change in Cost of Implementation with Polarization

We will now explore how polarization changes the power of the lobbyist. First, consider the cost to the lobbyist of implementing a given policy proposal  $x$ . We know from the previous

subsection that he will choose to bribe a minimal winning coalition of the legislators  $m, \dots, N$ . Hence, he must pay in bribes

$$\sum_{n=1}^N T_n = \sum_{n=m}^N \max \{f(s - x_n) - f(x - x_n), 0\}$$

in order to implement the policy proposal  $x$ . When  $f(s - x_n) \leq f(x - x_n)$  is satisfied, the legislator prefers the policy proposal to the status quo, so the bribe he will be paid is 0. When it is not satisfied, he must pay a bribe equal to the difference in utility between the policy proposal and the status quo. Finally, since he is only bribing a minimal winning coalition, he will only pay bribes to the median voter and the legislators with ideal points greater than the median legislator.

This last point is key: since the lobbyist only pays bribes to those agents on his ‘side’, an increase in polarization is helpful to him, as all the legislators who in equilibrium vote for the bill now require less money in order to be incentivized to do so. Of course, legislators on the other side of the median are now even less inclined to vote for the bill, but since their votes are not necessary, this does not increase the cost to the lobbyist.

To explore the policy implications of an increase in polarization, we will parameterize the change in polarization between two legislatures. Consider a legislature  $L = \{x_1, \dots, x_n\}$ . Given the definition of legislature, it is straightforward that an increase in  $x_n$  for  $n > m$  constitutes an increase in the polarization of the legislature.

Consider the cost of bribing any individual legislator  $n \geq m$  as a function of  $x$ ,  $T_n(x)$ .

$$\frac{\partial T_n(x)}{\partial x_n} = f'(x - x_n) - f'(s - x_n) \text{ if } f(s - x_n) > f(x - x_n)$$

in the region where the bribe is positive. Assuming  $x \geq s$ , since  $f$  is concave,  $f'(x - x_n) \leq f'(s - x_n)$ . Hence, the above derivative is negative. In the region where no bribe is paid, an increase in polarization will not matter. Since this is true for every legislator in the minimal winning coalition, we have that:

**Proposition 2** *With only one lobbyist and parallel preferences, if  $L'$  is more polarized than  $L$ , it is weakly more costly for the lobbyist to pass  $x$  under  $L$  than to pass  $x$  under  $L'$ .*

However, we should note that it is key that the legislators’ preferences are concave, and not just quasiconcave, in the policy space. Consider the following example. Let  $d \equiv |x - x_n|$  and let

$$f(x - x_n) = f(d) = \begin{cases} -2d & \text{for } d \leq 1 \\ -2 - (d - 1) & \text{for } d \geq 1 \end{cases}$$

so that while preferences are quasiconcave, they are not concave.

Now consider a legislature of three agents, whose ideal points are  $-r, 0, r$  where  $r \in [0, 1]$  and the status quo is  $s = 1$ . Consider the cost to pass the proposal  $x = 2$ ; the optimal strategy for the lobbyist, then, is to bribe the second and third agents. If  $r = 0$  ('low polarization') then the cost is 2 to pass the proposal; 1 is paid to each legislator, as moving from 1 to 2 is on the relatively flat part of that legislator's policy utility function when his ideal point is 0. However, if  $r = 1$ , so that we have a more polarized legislature, then the cost to pass the proposal is 3. Since the third legislator's ideal point and the status quo now coincide, it is more expensive to bribe him to vote for the proposal.

### 3.3 Change in Equilibrium Policy with Polarization

We now turn to the question of how the equilibrium policy will change as a function of polarization. The problem that the lobbyist solves is

$$\max_x \left\{ p(x) - \sum_{n=1}^N T_n \right\}$$

We know from Proposition 1 that the lobbyist will choose only to bribe a minimal winning coalition of the rightmost legislators, so the problem for the lobbyist reduces to

$$\max_x \left\{ p(x) - \sum_{n=m}^N \max \{ f(s - x_n) - f(x - x_n), 0 \} \right\}$$

Consider an example where the status quo  $s$  is a policy of 0. There are three legislators, with ideal points  $-r, 0$ , and  $r$ , and utility functions given by

$$u_n(x, T) = T_n - (x - x_n)^2$$

Then the cost to implement a given proposal  $x$  is falling as the third legislator becomes more polarized at the rate

$$\frac{\partial T_3}{\partial r} = -2(x - r) \text{ if } x > 2r$$

But now consider how this rate changes with a change in the proposal  $x$ :

$$\frac{\partial}{\partial x} \left( \frac{\partial T_3}{\partial r} \right) = -2 \text{ if } x > 2r$$

So we have that the larger the proposal  $x$ , the faster the cost to implement it falls with polarization. Since the cost to implement higher  $x$ 's falls faster, we are more likely to choose a higher  $x$  under higher polarization. Indeed, we can calculate the optimal proposal as a function of  $x$ , given  $p(x)$ . For instance, if  $p(x) = x$ , we have that

$$x^*(r) = \min \left\{ \frac{1 + 2r}{4}, \frac{1}{2} \right\}$$

and we can see that the optimal proposal is (weakly) increasing in the degree of polarization. Once  $r$  is high enough, we no longer have to bribe legislator 3 in order to vote for the optimal proposal, only the median legislator, and so the optimal proposal does not change with the degree of polarization.

