The Impact of the Internet on Advertising Markets for News Media

by

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In this paper, we explore the hypothesis that an important force behind the collapse in advertising revenue experienced by newspapers over the past decade is the greater consumer switching facilitated by online consumption of news. We introduce a model of the market for advertising on news media outlets whereby news outlets are modeled as competing two-sided platforms bringing together heterogeneous, partially multi-homing consumers with advertisers with heterogeneous valuations for reaching consumers. A key feature of our model is that the multi-homing behavior of the advertisers is determined endogenously. The presence of switching consumers means that, in the absence of perfect technologies for tracking the ads seen by consumers, advertisers purchase wasted impressions: they reach the same consumer too many times. This has subtle effects on the equilibrium outcomes in the advertising market. One consequence is that multi-homing on the part of advertisers is heterogeneous: high-value advertisers multi-home, while low-value advertisers single-home. We characterize the impact of greater consumer switching on outlet profits as well as the impact of technologies that track consumers both within and across outlets on those profits. Somewhat surprisingly, superior tracking technologies may not always increase outlet profits, even when they increase efficiency. In extensions to the baseline model, we show that when outlets that show few or ineffective ads (e.g. blogs) attract readers from traditional outlets, the losses are at least partially offset by an increase in ad prices. Introducing a paywall does not just diminish readership, but it furthermore reduce advertising prices (and leads to increases in advertising prices on competing outlets). JEL Classification Numbers: L11, L82

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1 Introduction

A recent report of the Federal Communication Commission found that U.S. Newspaper advertising revenues dropped 47% from 2005 to 2009. The ad revenue decline is pronounced even when controlling for obvious explanatory factors such as circulation, decline in revenues from classified ads and the business cycle. From a public policy perspective, the likely reduction in investigative, enterprise and beat reporting represents a serious source of concern. The average newsroom shrunk by a quarter with more than 50% due to heavy cuts of editorial costs. The report concludes that “in very real ways, the dramatic newspaper industry cutbacks appear to have caused genuine harm to American citizens.”

The decline in advertising revenue has been almost unanimously attributed to the rise of the Internet. However, the adverse impact of the web represents an economic puzzle because, in many respects, the forces influencing supply and demand appear to be as favorable for the industry, if not more so, than before. Online consumption of news media created new and improved advertising products and services that should be, in principle, more valuable to advertisers (e.g. enhanced ads, targeting capabilities, and improved measurement). Moreover, the Internet dramatically increased the accessibility of many outlets for a wider audience.

A variety of theories have been proposed to explain the drop in advertising revenue. A common theme is that there is a glut in the supply of advertising space (Rice, 2010). However, this argument fails to account for the fact that while there may be space for every advertiser on the web, those ads must be still viewed by actual consumers: human attention is naturally scarce which limits the amount of advertising that can be supplied. Another theme is that online or digital ads are far less effective than ads that are on paper. However, the evidence is not

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2 According to the Newspaper Association of America (www.naa.org), since 2000 total advertising revenue earned by its member US newspapers declined by 57% in real terms to be around $27 billion in 2009. Much of this decline was in revenue from classifieds but total display advertising revenue fell around 40%. In contrast, circulation over the same period declined by 18%. Ad revenue as a share of GDP also declined by 60%. According to ComScore, total US display advertising revenue online was around $10 billion in 2010 which includes all sites and not just newspapers.
3 The Internet has also created new types of opportunities such as “search ads.” However, many observers and regulators have noted that these new forms of advertising are complements rather than substitutes for the kind of advertising typically used by the news media; see Evans (2008, 2009). Chandra and Kaiser (2011) demonstrate that magazines who are better able to tailor content to specific consumer groups can continue to command a premium in ad rates and that this premium is associated with a consumer base with higher Internet use.
consistent with that hypothesis (see Dreze and Husherr, 2003; Lewis and Reiley, 2009; Goldfarb and Tucker, 2011b).

From an economics perspective, the industry-wide decline in advertising revenue remains a puzzle. A distinctive feature of the benchmark model in media economics pioneered by Anderson and Coate (2005) is that news outlets are the gatekeepers of their readers’ attention; that is, consumers are assumed to single home with their attention concentrated on one outlet. Thus, advertising revenues at the outlet and, hence, at the industry level reflect monopoly prices for access to those consumers. In particular, if the advertising space per outlet is constant, prices are independent of the number of outlets. Outlets compete for consumers by reducing advertising output. In this benchmark model, the comparative statics associated with changes in economic primitives are not consistent with the rise of the internet reducing aggregate ad revenue. For instance, increased competition for consumers due to lower search costs or increased entry by new outlets would lead to higher advertising prices, as those outlets scale back levels of annoying advertising to attract consumers, and then charge monopoly prices to advertisers for the reduced advertising space. In contrast to these predictions, there is evidence that competition is associated with falling prices (Anderson, Foros and Kind, 2011).

Our starting point is to consider seriously the impact of increased consumer switching (as facilitated by the Internet) that many have observed is an essential distinguishing feature of online news consumption (Fahri, 2009). Thus, while consumers may have spent 25 minutes reading the morning print newspaper, they may spend, on average, 90 seconds on a news website (Varian, 2010). This is not a reduction in the amount of consumption, but instead a reduction in ‘loyalty’ to any one outlet. Web browsers, search engines, aggregators and social network make it easy for consumers to move between outlets and increase consumer switching among outlets (Athey and Mobius, 2012), while free access removes other constraints. The basic question that follows is: how does increased consumer switching affect the advertisers’ choices and therefore aggregate advertising revenues? Can new technologies, such as those that track consumers across outlets, offset such changes?

To answer these questions we present a theory of advertising that has readers spreading their attention across multiple outlets. However, we do not assume that all consumers visit all outlets; instead, they switch outlets stochastically, that is, they multi-home but not fully (unlike the most of the existing literature on two-sided markets where agents either fully multi-home or
single-home). We deploy an equilibrium model featuring a set of heterogeneous advertisers who profit from informing readers about their products, a mass of identical (from the advertisers’ perspective) consumers with a fixed endowment of attention, and a finite number of outlets. Outlets use their consumers’ attention as an input to produce advertising inventory; the fixed number of ads that can be shown to a consumer (capacity) and advertisers purchase ads to reach them. Given the stochastic nature of consumer switching, an additional ad has uncertain benefits from the perspective of an advertiser. The ad either reaches already informed consumers (and, hence, wastes some of their attention) or informs a new one. The probability of success depends, among other things, on the outlets’ “tracking technology.” These technologies allow outlets to enhance the allocation of the ads and, hence, reduce wasteful duplication. We postulate that, as a baseline description of reality today, outlets have a superior ability to track the behavior of consumers within their outlets rather than between them (see, for example, Edelman, 2010). Finally ad prices are determined via a market clearing condition.