The example above can be made into the following general result regarding polarization and equilibrium policy:

**Proposition 3** *With only one lobbyist and parallel preferences, if  $L'$  is more polarized than  $L$ , then the equilibrium policy outcome is weakly higher under  $L'$  than  $L$ .*

The key observation is that the ‘price’ the lobbyist must pay in order to implement a given proposal  $x$  is decreasing faster the higher  $x$  is. This is both due to the concavity of  $f$ , as exemplified above, and the fact that for high values of  $x$ , polarization will continue to decrease the necessary bribe, instead of causing legislators who are already willing to vote for the bill for no cost to become even more willing to vote for the bill. Since the price decreases faster for higher values of  $x$ , the lobbyist is now more likely to buy these better values of  $x$ ; if the price difference between filet mignon and pot roast is \$20 per lb., you will probably get the pot roast; but if it falls to \$5 per lb., you are now more likely to get the filet mignon, even if the pot roast is cheaper than it was before.

## 4 Equilibrium with One Lobby and Orthogonal Preferences

We now turn to the case where the lobbyist’s goals are orthogonal to the dimension along which preferences are polarized. In this case, we may think of the lobbyist as either an agenda setting legislator, who has a policy goal (possibly pork for her district) that other legislators would object to. Or, the lobbyist may be working with an agenda setting legislator, making a proposal to the agenda setting legislator that moves policy along the general policy dimension in a way that the agenda setter would like but does not have the votes to do independent of the lobbyist’s help; however, by working with the lobbyist the agenda setter may achieve her goals.

We find that the optimal coalition is the same as in the case of parallel preferences. Consider any proposal  $(x, y)$  and, for concreteness, assume that  $x \geq s$ . Then, just as in the case of parallel preferences, it will be optimal to bribe the  $m$  most right wing legislators. The fact that policy is also moving in the  $y$  direction is immaterial; it simply adds a fixed amount to the cost of passing the proposal regardless of the coalition chosen.

**Proposition 4** *With only one lobbyist and orthogonal preferences, a minimal cost way to pass a given proposal  $(x, y)$  is to*

a. if  $x \geq s$ , bribe each legislator in  $\{m, m + 1, \dots, N\}$  an amount

$$\max \{f(s - x_n) - f(x - x_n) + g(q) - g(y), 0\}$$

or

b. if  $x \leq s$ , bribe each legislator in  $\{1, 2, \dots, m\}$  an amount

$$\max \{f(s - x_n) - f(x - x_n) + g(q) - g(y), 0\}$$

An immediate corollary of the above proposition is that the optimal coalition to implement any given  $y$  must be composed of either the  $m$  most right wing or the  $m$  most left wing legislators. Let  $x^*(y)$  be the policy choice in the  $X$  dimension that minimizes the costs for the lobbyist of implementing  $y$ . We know that for any choice of  $(x, y)$ , the optimal coalition is either the  $m$  most right wing or the  $m$  most left wing legislators, so whatever  $x^*(y)$  is, the optimal coalition for  $(x^*(y), y)$  must be one of these extreme coalitions.

**Corollary 5** *With only one lobbyist and orthogonal preferences, a minimal cost way to implement  $y$  as policy is to either*

a. bribe each legislator in  $\{m, m + 1, \dots, N\}$  an amount

$$\max \{f(s - x_n) - f(x^*(y) - x_n) + g(q) - g(y), 0\}$$

for some  $x^*(y) \geq s$  or

b. bribe each legislator in  $\{1, 2, \dots, m\}$  an amount

$$\max \{f(s - x_n) - f(x^*(y) - x_n) + g(q) - g(y), 0\}$$

for some  $x^*(y) \leq s$ .

So, given that the lobbyist will be choosing a coalition on one side of the political aisle, he will have incentives similar to the case of parallel preferences, even though he has no direct interest in policy changes along the dimension of polarization. By tying the two issues together, he can make it cheaper to implement his desired policy, as we show in the next subsection.

#### 4.1 Change in Cost of Implementation with Polarization

Consider an example of how the cost to implement a given policy in the dimension the lobbyist cares about varies with the amount of polarization. The status quo is  $(0, 0)$ , and

there are three legislators, with ideal points  $x_1 = -r$ ,  $x_2 = 0$ , and  $x_3 = r$  in the  $X$  dimension and utility functions given by

$$u_n(x, y, T_n) = T_n - (x - x_n)^2 - y^2$$

To implement a policy of  $y = 1$ , the lobbyist must bribe two of the legislators to vote for his proposal. Let us say he chooses to bribe the second and third legislators. Then the lobbyist can use a change in  $x$  to make the proposal more palatable to the third legislator. Hence, the lobbyist will not propose  $(0, 1)$ , but instead  $(x, 1)$  to minimize his cost. His problem is to minimize total cost subject to implementing  $y = 1$ ; he solves

$$\min_x \left\{ \left( y^2 + (x - 0)^2 - (0 - 0)^2 \right) + \max \left\{ \left( y^2 + (x - r)^2 - (0 - r)^2 \right), 0 \right\} \right\}$$

which reduces to

$$\min_x \{ 2y^2 + 2x^2 - 2rx \} \text{ for } r \leq \frac{2}{\sqrt{3}}$$

so the optimal choice of  $x^*$  is  $\frac{r}{2}$ .

However, this analysis is not complete. As  $r$  grows, a small increase in  $x$  increases the utility more and more for the third legislator, due to the concavity of his utility function over policy in that dimension. Hence, for large values of  $r$ , i.e. greater than  $\frac{2}{\sqrt{3}}$ , the lobbyist will not bribe the rightmost legislator at all. Since he can not extract payments from legislators, he will only choose an  $x$  large enough to make the third legislator indifferent between the proposal and the status quo. Hence, for large values of  $r$ , the policy chosen in the  $x$  dimension will be

$$\begin{aligned} x^*(y) &= f^{-1}(f(s - x_n) + g(q) - g(y)) + x_3 = -\sqrt{(0 - r)^2 + 0 - y^2} + r \\ x^*(1) &= r - \sqrt{r^2 - 1} \text{ for } r \geq \frac{2}{\sqrt{3}} \end{aligned}$$

Note that in this regime, the optimal  $x^*$  bends back towards the median as polarization increases. This is due to the fact that it takes only a small ‘policy bribe’ to satisfy the third legislator, and to make the policy bribe any larger is wasteful, as it will necessitate a larger bribe paid to the median legislator.

The above result is true for the general model: polarization will decrease the cost of implementing  $y$  for the lobbyist, even if he wishes to move policy in a direction that is orthogonal to the dimension of polarization. First, note that for any given policy  $(x, y)$ , the cost of implementation is cheaper under a more polarized legislature, just as in the previous section. The amount necessary to bribe a given legislator  $n \geq m$  to implement the policy  $(x, y)$  is given by

$$T_n(x, y) = \max \{ f(s - x_n) - f(x - x_n) + g(q) - g(y), 0 \}$$

We then have that

$$\frac{\partial T_n(x, y)}{\partial x_n} = f'(x - x_n) - f'(s - x_n) \text{ for } \frac{f(s - x_n) - f(x - x_n) + g(q) - g(y)}{g(q) - g(y)} > 0$$

and since  $f$  is concave, if  $x \geq s$ , the above expression is negative. Hence, since polarization increases  $x_n$  for all  $n \geq m$ , the total cost to pass  $(x, y)$  is falling with polarization.

**Proposition 6** *With only one lobbyist, if  $L'$  is more polarized than  $L$ , it is weakly more costly for the lobbyist to pass  $(x, y)$  under  $L$  than to pass  $(x, y)$  under  $L'$  for all  $(x, y) \in X \times Y$ .*

The intuition behind this result is similar to the case of parallel preferences: as preferences become more polarized, the agents on one side become more amenable to a proposal that favors their preferences. We also have the following corollary:

**Corollary 7** *With only one lobbyist, if  $L'$  is more polarized than  $L$ , it is weakly more costly for the lobbyist to pass  $y$  under  $L$  than to pass  $y$  under  $L'$  for all  $y \in Y$ .*

The key point is that since the cost of any given  $(x, y)$  is falling with the degree of polarization, then the cost to implement  $y$  for any given degree of polarization is falling. Let  $x^*(y, L)$  be the optimal choice of policy in the general policy dimension for the lobbyist to implement  $y$  in legislature  $L$ . Then the cost of  $(x^*(y, L), y)$  under a legislature  $L'$  that is more polarized than  $L$  must be less, and if the lobbyist were then to reoptimize his choice of  $x$  taking into account the more polarized legislature  $L'$ , it would lower his costs even more.

## 4.2 Change in Equilibrium Policy with Polarization

Now let us now return to our motivating example in the previous subsection, but let the utility function of the lobbyist be  $h(y) = 4y$ . The problem of the lobbyist is to solve

$$\max_{x, y} \left\{ 4y - (y^2 + x^2) - \max \left\{ y^2 + (x - r)^2 - r^2, 0 \right\} \right\}$$

So long as the third member requires a positive bribe to vote for the proposal, we have that  $x^* = \frac{r}{2}$ , and so the optimal proposal is  $x^* = \frac{r}{2}$ ,  $y^* = 1$  for  $r \leq \frac{2}{\sqrt{3}}$ . Note that the optimal proposal does not change along the policy axis that the lobbyist cares about with the level of polarization. This is because even though the cost is decreasing with  $r$ , the marginal cost of such a change remains the same. As long as both parties are receiving positive bribes in equilibrium, an increase in  $y$  will increase the bribe necessary to induce them to vote for the proposal an amount independent of  $r$ .

However, when  $r \geq \frac{2}{\sqrt{3}}$ , the situation is quite different. Since the third lobbyist is no longer receiving positive bribes, the marginal cost for an increase in  $y$  is now decreasing

as polarization increases. The optimal proposal now is characterized by letting the third agent be indifferent between the suggested policy and the status quo sans bribe. Hence  $x^*(y) = r - \sqrt{r^2 - y^2}$ , and we have that the problem for the lobbyist is

$$\max_{x,y} \left\{ 4y - \left( y^2 + (x^*(y))^2 \right) \right\} = \max_y \left\{ 4y - y^2 - \left( r - \sqrt{r^2 - y^2} \right)^2 \right\}$$

and so the first-order condition of this problem is

$$4 - 2y - 4y \left( \frac{r}{\sqrt{r^2 - y^2}} - 1 \right) = 0$$

While it is cumbersome to write a closed form expression for  $y$ , we can note that  $\frac{r}{\sqrt{r^2 - y^2}}$  is decreasing in  $r$ , and so the optimal  $y$  will increase as  $r$  gets larger. Essentially, as polarization increases, to stay on the indifference curve of the third legislator requires smaller and smaller increases in  $x$  for a given increase in  $y$ . Hence, the marginal cost to bribe the median legislator for an increase in  $y$  is less.

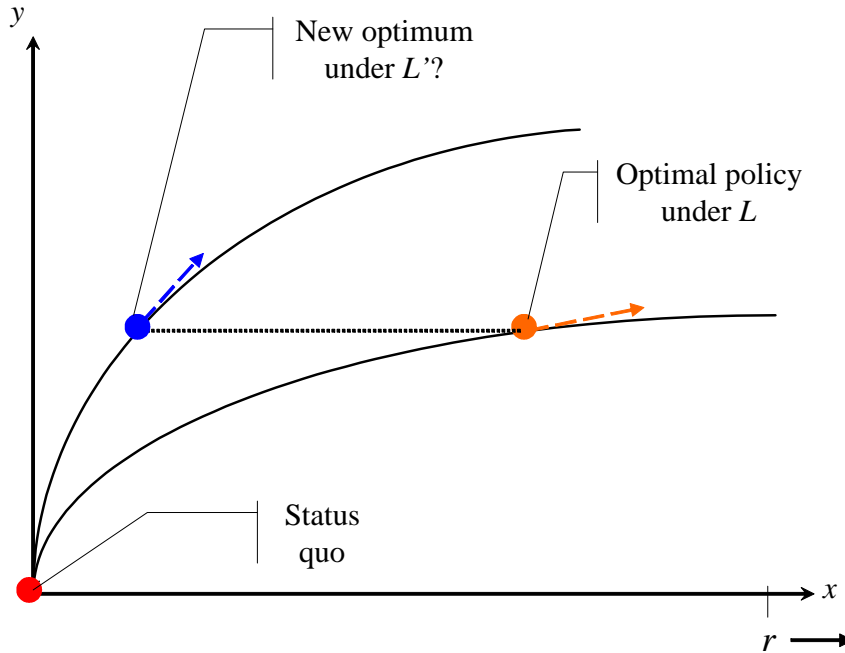


Figure 1: New optimal policy under the more polarized legislature.