A key feature of our model is that the multi-homing behavior of advertisers is determined endogenously. With no consumer switching and a single market-clearing price for advertising, advertisers should place ads on all outlets. Consumer switching together with imperfect tracking of consumers across outlets creates inefficiencies in duplicated impressions. Switching by consumers is, thus, a source of diminishing returns to buying ad space on additional outlets (multi-homing). Consequently, in equilibrium, higher value advertisers choose to multi-home, as they have a higher opportunity cost of not informing readers, while lower-value advertisers single home, avoiding wasted impressions. As a result of the subtle effects of the mixed homing behavior on market prices, ad prices do not necessarily fall with switching. We show that the marginal return from an additional ad is a convex combination of marginal returns on switching consumers and loyal readers. Increased switching decreases marginal returns for multi-homing advertisers. We show that whether switching reduces profits depends on the total available ad capacity per unit of attention. With low or moderate ad capacity, fewer advertisers multi-home, and a greater range of advertiser values is served, leading to lower advertising prices. However, with high ad capacity, increased switching induces high value advertisers to purchase multiple impressions on each outlet, leading to a higher-value set of advertisers being served and higher advertising prices. Indeed, profits may exceed levels that can be achieved when either switching
or imperfect tracking is not a problem. Interestingly, this implies that outlets can have suboptimal incentives to invest in technology.

Next, we consider several applications of the theory. We show that it offers a natural solution to a number of long-standing puzzles in media economics. First, there is evidence that larger outlets command a premium, and that advertisers are willing to pay for “reach” which refers to the number of users who can be impressed through an outlet.\(^4\) However, the benchmark model with no switching predicts that prices per viewer equalize across outlets in equilibrium. We show that consumer switching makes larger outlets relatively more attractive to those advertisers who cannot afford the waste that comes with large (i.e. multi-homing) campaigns. Consequently, higher valued, single-homing advertisers sort onto the high readership outlet first, giving larger outlets a “positional advantage.” Second, rather than welcome regulation that requires public media to raise revenue from ads as opposed to be subsidized, existing outlets have typically lobbied against the lifting of advertising restrictions.\(^5\) (Public subsidies, the argument goes, should make state-owned media tougher competitors on the market for readers). We demonstrate that, when some outlets cannot sell ads (as they might if they are regulated public broadcasters or smaller blogs), ad prices will be higher. The more obvious effect behind that result is that when outlets capture consumer attention without selling ads, this reduces the capacity that can be sold to advertisers in the market, raising prices (but note, this effect is absent in traditional models). Further, because movements to and from such outlets do not create wasted impressions, efficiency and prices typically go up.

On the policy side, our model sheds light on a number of issues that we believe are important for antitrust policy. Specifically we discuss the impact of a merger (in terms of better technology and stronger discrimination power) on the allocative efficiency of consumer attention. Also, we discuss the impact of privacy regulation that reduces the extent of tracking.

Finally, we explore strategic implications arising from our model. The positional advantage arising from having a larger readership share can drive competition for consumers and, indeed, may cause outlets to invest more in quality than they would under benchmark cases or perfect tracking. This result is consistent with the stylized fact that media outlets that provide

\(^4\) Recently, this has been referred to the “ITV Premium Puzzle.” (Competition Commission 2003). However the relationship has been noted previously by Fisher et al (1980) and Chwe (1988).

\(^5\) For example see Filistrucchi, Luini and Mangani (2011) for an empirical analysis of the French advertising ban on prime-time state television.
greater “reach” command higher ad prices, all else equal.\textsuperscript{6} We also demonstrate that an outlet can gain a positional advantage by having limited content, but content that consumers visit reliably – something we term ‘magnet content.’ If outlets can ensure that a high share of consumers will at some point allocate attention to them, those outlets can command a premium in advertising markets. This suggests that outlets may focus their efforts on producing offerings that regularly attract the attention of many consumers rather than the focused attention of fewer consumers. Relatedly, we demonstrate that paywalls unilaterally imposed by an outlet can have the effect of reducing their positional advantage or giving their rivals a positional advantage in advertising markets. As a result, we identify additional competitive costs to outlets from introducing paywalls.

We have focused thus far on comparing our model to the standard setup for analyses of media markets. While most models in the media economics literature assume that consumers single-home – that is, choose to allocate attention to only one outlet – there are a few recent papers that have considered what happens when consumers multi-home, including Ambrus, Calvano and Reisinger (2011) as well as independent contributions from Anderson Foros and Kind (2011), Anderson, Foros, Kind and Peitz (2011), and George and Hogendorn (2011) among others. There are a few important distinctions between our model and the ones studied in the literature. First, our model explicitly models the consumer switching process in an environment with a fixed amount of consumer attention, allowing us to perform comparative statics with respect to the extent of switching. The alternative, where multi-homing consumers consume more media, are not as well suited for understanding trends in market prices for advertising, since they implicitly assume that total consumer attention and, thus, potential ad capacity increases with switching. Second, our model introduces a new force, which is the (potential) inefficiency created by consumer switching in the absence of perfect tracking technologies. We study the implications of the inefficiency for the advertising market equilibrium, and, in particular, for advertiser strategies and willingness to pay for ad space. In contrast, the existing literature focuses on the outlets’ choice of advertising space and the tradeoff between the revenue gained from additional advertising space and consumer disutility for ads. Consumer switching

\textsuperscript{6} A countervailing effect outside our model is that with more data about consumers, outlets can sell more targeted advertising. See Athey and Gans (2010) for an analysis of the impact of targeting technology on ad prices. See also Bergmann and Bonatti (2011) for an analysis of the interaction between online and offline media competition and targeted advertising.
increases competition between outlets and, thus, increases equilibrium ad space. Our paper treats the ad space as exogenous for much of the analysis, endogenizing it as an extension to the model, in order to highlight more clearly the novel forces introduced in our model. We, thus view our model as complementary to prior literature. Moreover, by modeling explicitly the allocation process of scarce attention, we are able to identify and characterize additional outlet reactions as well as discuss the impact of different government policies towards the news media.\footnote{This method of dealing with two-sided markets is itself novel. Rather than the outlet (or platform) choosing prices in a monopolistic or oligopolistic fashion (e.g., see the general result of Weyl, 2010), on the advertising side, revenues to outlets are determined by market clearing prices. Thus, we can analyze how technology and other factors impact on the efficiency of advertising market outcomes and, in turn, how this impacts on outlet revenues.}

We share the finding that larger outlets command a premium with the work of Crampes, Haritchabalet and Jullien (2009) and Anderson, Foros and Kind (2011). The former argue that by exploiting information from a large customer base, larger outlets have superior possibilities in targeting leading to increasing returns. In Anderson, Foros and Kind (2011), having relatively more exclusive viewers allows outlets to charge higher prices. In contrast, we show that a “positional advantage” can obtain regardless of the composition of one’s viewership and absent returns to scale. In accord with conventional wisdom among practitioners, large outlets command a premium because they reach relatively more viewers while minimizing duplication.