This is for two reasons, as exemplified in figure 1: first, as partisanship increases, the requisite movement to the right in the  $x$  direction to stay on the third legislator's indifference curve goes down. Since the third legislator's utility is concave in the general policy dimension, as he becomes more polarized, smaller shifts in policy along that dimension are

necessary to offset a shift along the lobbyist policy dimension. Second, as the third legislator becomes more polarized, the original choice of  $x$  is smaller for a given value of  $y$ : hence, shifts to the right in the general policy dimension (in order to stay on the indifference curve of the third legislator) discomfit the median legislator less, as the starting point is closer to his ideal point in the general policy dimension.

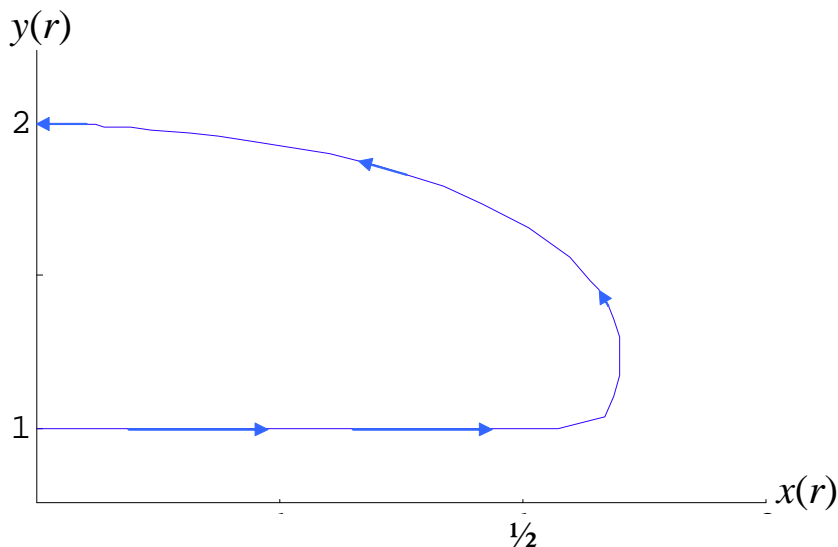


Figure 2: Equilibrium policy as a function of polarization. At first, policy in the lobbyist policy dimension is unchanged with respect to polarization, but if the legislature becomes very polarized, policy will move further towards the lobbyist’s ideal point.

The optimal policy proposal by the lobbyist is shown in Figure 2. Note that for small values of polarization, the optimal proposal along the lobbyist dimension does not change, but for larger values it is increasing in polarization. Furthermore,  $x$  is increasing not only for small values of polarization, but for larger values as well: in this example, polarization is increasing even as the value in the lobbyist dimension is increasing as well. However, for large values of polarization the value along the  $x$  dimension falls again, as the lobbyist exploits the very polarized policy position of the third legislator.

The result that policy moves to favor the lobbyist as polarization increases is quite general. Consider the case when the optimal policy in the dimension of polarization is greater than the status quo, and consider the effect of an increase in polarization among right-wing legislators. Intuitively, there are two distinct possible situations for the lobbyist. The first is when all agents receive a positive bribe in equilibrium or are receiving a positive

surplus in equilibrium (and therefore no bribe). In this case, a small increase in polarization of one legislator will not change the optimal value of  $y$ . If that legislator is receiving a positive bribe in equilibrium, then by becoming slightly more polarized, the total amount necessary to bribe that legislator will fall, but the marginal cost to increase  $y$  will remain the same: hence, the former choice of  $y$  is still optimal. If that agent is receiving a positive surplus and no bribe, then an increase in his polarization will have no effect on the optimal choice of  $x$  or  $y$ .

In the second case, there is one agent  $n$  who is receiving no bribe and is exactly indifferent between the status quo and the proposal. This is the second case in the example above. Again, if any other agent  $n'$  becomes slightly more polarized, the optimal choice of  $y$  does not change, for the reasons similar to those in the first case above: if  $n'$  is receiving a positive surplus, then this increase in his polarization does not change the problem of the lobbyist. If  $n'$  is receiving a positive bribe, then the increase in his polarization does decrease his bribe, and hence the cost to the lobbyist: however, the marginal cost of an increase in  $y$  remains the same. However, if legislator  $n$  becomes more slightly more polarized, then the lobbyist will wish to stay on agent  $n$ 's indifference curve. To do this, he will not only decrease  $x$  but also increase  $y$ , as the marginal cost of increasing policy in the dimension of concern to the lobbyist is decreasing in  $x_n$ . If the optimal proposal was the same, then agent  $n$  would be now be receiving a positive surplus from voting for the bill, and so a small increase in the  $y$  direction now has a lower marginal cost, as there is one less legislator who must be compensated for such a change. Hence, it will be optimal to slightly increase  $y$  when  $x_n$  increases slightly.