Our paper also relates to a number of price-theoretical papers on multi-sided markets explore the equilibrium implications of having one (or more) sides multi-homing.\footnote{A (partial) list includes Caillaud and Jullien (2003), Anderson and Coate (2005), Armstrong (2006), Gabszewicz and Wauthy (2004), Anderson Foros and Kind (2011) and Reisinger (2012)).} A common theme in these works is the idea that increased multi-homing on one side of the market, in equilibrium, reduces (and, in the limit, annihilates) the incentives to multi-home of those on the opposite side. These latter become “competitive bottlenecks” leading to a number of important positive and normative implications. This paper shows a natural instance in which increased multi-homing on one side could lead to increased multi-homing on the opposite side. Contrary to the above papers, multi-homing readers are relatively harder to impress. This highlights a fundamental matching problem at the heart of advertising markets that the Internet potentially has disrupted but also, in the future, could resolve.
2 Model Set-up

2.1 Consumer Attention and Advertiser Value

There is a unit mass of consumers. Consumer attention is allocated to media for a number (2) of periods and then they make purchases; e.g., a consumer might need to shop for an item of clothing online on a given day, but have numerous opportunities to consume media prior to that. Formally, each consumer consumes media (e.g., a web page or a series of web pages viewed within a specified time interval) in both periods. In each period, they visit outlet $i$, a consumer is impressed by $a_i$ ads (which is assumed exogenous but is endogenized in an extension). We refer to $a_i$ as the ad capacity of outlet $i$ in each period. This characterizes the supply-side of advertising markets.

On the demand-side, advertisers want each consumer to see their ad sometime over the two periods but are assumed to be indifferent about precisely when. Advertisers are characterized by a parameter, $v$, which is the value of an impression put in front of a consumer (i.e., the expected value of a lead). For the given advertiser, $v$ is the same for all consumers and independent of the number of distinct consumers receiving an impression. The value to the advertiser does not increase if the same consumer sees more than one ad impression from a given advertiser. Advertisers are heterogeneous in their valuations, and the cumulative distribution function of advertiser valuations is $F(.)$ for $v \in [0, V]$. We restrict attention to cases where $\max_i a_i < \frac{1}{2}$ so that no outlet can serve all advertisers over both attention periods.

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9 We could model the consumer has having $T$ periods where they could read one unit of content per period but that leads to a considerable cost in complication and notation with regard to the results emphasized in this paper. (so it would correspond to something like a view of an online web page or a series of web pages viewed within a short time interval where the outlet can recognize the consumer throughout the time interval).

10 If advertisers placed only a single ad on an outlet, $2a_i$ is the maximum quantity of advertisers who could possibly reach an individual consumer that stays with outlet $i$ for 2 periods.

11 We assume that all advertising is equally effective regardless of the quantity, and we assume away consumer disutility of ads (cf. Anderson and Coate, 2005).

12 An alternative specification might have advertisers aiming to reach a specific number of consumers (Athey and Gans, 2010) or a specific consumer type (Athey and Gans, 2010; Bergemann and Bonatti, 2010).

13 More generally the results here require that there is an optimal number of ads an advertiser would like to impress a consumer with. Here we have effectively assumed that that optimal number is 1.
2.2 Outlet Demand and Advertising Inventory

How do consumers allocate attention to different media outlets? We assume that whenever a consumer has an opportunity to choose, outlet $i$ is chosen with (exogenous) probability $x_i$. Thus, $x_i$ is a measure of an outlet’s intrinsic quality.\(^{14}\)

Between attention periods, an opportunity for a consumer to switch outlets arrives (independently) with probability, $\rho$.\(^{15}\) Thus, the total expected amount of attention going to $i$ is $x_i + x_i ((1 - \rho) + \rho x_i) + (1 - x_i) \rho x_i = 2x_i$. We let $D_i^l = x_i - x_i(1 - x_i)\rho$ denote the share of consumers who end up using the same outlet in each period (single-homers), that is, are (ex post) loyal to $i$ and $D_i^s = 2\rho x_i x_j$ denote the share consumers who switch between outlets $i$ and $j$ (i.e., multi-homers) in any given period. When there are no switching opportunities (i.e., $\rho = 0$), $D_i^l = x_i$ and $D_i^s = 0$ for all $\{i, j\}$. In this model, if outlets have asymmetric ad capacity, then different consumer “switching types” will generate different advertising inventories. Consumers loyal to an outlet $i$ will generate $2a_i$ in advertising inventory while a consumer switching between outlets $i$ and $j$ will generate $a_i + a_j$ in advertising inventory.

2.3 Benchmark

Given this set-up, it is useful to consider an efficient outcome for the allocation of advertisers to consumers. A first-best allocation would ensure that highest value advertisers are allocated with priority to scarce advertising inventory. Let $v_i$ denote the marginal advertiser allocated to consumers loyal to outlet $i$ and let $v_{s,ij}$ denote the marginal advertiser allocated to consumers who switch between outlets $i$ and $j$. An efficient allocation of advertisers to consumers involves allocating all advertisers with $v \geq v_i$ to outlet $i$’s loyal consumers and those with $v \geq v_{s,ij}$ to those who switch between $i$ and $j$. Thus, the marginal advertisers will be determined by: $2a_i = 1 - F(v_i)$ and $a_i + a_j = 1 - F(v_{s,ij})$.

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\(^{14}\) In our baseline model it is exogenous, but in Section 5.1 we endogenize the quality.

\(^{15}\) Here we treat this probability as independent of history (i.e., outlets a consumer may have visited earlier) or the future (i.e., outlets that they may visit later). In Section 5.2, below, we explore the implications of relaxing this assumption.
To see how this first best might be implemented in practice, consider an ad platform with a technology – the elements of which currently exist (at least online) but the implementation is far from achieving its ideal – where the following conditions hold. First, consumers can be tracked both within and across outlets with information kept as to the ads they have seen. In this situation, a consumer could be impressed by an ad at most once and advertisers could, with certainty, pay for an impression to a consumer and receive it. We term this perfect ad tracking, as the ad platform is able to track consumers across web-sites and control the ads they see in a given period of time. The second condition requires that outlets can set prices specific to the type of consumer that visits them – loyal or switching. That is, the platform can price discriminate based on consumer-type (the platform can do this, because it sees behavior of consumers on all outlets). Third, suppose that the outlets have a single level of ad capacity for all consumers and sells all of its impressions (those for loyal users and those for switching users) using the ad platform.\footnote{An alternative (but probably less realistic) assumption would be that the ad platform shares information with the outlet about the consumer type, so that the outlet can set different capacities for different types. This additional flexibility would lead to a scenario with essentially distinct markets, so that firms compete for switchers and have a monopoly over access to loyal users. It is a bit more complicated to think how this would work in practice, since consumer types would only be fully determined in the second period, after the consumer had already experienced a first-period ad capacity. We omit the formal analysis of this case.}

Now consider the consequences of these three conditions. The second and third conditions together require that prices are set to equate supply and demand for the two distinct products, impressions from loyal users and impressions from switching users. But the prices $p_i = v_i$ and $p_{ij} = v_{ij}$ defined above are exactly those prices, since advertisers will choose to advertise to a consumer so long as their value exceeds the impression price. Note that if $a_i = a_j$, then $p_i = p_j = p_{ij}$ while if $a_i > a_j$, then $p_i < p_{ij} < p_j$.