**Proposition 8** *With only one lobbyist and orthogonal preferences, let  $(x^*(L), y^*(L))$  be the optimal proposal under  $L$ . Then if  $L'$  is more polarized than  $L$ ,  $y^*(L) \leq y^*(L')$ .*

We note two implications of these results. The first is that the lobbyist always strictly prefers more polarization, so that if there are fixed costs to starting a lobby, polarization will make lobbies more likely to form and influence policy. So not only does increase in polarization move policy farther from the ideal point if a lobby, it also makes it more likely lobbies will form in the first place.

The second point to note is that as the legislature becomes very polarized, the lobbyist does not need to choose policies that are far from the median's ideal point along the axis of polarization in order to implement proposals that he likes. As in the example, as the legislature become very polarized, smaller deviations from the median legislator's ideal point are optimal for the lobbyist, even as he chooses larger increases in the policy space he is concerned with. Hence, the theory does not predict how policy will change along the

dimension of polarization as a function of polarization: we do predict, however, that policy along the lobbyist's dimension will move farther from the ideal point of the legislature and closer to that of the lobbyist.

## 5 Equilibrium with Two Lobbies

### 5.1 Change in Cost of Implementation with Polarization

The case of two lobbyists is considerably more complex. Consider an example, where we return to the case of parallel preferences, and where the policy choices are set: the status quo  $s = 0$  and the proposed policy  $x = 1$ . There are three legislators, with ideal points  $-r, 0$ , and  $r$ , where  $r \in [0, \frac{1}{2}]$ , and utility functions are given by

$$u_n(x, T) = T_n - |x - x_n|^3$$

There is also a second lobby willing to pay an amount  $W = 4$  in order to retain the status quo. In this case, the minimal amount that the first lobbyist must pay in bribes in order to ensure passage is non-monotonic in  $r$ . From the perspective of the first lobbyist, there are two ways to ensure passage of the bill. The first is to bribe the lowest cost minimal winning coalition, in this case legislators 2 and 3. To stop the second lobbyist from counterbribing either of these legislators, each legislator that is part of the winning coalition must receive a surplus of  $W = 4$ . Hence, the total cost of bribes under this strategy is

$$\begin{aligned} & \left(4 + |x|^3 - |s|^3\right) + \\ & \left(4 + \left(|x - r|^3 - |s - r|^3\right)\right) = 10 - 3r(1 - r) - 2r^3 \end{aligned}$$

Note that the cost is decreasing as the legislature becomes more polarized. Since the first lobbyist is only bribing the legislators with views closer to his, the logic is the same as in the previous sections: as the legislature becomes more polarized, it is less costly to pass a given bill since the lobbyist only bribes those who favor his cause (relative to the median legislator). This cost is the thick line in Figure 2.

The second strategy, is to bribe a supermajority of legislators, in this case all three. To stop the second lobbyist from counterbribing two legislators to vote against the bill, each legislator must receive a surplus of  $\frac{W}{2} = 2$ . Hence, the total cost of bribes under this strategy is

$$\begin{aligned} & \left(2 + \left(|x + r|^3 - |s + r|^3\right)\right) + \\ & \left(2 + |x|^3 - |s|^3\right) + \\ & \left(2 + \left(|x - r|^3 - |s - r|^3\right)\right) = 9 + 6r^2 - 2r^3 \end{aligned}$$

Note that the cost in this case is increasing in the polarization of the legislature. This is because the lobbyist is now bribing all the legislators, and as their ideal points move away

from the median, the shape of the preferences dictates that the cost to bribe the majority of the whole enough that the second lobbyist does not wish to counterbribe is increasing with polarization. This cost is the thin line in Figure 3.

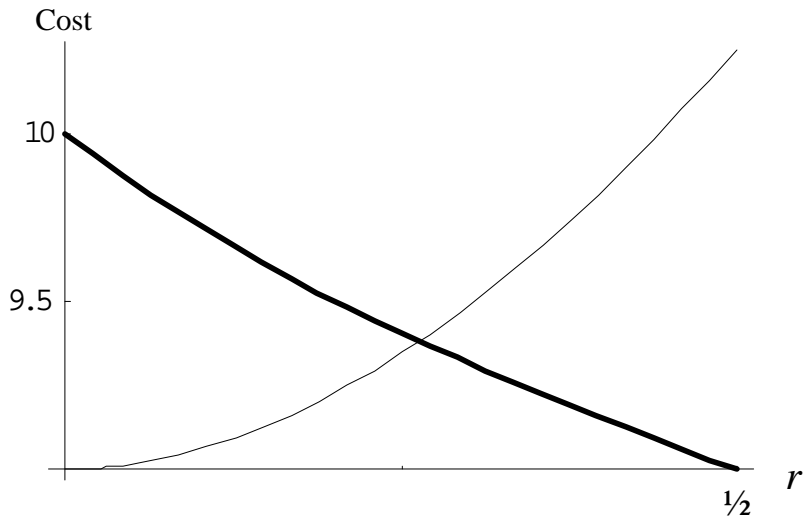


Figure 3: Cost of implementation as a function of  $r$ , the degree of polarization. The cost to bribe the minimal winning coalition (the thick line) is falling with  $r$ , while the cost to bribe the coalition of the whole (the thin line) is increasing in  $r$ .

The graph of the minimal cost is the lower envelope of the two lines in Figure 3. For low values of  $r$ , it is optimal to bribe all three legislators, and so the cost of passing the policy proposal is increasing in the polarization of the legislature. At  $r^* = \frac{\sqrt{21}-3}{6} \approx .264$  the costs for both coalitions are equal, and for a more polarized legislature, it is cheaper to bribe a minimal winning coalition, and so the cost of passing the policy proposal is now decreasing in the polarization of the legislature. In general, then, we can not say how an increase in polarization will change the power of lobbyists, nor how an increase in polarization will affect the equilibrium policy.

The key issue is that with two lobbyists, we can not determine how a change in polarization will affect the price of different policies. We shall let

$$W^{\parallel}(x) \equiv \hat{p}(s) - \hat{p}(x)$$

be the amount the second lobbyist is willing to spend in order to stop the bill; clearly,  $W(x)$  is increasing in  $x$ . If the first lobbyist chooses a proposal  $x$  and a winning coalition of size

$m + k$ , then his total cost to ensure passage is

$$\sum_{n=m-k}^N \max \left\{ f(s - x_n) - f(x - x_n) + \frac{W^{\parallel}(x)}{k+1}, 0 \right\}$$

Hence the effect on cost is ambiguous: an increase in polarization makes legislators with ideal points above the mean less expensive to bribe, but ones with ideal points below the mean more expensive to bribe, so that the total effect on cost is ambiguous.

The result is similar for the case of orthogonal preferences. Since the first lobbyist may now bribe a supermajority, the effect of a change in polarization is still ambiguous. Letting

$$W^{\perp}(y) \equiv \hat{h}(q) - \hat{h}(y)$$

we have that the total cost to ensure passage is

$$\sum_{n=m-k}^N \max \left\{ f(s - x_n) - f(x - x_n) + g(q) - g(y) + \frac{W^{\perp}(y)}{k+1}, 0 \right\}$$

in the case where it is cheaper to assemble a coalition of high types than low types. So again, polarization will increase the cost of high types and decrease the cost of low types, so the effect of a change in polarization is ambiguous.

In the special case where  $g(\cdot) = 0$ , legislators are not concerned with policy along the lobbyist's dimension *per se*. Groseclose and Snyder (1996) show that in this case the cost to implement a proposal  $y$  is approximately  $2W^{\perp}(y)$ . However, if the legislature is polarized along other dimensions, the first lobbyist may be able to take advantage of this to lower his costs; if he chooses the proposal  $(s, y)$ , then the problem is exactly as in Groseclose and Snyder (1996). Hence,  $2W^{\perp}(y)$  is only an upper bound on the transfers necessary to enact passage.

## 5.2 Size of Winning Coalitions

We now consider how the size of winning coalitions changes with a change in polarization in the case of parallel preferences. One way of thinking about the Groseclose-Snyder model is as a composite of the Snyder (1991) model and a sequential ‘‘Colonel Blotto’’ game, where the first lobbyist must defend at least half of the legislators from being influenced by the second lobbyist. As a legislature becomes more polarized, the optimal winning coalition becomes smaller, as the preferences of the legislators become more important, so the game becomes more like the Snyder (1991) game than the sequential Colonel Blotto game.

To understand why an increase in polarization decreases the size of the optimal winning coalition, first note that any median-preserving spread can be broken into a median-preserving spread of high types and a median-preserving of low types. Consider the change

in the cost of various coalitions when one agent whose ideal point is less than the median becomes more polarized. Any coalition involving that agent increases in cost, while the cost of any coalition not involving that agent stays the same. Hence, the lobbyist is more likely to choose a smaller coalition, i.e. one not involving that agent, as that agent's polarization increases. So we have that the size of the minimal winning coalition can only fall as polarization among low types increases. Now consider an increase in the polarization of one high type legislator. If he is receiving positive bribes in any winning coalition, then the price of passing the proposal  $x$  falls an equivalent amount for any winning coalition. However, since the size of the bribe he receives is

$$\max \left\{ f(x - x_n) - f(x - x_n) + \frac{W^{\parallel}(x)}{k+1}, 0 \right\}$$

he may only receive bribes for small choices of  $k$ . Hence, as a high type becomes more polarized, the cost of smaller coalitions may go down, since that legislator is receiving positive bribes in equilibrium, while the cost of larger coalitions may not, since that agent is not receiving a bribe under those conditions. Hence we have the following proposition.

**Proposition 9** *With two lobbyists and parallel preferences, if  $L'$  is more polarized than  $L$ , for a given proposal  $x$  the optimal winning coalition is (weakly) smaller under  $L'$  than under  $L$ .*

Note that we do not claim that equilibrium winning coalitions will decrease in size with polarization. While the direct effect of a change in polarization will decrease the size of the minimal winning coalition for a given proposal, we do not know how a change in polarization will affect the optimal proposal. Further, we do not know how changes in the proposal will affect  $W(x)$  in comparison to how it changes the price of various legislators. Thus, we can not say how the equilibrium winning coalition will change in size with respect to polarization.

## 6 Conclusions

We have shown that polarization results in lobbyists becoming more powerful: the cost of moving policy away from the median legislator is decreasing as the legislature becomes more polarized. Furthermore, an increase in polarization has policy effects: equilibrium policy will move farther away from the median legislator as more extreme policies become cheaper faster. Note that there is also a secondary effect to an increase in polarization: it will encourage the formation of more lobbies. If we assume there is fixed cost to forming a lobby (perhaps to organize an enforcement mechanism to solve the collective action problem), an

increase in polarization will give stronger incentives for lobbies to form. Hence, polarization will lead to policy more oriented to the wishes of lobbyists as opposed to the wishes of the voters, as expressed through elected representatives.

Many political scientists and economists have argued that the effects of lobbying are substantial and harmful to general welfare. While stopping lobbying altogether is almost certainly a pipe dream, we can consider how the design of political institutions will affect lobbying outcomes. While some work in this regard has been done,<sup>5</sup> much work remains. This essay has shown that polarization is likely to lead to more powerful lobbyists: hence, institutional features to reduce the polarization of the legislature (for a given set of voters) may well lead to superior legislative outcomes, by reducing the influence of lobbyists.

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<sup>5</sup>See, for instance, Deirmeier and Myerson (2000) and Bardhan & Mookherjee (2000), among others.