In a given period, outlet $i$ receives all of its loyal consumers, $D^l_i$, and half of the switchers between it and a given outlet $j$, $D^s_{ij}$. Given this specification, the producer surplus attributable to outlet $i$ is: $\pi_i = \sum_{j \neq i} P(a_i + a_j) a_i D^l_{ij} + P(2a_i) 2a_i D^l_j$. From this, it is clear that outlet surplus is affected by the type of consumers it attracts only if its ad capacities differ from other outlets. If $a_i = a$ for all $i$, then $\pi_i = P(2a) a \left( \sum_{j \neq i} D^l_{ij} + D^l_j \right) = 2x_i P(2a) a$. Note that these profits are independent of the mix of loyal and switching customers.
3 Market Equilibrium

We now turn to consider the market equilibrium that arises when tracking is not perfect. We focus here on the case where there are two outlets, 1 and 2; consequently we let $D^i_{12} = D'$. Each advertiser makes a choice $(n_1, n_2)$ where $n_i$ is the number of impressions purchased per customer on outlet $i$ over the two attention periods. Let $\varphi(n_1, n_2) : \{0,1,2\}^2 \rightarrow [0,1]$ denote the number of unique consumers, any advertiser choosing $(n_1, n_2)$ expects to impress. As before, the expenditures required to implement that choice is $p_1 n_1 x_1 + p_2 n_2 x_2$. A market equilibrium is described by 

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\left\{ (n_1^*, (\hat{p}_1, \hat{p}_2), n_2^* (v, \hat{p}_1, \hat{p}_2)) \right\}_{v \in [0,1], (\hat{p}_1, \hat{p}_2)} \]

where (i) for each advertiser, $v$, $(n_1^* (v, \hat{p}_1, \hat{p}_2), n_2^* (v, \hat{p}_1, \hat{p}_2)) = \arg \max_{n_1, n_2} \varphi(n_1, n_2) v - \hat{p}_1 n_1 x_1 - \hat{p}_2 n_2 x_2$ and (ii) for each outlet, $i$, $\hat{p}_i$ is such that $\int_0^1 n_1^* (v, \hat{p}_1, \hat{p}_2) dF(v) = 2 x_i a_i$. The first condition says that advertisers optimize their impression choices taking prices as given while the second condition says that the market for each outlet clears. We begin with a situation where both outlets are symmetric (i.e., $x_1 = x_2 = \frac{1}{2}$, $D^i_1 = D^i_2 = D'$ and $a_1 = a_2 = a$). In this case, it can be readily seen that $\hat{p}_1 = \hat{p}_2 = \hat{p}$.

3.1 Imperfect tracking

$\varphi$ captures the structure of tracking and to proceed we must specify that.\textsuperscript{17} The main assumption underpinning imperfect tracking is that outlets cannot track consumers – and specifically, the ads they see – across outlets but they have some scope to track them internally. Here, as attention is a primitive of the model, we need to be very specific as to how that attention becomes matched to ads – something that is usually not explicitly considered in the economics of advertising literature.

There are many different ways to specify imperfect tracking, each drawing from some aspects of the constraints faced by outlets in reality.\textsuperscript{18} In the (online) appendix, we discuss a

\textsuperscript{17} Athey, Calvano and Gans (2013) explore a situation where the number of impressions is per outlet and a continuous variable. Without loss in generality, we focus on a restricted domain here.

\textsuperscript{18} One might wonder whether a pay-per-click model of advertising would alleviate the inefficiencies created by switching. The answer is no: whatever the payment model, displaying one advertisement necessarily displaces another. For this reason, most pay-per-click advertising networks charge advertisers a price per click that is inversely proportional to the click-through rate of the ad. Thus, the overall payment of the advertiser is “per
number of alternatives, most of which lead to qualitatively similar results to the model we focus on. In this paper we select one particular formulation of imperfect tracking. Specifically, we assume the following:

(i) (Single-home) \( \varphi(1,0) = \varphi(0,1) = D' + \frac{1}{2} D^*; \)
(ii) (Intense single-home) \( \varphi(2,0) = \varphi(0,2) = D' + D^*; \)
(iii) (Multi-home) \( \varphi(1,1) = \varphi(1,1) = 2D' + \frac{1}{2} D^*; \)
(iv) (Targeted multi-home) \( \varphi(2,1) = \varphi(1,2) = 2D' + D^*; \)
(v) (Intense multi-home) \( \varphi(2,2) = \varphi(2,2) = 2D' + D^* \)

This structure captures the notion that outlets can track impressions internally across periods but not externally. The easiest context to understand this is to imagine that each outlet has two units of content (e.g., web pages or articles) and consumers do not read the same content twice. Thus, duplication to a loyal consumer on an outlet can be avoided by associating an advertisement with a unit of content. Of course, if an advertiser allows two impressions on an outlet, loyal consumers will see duplicated impressions. The reason an advertiser might choose to do this can be seen by comparing (i) and (ii) above. By moving from 1 to 2 impressions on one outlet, the advertiser impresses more switchers. With just a single impression, the outlet may place that impression on all consumers in the first period. As half of the switchers will come from the other outlet in the second period, the advertiser will only capture those if it also has a second impression that the outlet optimally places in the second period. Thus, more intense advertising on a given outlet, allows the advertiser to capture more switchers (in this case, all switchers).

While more intense single-homing on an outlet can increase the number of switchers impressed by an advertiser, it is only by multi-homing that an advertiser can impress loyals across both outlets. This can be seen by comparing (i) to (iii). In (iii) when advertisers opt to place a single impression on each outlet, the impress all of the loyals but still miss some switchers. This is because the outlets do not coordinate impressions and so, they may choose to place impressions in different time periods. Thus, there is a 50 percent chance multi-homing advertisers will impress all switchers and a 50 percent chance they will only impress half of them (or, more specifically, impress those switchers twice) so that, expectation, multi-homing

— an ad that is not clicked on often (perhaps because it is wasted, if the advertiser multi-homes) has to pay a proportionally higher price per click to justify displacing another advertiser.
impresses \( \frac{3}{4} \) of the switchers. The key assumption here is that advertisers cannot themselves coordinate their ad campaigns to concentrate on a single time period. The idea is that periods are consumer specific. For instance, different users might log in at different times, so that when a user arrives, there is no way of knowing whether that user already had a browsing session earlier in the day. We thus rule out a phenomenon that sometimes occurs in radio or TV advertising, where advertisers might seek to avoid duplicating impressions by advertising in the same time block on multiple stations.\(^{19}\) This practice is not commonly used in online advertising.\(^{20}\)

By both intensifying impressions on a given outlet and by multi-homing, an advertiser can impress all consumers. Indeed, under our assumptions, this is achieved by targeted multi-homing and so there is no extra benefit from intense multi-homing. However, ensuring it can impress all consumers is costly for the advertiser as they must purchase expected duplicated impressions on all loyal consumers on one outlet and half of the switchers.

3.2 Pure Single-Homing Consumers

To begin, it useful to consider the case where consumers are all loyal (i.e., where \( \rho = 0 \)) and single-home on a single outlet (as in Anderson and Coate, 2005). When there is no switching, outlets have a monopoly over access to a share of consumers, and advertising pricing will reflect that.

Given our specification of \( \varphi \), an advertiser need only place a single impression per consumer on an outlet to reach all of its consumers. As advertisers place the same marginal value per consumer on reaching any number of consumers and given that there are no fixed costs of advertising with different outlets, an advertiser, \( v \), will advertise on any outlet whose impression price, \( p_i \), is less than \( v \). Market clearing implies that, in equilibrium, for each \( i \), \( \hat{p}_i \) satisfies \( 1 - F(\hat{p}_i) = 2a_i \). If \( P(z) \equiv F^{-1}(1 - z) \) then \( \hat{p}_i = P(2a_i) \). Outlet \( i \)’s profits will be: \( \pi_i = x_iP(2a_i)2a_i \). Thus, in this case, no switching yields the same profits as the perfect tracking benchmark. Thus,

\(^{19}\) In the context of coordinating attention, the Superbowl commands such a large share of attention at a given period of time that advertisers can be assured of impressing that share of consumers. Consequently, the coordination opportunity afforded by this may be a reason why ad space commands such high payments per viewer during that event. We explore a similar effect below.

\(^{20}\) As we discuss in the appendix, the assumption of no tracking between outlets across periods may seem extreme, since cookies can in principle be used to track a consumer on a web browser, avoiding this duplication. Moreover, a consumer might return to the outlet from a different device or browser, or might clear their cookies. Thus, a more realistic assumption for outlets who do use cookies to avoid duplication is that there is less duplication of ad impressions within an outlet across periods than across outlets (Athey, Calvano and Gans, 2013).
contingent upon the assumption that $\rho = 0$, this is an efficient allocation of advertisers to consumers. Moreover, here, profits are invariant to the number of outlets. \(^{21}\)

### 3.3 Switchers and Outlet Advertising Demand

When $D^* = 0$ (all consumers single-home) and outlet ad capacities are equal, all advertisers multi-home. This is because an advertiser’s “reach” (incremental unique consumers) is additive across outlets. When $D^* > 0$ (some consumers single-home and others multi-home), an advertiser’s reach depends on its allocation of impressions across outlets. In particular, increasing reach across outlets faces diminishing returns and so different advertisers will aim at different levels of reach depending on the value they receive from impressions. In equilibrium, given impression prices, advertisers will pursue different strategies and will sort, in equilibrium, with higher value types purchasing weakly more impressions.

The following table characterizes the sorting that arises. It demonstrates the threshold values (continuing our assumption of symmetric outlets with $D_1^* = D_2^* = D^*$ and $a_1 = a_2 = a$) which determine an advertiser’s ($v$’s) optimal choice given prices.

<table>
<thead>
<tr>
<th>$(n_1^<em>, (v, p), n_2^</em>(v, p))$</th>
<th>$D^* \leq \frac{2}{3}$</th>
<th>$D^* \geq \frac{2}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0,0)$</td>
<td>$v &lt; v_1 \equiv p$</td>
<td>$v &lt; v_1 \equiv p$</td>
</tr>
<tr>
<td>$(1,0)$ or (0,1)</td>
<td>$v_1 \equiv \frac{2}{2-D^*} p \geq v_1 \equiv p$</td>
<td>$v_2 \equiv \frac{1}{D^*} p \geq v \equiv v_1 \equiv p$</td>
</tr>
<tr>
<td>$(1,1)$</td>
<td>$v_2 \equiv \frac{2}{2-D^*} p \geq v \equiv v_2 \equiv p$</td>
<td>-</td>
</tr>
<tr>
<td>$(2,0)$ or (0,2)</td>
<td>-</td>
<td>$v_2 \equiv \frac{2}{1-D^<em>} p \geq v \equiv v_2 \equiv \frac{1}{D^</em>} p$</td>
</tr>
<tr>
<td>$(2,1)$ or (1,2)</td>
<td>$v \equiv v_2 \equiv \frac{2}{D^*} p$</td>
<td>$v \equiv v_2 \equiv \frac{2}{1-D^*} p$</td>
</tr>
</tbody>
</table>

Note first that advertisers with a higher $v$ choose to purchase more impressions both on and across outlets. This is because relatively high value advertisers are more willing to pay for wasted impressions in order to advertise to incremental consumers. However, the extent they are willing to pay for the opportunity to impress incremental consumers is dependent on price. For example, the highest value advertiser ($V$) will only engage in a targeted multi-home strategy if

\(^{21}\) Of course, if capacity were endogenous, each outlet has an incentive to restrict capacity relative to what might be socially optimal. The capacity choice would be determined as in a Cournot model, and profits would change with the number of outlets as is standard in oligopoly models.
or \( D' \leq \frac{1}{2} (1 - D') V \). Relatively medium value advertisers will choose to purchase two impressions either on the same or across outlets depending upon whether \( D' \) is higher or lower than 2/3. Finally, lower value customers will target loyal customers and hence, single-home with one impression on one outlet.\(^{22}\)

Solving for the market equilibrium involves taking these individual demands and summing across them to derive outlet demand. Specifically, an outlet receives, for each consumer it expects to attract, a share of single-homing advertisers \(( F(v_{i1}) - F(v_i) )\) and, if \( D' \) is relatively low, an impression from each multi-homer \((1 - F(v_{i1}))\) or \( F(v_{2i}) - F(v_{1i})\) as the case may be) and a further half (under symmetry) of multi-homers (if any) who have 2 impressions on one outlet \((1 - F(v_{2i}))\). Thus, advertiser demand on an outlet is:

\[
(D' + \frac{1}{2} D')(\sigma_i (F(v_{i1}) - F(v_i)) + (1 - F(v_{i1}) + \frac{1}{2} (1 - F(v_{2i}))) \quad V > v_{2i}
\]

\[
(D' + \frac{1}{2} D')(\sigma_i (F(v_{i1}) - F(v_i)) + (1 - F(v_{i1}))) \quad V \leq v_{2i}
\]

where

\[
\sigma_i = \begin{cases} 
\max \left[ \frac{2a_i -(F(v_{2i}) - F(v_{1i}) + \frac{1}{2} (1 - F(v_{2i})) + 2a_i -(F(v_{2i}) - F(v_{1i}) + \frac{1}{2} (1 - F(v_{2i})) )}{2a_i -(1 - F(v_{1i})), 0} \right] \quad \text{if } V > v_{2i} \\
\max \left[ \frac{2a_i -(1 - F(v_{1i})))}{2a_i -(1 - F(v_{1i})), 0} \right] \quad \text{if } V \leq v_{2i}
\end{cases}
\]

That is, \( \sigma_i \) is outlet i’s share of spare capacity after sales to multi-homing advertisers. We assume that single-homers are allocated in equilibrium to each outlet according to their spare capacity (if any). When \( a_i = a_2 \), \( \sigma_i = \frac{1}{2} \).