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## 7 Appendix

**Proof of Proposition 1.** The proof of the first proposition is essentially the one given in Snyder (1991). Consider any cheapest winning coalition that does not include legislator  $n$ ,  $n \geq m$ . Then

$$f(s - x_n) \geq f(x - x_n)$$

and since the bill passes, there exists a  $n' < m$  such that

$$\begin{aligned} T_{n'} &= \max \{f(s - x_{n'}) - f(x - x_{n'}), 0\} \\ &= f(s - x_{n'}) - f(x - x_{n'}) \\ &\geq f(s - x_n) - f(x - x_n) \end{aligned}$$

from the concavity of  $f$ . Hence, a cheaper winning coalition could be constructed by excluding  $n'$ , and adding  $n$ , lowering the cost of the winning coalition. Hence any cheapest winning coalition includes  $\{m, m + 1, \dots, n\}$ . ■

To simplify the following proofs, we first prove a short lemma that if  $L'$  is more polarized than  $L$ , we can construct intermediate legislatures where only one agent has changed his ideal point, without changing the order of the agents.

**Lemma 10** *For a legislature  $L$  and a more polarized legislature  $L'$ , we have that there exist a set of intermediate legislatures  $L = L_0, L_1, L_2, \dots, L_N = L'$  such that  $L_i$  is less polarized than  $L_{i+1}$  and differs by only one ideal point.*

**Proof.** Let  $L_i = \{x'_1, \dots, x'_i, x_{i+1}, \dots, x_N\}$  for  $1 \leq i \leq m$ . Note that the order does not change, as the more polarized agent is moved away from the median first, so only the ideal point of  $i$  changes. And since  $L'$  is more polarized than  $L$ , we have that  $L_i$  is more partisan than  $L_{i-1}$ . Now let  $L_i = \{x'_1, \dots, x'_m, x_{m+1}, \dots, x_{N-i}, x'_{N-i+1}, \dots, x'_N\}$  for  $m+1 \leq i \leq N$  and the same argument shows that the result holds. ■

**Proof of Proposition 2.** We will consider the case where  $x \geq s$ , and the optimal winning coalition  $\{m, \dots, N\}$ . For  $x \leq s$ , the optimal winning coalition is  $\{1, \dots, m\}$ , and the proof is symmetric.

Consider an increase in the polarization of agent  $n$ . By the Lemma, this is sufficient to show the result. If  $n < m$ , then we are done. If  $n > m$ , then<sup>6</sup>

$$T_n(x; x_n) - T_n(x; x'_n) = \left( \frac{f(s - x_n) -}{f(x - x_n)} \right)^+ - \left( \frac{f(s - x'_n) -}{f(x - x'_n)} \right)^+$$

First, note that due to the concavity of  $f$ , we have that

$$\begin{aligned} \left( \frac{f(s - x_n) -}{f(x - x_n)} \right) - \left( \frac{f(s - x'_n) -}{f(x - x'_n)} \right) &\geq 0 \\ f(s - x_n) - f(x - x_n) &\geq f(s - x'_n) - f(x - x'_n) \end{aligned}$$

due to the concavity of  $f$ . Hence, there are three cases to consider:

1. If

$$\left( \frac{f(s - x_n) -}{f(x - x_n)} \right)^+ = \left( \frac{f(s - x'_n) -}{f(x - x'_n)} \right)^+ = 0$$

then

$$T_n(x; x_n) - T_n(x; x'_n) = 0$$

2. If

$$\left( \frac{f(s - x_n) -}{f(x - x_n)} \right)^+ > 0 = \left( \frac{f(s - x'_n) -}{f(x - x'_n)} \right)^+$$

then

$$T_n(x; x_n) - T_n(x; x'_n) = f(s - x_n) - f(x - x_n)$$

3. If

$$\left( \frac{f(s - x_n) -}{f(x - x_n)} \right)^+ > 0 \text{ and } \left( \frac{f(s - x'_n) -}{f(x - x'_n)} \right)^+ > 0$$

then

$$T_n(x; x_n) - T_n(x; x'_n) = f(s - x_n) - f(x - x_n) - (f(s - x'_n) - f(x - x'_n)) \geq 0$$

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<sup>6</sup>We will use the notation  $y^+$  to denote  $\max\{y, 0\} = y\mathbb{I}[y \geq 0]$ .

■

**Proof of Proposition 3.** Consider an increase in the polarization of agent  $n$ . By the Lemma, this is sufficient to show the result.

To prove the proposition, consider

$$\varphi(x, x_n) \equiv p(x) - \sum_{j=m}^N \left( \frac{f(s - x_j) -}{f(x - x_j)} \right)^+$$

We have that

$$\frac{\partial \varphi(x, x_n)}{\partial x} = \begin{cases} r(x) & \text{for } f(s - x_m) < f(x - x_m) \\ r(x) + f'(x - x_j) & \text{for } f(s - x_m) > f(x - x_m) \text{ and } f(s - x_{m+1}) < f(x - x_{m+1}) \\ \vdots & \vdots \\ r(x) + \sum_{j=m}^N f'(x - x_j) & \text{for } f(s - x_N) > f(x - x_N) \end{cases}$$

since  $\varphi(x, x_n)$  is differentiable almost everywhere. Now, note that, where it exists,  $\frac{\partial \varphi(x, x_n)}{\partial x} = K(x) + f'(x - x_n) \mathbb{I}[f(s - x_n) > f(x - x_n)]$  where  $K(x)$  is a function of  $x$  but not  $x_n$ . Note that  $f'(x - x_n)$  is always negative as, if  $f(s - x_n) > f(x - x_n)$  and  $s \leq x$ , then  $x_n < x$ . Furthermore,  $K(x) + f'(x - x_n) \mathbb{I}[f(s - x_n) > f(x - x_n)]$  is constant with respect to  $x_n$  if  $\mathbb{I}[f(s - x_n) > f(x - x_n)] = 0$  and increasing if  $\mathbb{I}[f(s - x_n) > f(x - x_n)] = 1$  as  $f$  is concave. Hence,  $\frac{\partial \varphi(x, x_n)}{\partial x}$  is an increasing function of  $x_n$  everywhere it exists, which is almost everywhere and by the standard arguments of comparative statics, the optimal choice of  $x$  by the lobbyist is increasing in  $x_n$ .<sup>7</sup> ■