This allows us to prove our first proposition.

**Proposition 1.** Outlet (and aggregate) demand is decreasing with \( D' \) around \( D' = 0 \).

The proof is relatively straightforward. Note first that as \( D' \to 0, v_{2i} \to \infty \). Thus, around \( D' = 0 \), no advertiser chooses to purchase more than two impressions across outlets. Note also that at \( D' = 0 \), \( v_{1i} = v_i = p \) and, total demand for an outlet, \( q(p) = 1 - F(p) \). If \( D' > 0 \) while

\(^{22}\) Note that this stands in contrast to Anderson, Foros and Kind (2011) who find that in the symmetric case all advertisers multi-home even when they differ in their value of a lead. As noted earlier, this comes from their assumption that multi-homing consumers see ads twice but, in equilibrium, outlets cannot charge for ads impressed on those consumers.
$v_{i1} > V$, then $v_{i1} > v_i = p$ and $q(p) = 1 - F(p) - \frac{1}{2}(F(v_{i1}) - F(p))$. Thus, outlet demand falls; that is, for any given price, $p$, fewer impressions are purchased.

### 3.4 Switchers and Outlet Profit

We are now in a position to examine the impact of a greater share of switchers on outlet profit. To solve for the market equilibrium, each outlet’s demand has to equal its supply. For an outlet, its total supply of advertising inventory is given by:

$$2a_iD_i + a_{i'}$$

(3)

It will often be convenient in what follows to express variables in a per customer basis. In this case, advertising inventory on outlet $i$ is $2a_i$.

Given this supply, we now consider possible equilibrium allocations of advertisers to outlets. First, is it possible that $\sigma_i = \sigma_2 = 0$ and there are only multi-homing advertisers in the market? For this to be an equilibrium, the willingness to pay of a multi-homing advertiser for an impression on an outlet must exceed the willingness to pay of a single-homing advertiser for an impression on an outlet. That is, the following two inequalities must hold:

$$(D_i' + \frac{1}{4} D')v_{i1} - (D_i' + \frac{1}{2} D')p_1 \geq (D_i' + \frac{1}{2} D')(v_1 - p_1)$$

(4)

$$(D_i' + \frac{1}{4} D')v_{i1} - (D_i' + \frac{1}{2} D')p_2 \geq (D_i' + \frac{1}{2} D')(v_2 - p_2)$$

(5)

Note that the marginal advertiser on each outlet would have to be a multi-homer and so $v_i = v_{i1}$. Note also that because the ‘just excluded advertiser’ (with value $v_{i1} - \varepsilon$) would be willing to pay that for a single impression on an outlet, $p_i > v_{i1} - \varepsilon$ for each outlet. It is clear that as $\varepsilon$ goes to zero, the willingness to pay of the just excluded advertiser to single-home exceeds the willingness to pay of the marginal multi-homing advertiser for its marginal impression. If $D' > 0$, at least one outlet must, in equilibrium, sell to single-homing advertisers. That advertiser sets the marginal price in the market.\(^{23}\) If $D' = 0$, (4) and (5) hold with equality and so a pure multi-homing equilibrium can arise.

\(^{23}\) This result stands in contrast to Anderson, Foros and Kind’s (2011) “principle of incremental pricing.” In their model, they found that an outlet could only charge an incremental price for access to its exclusive consumers. Here, in contrast, the market price is set by single-homing advertisers who compete with other advertisers for scarce advertising inventory. The marginal advertiser pays a willingness to pay for ads to impress all of the outlet’s consumers in a period and not just its loyal consumers.
Second, is an equilibrium where each outlet has both multi-homing and single-homing advertisers possible? That is, is there an equilibrium involving $\sigma_i > 0$ for all $i$? For this to arise, demand from (1) must equal supply from (3) with symmetry implying that $\sigma_i = \frac{1}{2}$. Thus, this type of equilibrium can arise.

Finally, is it possible that there are only single-homing advertisers in equilibrium? This would arise if for the highest value advertiser ($V$), its willingness to pay for an additional impression on an additional outlet were negative; that is, $v_{11} = \frac{1}{1-D^s} p > V$. In this case, $\sigma_i = 1$ and equating supply to demand implies $2a = \frac{1}{2}(1-F(p))$ or $\hat{p} = F^{-1}(1-4a)$. Thus, this equilibrium will arise if $1-F(V(1-\frac{1}{2}D^s)) > 4a$. Note, however, that as $D^s$ approaches 0, this equilibrium allocation cannot arise.

Using this, we can prove the following.

**Proposition 2.** Equilibrium prices and profits are decreasing in $D^s$ around $D^s = 0$.

This directly follows from Proposition 1 and (3); that is, aggregate demand decreases while supply stays constant for each outlet. Intuitively, when there is a small but positive set of switchers, the marginal impression of a higher valued advertiser (on a second outlet) is out-bid by the first impression of the just excluded advertiser. Consequently, the marginal advertiser in the market is of lower value as $D^s$ rises. Note also that this implies that the total number of advertisers purchasing impressions increases.

While Propositions 1 and 2 characterize changes in prices and profits as the number of switchers increases from $D^s = 0$, it is also the case that a greater number of switchers changes the composition of advertiser choices. In particular, an increase in $D^s$ increases $v_{11}$ (with marginal multi-homers becoming single-homers) and decreases $v_{21}$ (with high value multi-homers increasing their frequency on one outlet). Depending upon the rate of changes of these sets, aggregate demand may increase or decrease. Indeed, an increase could occur such that profits eventually become higher than profits when $D^s = 0$.

To demonstrate this, we assume here a specific uniform distribution of advertisers, $F(v) = v$ with $V = 1$. Under this assumption, market clearing impression prices are:
\[ p = \begin{cases} 
\frac{D^s(2-D^s)}{4+D^s(2-D^s)} (3-4a) & \text{if } \frac{1}{2} D^s > p \\
\frac{2(2-D^s)}{4-D^s} (1-2a) & \text{if } \frac{1}{2} D^s \leq p 
\end{cases} \]

Note that under symmetry, \( D^s = 2 \rho x^2 < \frac{1}{2} \). Thus, the number of switchers cannot exceed that level. When there are no advertisers purchasing multiple impressions on a single outlet, price declines with \( D^s \). However, as \( D^s \) rises, there comes a point at which price is low enough that advertisers do purchase multiple impressions. The ones that do so are the inframarginal advertisers, and so as \( D^s \) rises beyond this point, price, and hence, outlet profits, \( p(D^s 2a + D^s a) = pa \), rise.\(^{24}\)

**Figure 1 (a): Outlet Profits as a function of \( D^s \) (a = 0.4)**

\(^{24}\) It is useful to check whether multiple equilibria are possible. To rule this out as a concern note that market clearing prices in both cases above are equal if:

\[ \frac{D^s(2-D^s)}{4+D^s(2-D^s)} (3-4a) = \frac{2(2-D^s)}{4-D^s} (1-2a) \Rightarrow D^s = 2 \left( 2(1-a) - \sqrt{2(1-2a) + 4a^2} \right) \]. At this level of \( D^s \),\n
\[ p = 2(1-a) - \sqrt{2(1-2a) + 4a^2} \]; i.e., \( D^s / 2 \). So, for given ad capacities, there is no issue of multiple equilibria.
In this situation, we can calculate profits precisely. Specifically, in equilibrium,

(i) For \( D' \leq \min\{8a, 4(1-a)-2\sqrt{2(1-2a)+4a^2}\} \), advertisers single-home or multi-home with an outlet earning profits \( \pi_t = \frac{1}{2} \frac{2(2-D')}{4-D'} (1-2a)2a \);

(ii) For \( D' \in \left[ 4(1-a)-2\sqrt{2(1-2a)+4a^2}, 8a \right] \), advertisers single-home, multi-home or targeted multi-home with an outlet earning profits \( \pi_t = \frac{1}{2} \frac{D'(2-D')}{4+D'(2-D')} (3-4a)2a \);

(iii) For \( D' \geq 8a \), all advertisers single-home with an outlet earning profits \( \pi_t = \frac{1}{2} (1-4a)2a \).

Figure 1 (a) illustrates how these equilibrium outlet profits vary with \( D^s \) and hence, the impact of the Internet on the news media. To the extent that the Internet has facilitated switching, these results suggest that, for a given \( a \), profits will decline but will eventually rise as switching becomes easier (see Figure 1 (a)). Figure 1 (b) demonstrates, however, that the interaction between switching and ad capacity is subtle.

To see the intuition, note that when ad capacity is very low (case (iii) above), switching does not affect outlet profits. In this situation, capacity is so scarce that even high value advertisers choose to single-home on one outlet. Consequently, no advertiser suffers wasted impressions as a result of switching consumers and hence, their willingness to pay for a single impression on each from a given outlet does not change with the share of switchers. In contrast, for moderate amounts of ad capacity (case (ii) above), high valued advertisers find it profitable to outbid low value advertisers for an impression on consumers on a second outlet in order to impress loyal consumers there. In this case, high valued advertisers multi-home while low valued...
advertisers remain single-homers. The marginal advertiser is a single-homer because, with a positive number of switching consumers, there is some wasted impressions from multi-homing as some switchers see an impression twice. Consequently, the marginal return (in terms of expected unique impressions) of advertising on a second outlet is lower than the return to advertising on one outlet. Thus, in this case, as the share of switchers rises, the ‘cost’ in terms of wasted impressions from multi-homing rises, so high value advertisers choose to single-home, freeing up space for a lower value marginal advertiser. Hence, prices and profits fall.

In the final case, where ad capacity is relatively high, the expansion in supply means that high value advertisers who were multi-homing (but still missing some share of switchers as every wasted impression on them is also a missed impression) now increase the frequency of their impressions. They do this by purchasing a second impression on one of the outlets so that they can impress all consumers in the market (at the cost of wasted impressions on loyal consumers). For this reason, as the share of switchers grows, high value advertisers purchase two impressions per consumer on one outlet and one on the other in order to avoid missing those switchers but also because the ‘cost’ in terms of wasted impressions on loyals also falls. These high value advertisers are infra-marginal since low valued advertisers who single-home still set the market clearing price. However, as more high valued advertisers purchase additional impressions, this displaces low valued ones, causing impression prices to rise as the share of switchers increases.

It is important to note, however, that the result that profits will rise with $D^*$ relies on ad capacity being high enough. If ad capacity is scarce, impression prices never fall to a level that makes it worthwhile for infra-marginal advertisers to purchase multiple impressions on individual outlets. And though endogenous ad capacity is not the focus of this paper, we highlight the fact that if it were endogenized in the uniform distribution case, profits always fall when switching increases.

The possibility that advertisers will purchase multiple impressions at a rate that likely leads to likely waste is borne out by the ComScore data. For instance, they estimate that in the first quarter of 2011, almost 1.1 trillion display ads were delivered in the US. Of these, 19.5 billion were purchased by AT&T, 16.6 billion by Experian Interactive and 11.2 billion by

Scottrade. If the entire US population surfed the net daily during that time, they would see one AT&T ad per day.

### 3.5 Asymmetric ad capacities

While the above analysis allowed for some differences between outlets in ad capacities, the main results on imperfect tracking assumed symmetry. Here we consider what happens when ad capacities can be asymmetric. We study whether asymmetry can permit a single market clearing price for advertising and, if not, what do prices look like? Importantly, does an outlet have an incentive to reduce ad capacity in order to exercise market power in advertising markets?

The following proposition summarizes the equilibrium outcomes, showing that for sufficiently asymmetric capacities, prices are higher for the low-capacity outlet.

**Proposition 3.** Suppose that outlets are symmetric in readership, \( F(v) : U[0,1] \) but that \( a_1 < a_2 \). When ad capacities are sufficiently asymmetric ( \( a_1 \in \left[0, \frac{4a_2-D'}{2(2-D')}\right] \) and \( a_2 \in \left[\frac{1}{4}(2a_1(2-D') + D'), 1\right] \) ), then, in equilibrium, \( \hat{p}_1 > \hat{p}_2 \). Otherwise, \( \hat{p}_1 = \hat{p}_2 \).

The proof (in the appendix) demonstrates that profits are:

\[
\pi_1 = (1 - \frac{1}{2}D')(1-2a_1)2a_1
\]

\[
\pi_2 = \begin{cases} 
(1-2a_2)2a_2 & \text{if } a_2 \leq \frac{2-D'}{4} \\
\frac{2D'}{2+D'}(1-a_2)2a_2 & \text{if } a_2 > \frac{2-D'}{4}
\end{cases}
\]

Here it is clear that having a smaller ad capacity is not necessarily an advantage for outlets even if it does result in a higher impression price.

What does this imply for the incentive of an outlet to use capacity to exercise market power? When ad capacities are symmetric, outlets have incentives akin to those of quantity duopolists in choosing their ad capacities. However, while locally this may be the case, each can unilaterally generate an asymmetric equilibrium of the form described in Proposition 3. When its rival’s capacity is low, an outlet has an incentive to expand capacity so that there are no single-homers on the rival outlet. In contrast, when a rival outlet has very high capacity, an outlet may choose a low capacity so as to only sell to multi-homing advertisers. Over a non-trivial range of \( D' \), no pure strategy equilibrium exists (see the online appendix). However, if outlets choose capacities sequentially, the resulting equilibrium is asymmetric with one outlet choosing a low
and the other a high ad capacity converging to symmetry as $D'$ becomes small. Nonetheless, if each ad capacity is constrained to be no greater than $\frac{1}{4}$, then both choosing $\frac{1}{4}$ is the resulting equilibrium and no asymmetric outcome occurs. In the online appendix, we discuss the issue of endogenous ad capacities in more detail.