**Proof of Proposition 4.** Consider any proposal  $(x, y)$  where  $x \geq s$ . Then, by exactly the same logic as in Proposition 1, the cheapest winning coalition includes  $\{m, m + 1, \dots, n\}$ . For any proposal  $(x, y)$  where  $x \leq s$ , by Proposition 1, the cheapest winning coalition includes  $\{1, 2, \dots, m\}$ . ■

**Proof of Proposition 6.** The proof here follows exactly as for Proposition 2. ■

**Proof of Proposition 8.** Consider the case when the original proposal under  $L$  is  $(x, y)$ , where  $x \geq s$ . Then the winning coalition is given by  $\{m, m + 1, \dots, N\}$ . By the Lemma, we need only consider an increase in the type of one agent  $n > m$ . If  $f(s - x_n) + g(q) < f(x - x_n) + g(y)$ , then legislator  $n$  is receiving no transfer in equilibrium, and so a small change in  $x_n$  has no effect on the equilibrium outcome. If  $f(s - x_n) + g(q) > f(x - x_n) + g(y)$ , then the effect on the marginal cost of an increase in  $y$  of a change in  $x_n$  is 0, and so the change in  $y$  will be 0. Hence, the only case we are concerned with is the case when  $n$  is the legislator who is just indifferent between the proposal and the status

<sup>7</sup>The sufficiency of increasing differences for the maximizer to be weakly increasing in a parameter is a well-known result in comparative statics. Milgrom and Shannon (1994) is the classic reference; Ashworth and Bueno de Mesquita (2006) is an excellent primer.

quo, and it continues to be optimal (for a small enough change in  $x_n$ ) for the lobbyist to choose a policy that keeps him indifferent. Hence, in this case, the lobbyist will solve

$$\max_y \left\{ h(y) - \sum_{n'=m}^{n-1} [f(s - x_{n'}) - f(x^*(x_n, y) - x_{n'}) + g(q) - g(y)] \right\} \equiv \max_y \{Z(y, x_{n'})\}$$

as if the agent with ideal point  $x_n$  is indifferent, every agent with a higher ideal point must strictly prefer the proposal and every agent with a lower ideal point must strictly prefer the status quo. Furthermore,  $x^*(x_n, y)$  is given by

$$x^*(x_n, y) = f^{-1}(f(s - x_n) + g(q) - g(y)) + x_n$$

so we have that

$$\begin{aligned} \frac{\partial x^*}{\partial x_n} &= 1 - \frac{f'(s - x_n)}{f'(x^* - x_n)} < 0 \\ \frac{\partial x^*}{\partial y} &= -g'(y) (f^{-1})'(f(s - x_n) + g(q) - g(y)) = \frac{-g'(y)}{f'(x^* - x_n)} \end{aligned}$$

using the inverse function theorem to obtain the latter result.

From the objective function, we have that

$$\begin{aligned} \frac{\partial Z}{\partial y} &= h'(y) + \sum_{n'=m}^{n-1} \left[ f'(x^*(x_n, y) - x_{n'}) \frac{\partial x^*}{\partial y} + g'(y) \right] \\ &= h'(y) + \sum_{n'=m}^{n-1} g'(y) \left[ 1 - \frac{f'(x^* - x_{n'})}{f'(x^* - x_n)} \right] \end{aligned}$$

and so

$$\frac{\partial^2 Z}{\partial x_n \partial y} = - \sum_{n'=m}^{n-1} g'(y) \left[ \frac{f''(x^* - x_{n'}) f'(x^* - x_n) \frac{\partial x^*}{\partial x_n} - f''(x^* - x_n) \left( \frac{\partial x^*}{\partial x_n} - 1 \right) f'(x^* - x_{n'})}{(f'(x^* - x_n))^2} \right]$$

However, since  $g'(y) < 0$ , we wish to show the term in the brackets is positive. Note that  $f''(x^* - x_{n'}) < 0$ ,  $f'(x^* - x_n) > 0$  (as agent  $n$  is indifferent between the status quo and the higher policy  $x$ , so the slope of his utility with respect to an increase in policy along the general policy dimension must be positive), and  $\frac{\partial x^*}{\partial x_n} < 0$ , so the first term of the sum is always positive. For the second term, since  $f''(x^* - x_n) < 0$  and  $\frac{\partial x^*}{\partial x_n} - 1 < 0$ , we need to show that  $\sum_{n'=m}^{n-1} f'(x^* - x_{n'}) \leq 0$ . This follows from the maximization problem of the lobbyist over the choice of  $x^*$ . The cost to bribe agents  $m, \dots, n-1$  is

$$\sum_{n'=m}^{n-1} \left( g(q) - g(y) + f(s - x_{n'}) - f(x^* - x_{n'}) \right)$$

which can not be strictly decreasing in  $x^*$ : otherwise the lobbyist could increase  $x^*$  and lower his total costs. (Note that agents  $n, n+1, \dots, N$  all strictly prefer a higher value of  $x^*$ , and so the cost to bribe them will not rise with  $x^*$ .) Hence

$$\begin{aligned}
 0 &\leq \sum_{n'=m}^{n-1} -f'(x^* - x_{n'}) \\
 0 &\geq \sum_{n'=m}^{n-1} f'(x^* - x_{n'})
 \end{aligned}$$

Hence, the optimization problem exhibits increasing differences in  $x$  and  $x_n$ , and by the standard arguments of comparative statics, the optimal choice of  $y$  by the lobbyist is increasing in  $x_n$ . ■

**Proof of Proposition 9.** As given in the text. ■