### 3.6 Asymmetric readership share

Asymmetric capacity choices can lead to differential prices but do not confer absolute positional advantages on outlets; depending on circumstance having either high or low capacity can result in greater profits. We now consider what happens when outlets have different content quality with one outlet being able to generate a higher readership share than the other: what happens when $x_1 > x_2 \Rightarrow D_1 > D_2$. In this case, we demonstrate that outlet 1 commands a positional advantage in the advertising market that leads to it being able to earn higher impression prices than outlet 2 alongside having a higher readership share.

To see this, observe that, if there is sufficient capacity on both outlets, single homing advertisers will sort on to outlet 1 first, since they can reach more unique viewers that way. Formally, for a given $v$, if impression prices were the same on each outlet (equal to $p$) then $(D_1 + \frac{1}{2}D')(v-p) > (D_2 + \frac{1}{2}D')(v-p)$. However, as impression prices will differ in equilibrium (specifically, the previous equation implies that it must be the case that $p_1 > p_2$ if there are single homers on outlet 2), the marginal single-homer on outlet 1 must be indifferent between the two outlets, so that $v_1 = \frac{2(D_1 p_2 - D_2 p_1)}{2(D_1 - D_2)}$, while the marginal single-homer on outlet 2 is indifferent between outlet 2 and not advertising at all, so that $v_2 = p_2$. Note that given these expressions, $v_1 > v_2 \Rightarrow (2D_1' + D_2')(p_2 - p_1) < 0$; this confirms the price premium of outlet 1.\footnote{Of course, in equilibrium, there may be no single-homers on outlet 2 which will alter this intuition as we discuss below.}

It is important to emphasize that it is the existence of switching consumers (i.e., $D' > 0$) that generates this sorting. If there are no switchers, then the marginal advertiser on each outlet is competing with a multi-homing advertiser for their marginal impression. In this case, as there are no diminishing returns to additional impressions, a higher value multi-homing advertiser will outbid a smaller value single-homing advertiser for that slot. It is only when there are switchers
that single-homing advertisers – competing against one another – determine the impression price on an outlet.

In addition to single-homers on each outlet, some advertisers will multi-home. The marginal multi-homing advertiser \((v_{11})\) will be determined by:

\[
(D_i^l + D_i^l + \frac{1}{2} D^r) v_{11} - (D_i^l + \frac{1}{2} D^r) p_i - (D_i^l + \frac{1}{2} D^r) p_2
= \max \left[ (D_i^l + \frac{1}{2} D^r)(v_{11} - p_i), (D_i^l + \frac{1}{2} D^r)(v_{11} - p_2) \right]
\] (9)

Note that if \(p_1 \leq p_2\) or there are single-homers on outlet 1, then \((D_i^l + \frac{1}{2} D^r)(v_{11} - p_1) \geq (D_i^l + \frac{1}{2} D^r)(v_{11} - p_2)\) implying that \(v_{11} = \frac{D_i^l + \frac{1}{2} D^r}{D_i^l + \frac{1}{2} D^r} p_2\). Of course, it is also possible that some advertisers will multi-home with 2 impressions on one outlet. Note that, in this case, the outlet receiving the additional impression will be outlet 2 as it has the smallest number of loyal consumers. Hence, the threshold, \(v_{12} = \frac{2(2D_i^l + D^r)}{(1-D_i^l - D_i^l - D^r)} p_2\).

Given this, market clearing implies that the following equations (for each outlet) be simultaneously satisfied:

\[
F(v) = 2a
\] (10)

\[
F(\min\{v_{11}, v_{12}\}) + F(\min\{v_{11}, v_{12}\}) - F(v_{11}) - F(v_{12}) - F(v) = 2a
\] (11)

Thus, high-valued advertisers sort on to outlet 1, while outlet 2 attracts low-valued advertisers who single-home, as well as a set of high valued advertisers who multi-home. There is an intermediate interval of advertisers who do not advertise on outlet 2. The following proposition formalizes how asymmetries between the outlets in terms of readership translate into differences in their ability to profit in the advertising market.

**Proposition 4.** Let \(F(v) : U[0,1], a_1 = a_2 = a\) and \(x_1 > x_2\). Then, \(\pi_1 > \pi_2\) if and only if

\[
\frac{1}{2} - \frac{a}{16} \left( \frac{4}{1-x_1} + \frac{4}{4(1-x_1) - D^r} \right) > a; \text{ that is, } D^r \text{ and } a \text{ are sufficiently small.}
\]

The proof of the proposition demonstrates that the asymmetric outlet case operates similarly to the symmetric outlet case but with an important difference: in general, the ‘larger’ outlet in terms of readership share can command a premium for its ad space. This is a known puzzle in traditional media economics, where in a typical model consumers are equally valuable regardless of the outlet they are on, yet in practice advertising rates are typically higher on larger outlets.
Here, because ads are tracked more effectively internally, placing ads on the larger outlet only involves less expected waste than when you place ads on the other outlet or spread them across outlets. Hence, the larger outlet can command a premium.

However, we also find one exception to this pattern when both switching and ad capacity are large. In this case, outlet 2 is a more attractive outlet for high value advertisers who multi-home with an additional impression on one outlet. These advertisers outbid single homing advertisers on outlet 2. Hence, the lowest value advertisers reside, in that case, on outlet 1 that, in turn, implies that, in equilibrium, \( p_1 < p_2 \). Thus, 1’s profit per reader may be lower than 2’s.

4 Policy Implications

4.1 The Impact of Prohibiting Tracking

In 2010, the Federal Trade Commission was exploring a policy that would give consumers the right to ‘opt out’ of tracking of any kind by websites. If widely adopted, this would eliminate tracking options for media outlets. The analysis here allows us to examine the impact of that on advertising markets.

The impact of prohibitions on tracking depends on incentives to adopt such tracking in the first place. Our analysis provides some insight into this by examining what happens to outlet profits as we move from imperfect to perfect tracking.

**Proposition 5.** Assume that \( F(v) : U[0,1] \) \( a_i = a_2 \) and \( x_i = x_2 \). For low levels of \( D^i \), outlet profits under perfect tracking exceed profits with imperfect tracking. For high levels of \( D^i \), profits under perfect tracking may be lower than profits with imperfect tracking.

This result is depicted in Figure 1(a). Our earlier analysis identified that outlets with symmetric capacities, perfect tracking yields the benchmark profit outcome. Nonetheless, here we have demonstrated that when ad capacities are sufficiently high, profits for both outlets may be higher under no tracking than under perfect tracking. The reason is that higher value advertisers are induced to purchase more impressions. This crowds out lower value advertisers who are setting price at the margin and consequently, impression prices are higher. This suggests that perfect
tracking technology might not be adopted despite their ability to generate efficient outcomes in advertising markets.\textsuperscript{27}

It is useful to note that outlets do not have a unilateral incentive to adopt perfect tracking as it has no value unless the other outlet is on board. This fact also makes it challenging for a provider of perfect tracking services to appropriate the rents from that activity as we would expect each outlet to have some hold-out power.

\textbf{4.2 The Impact of Mergers}

The evaluation of mergers between media outlets has always posed some difficult issues for policy-makers. On the one hand, if it is accepted that outlets have a monopoly over access to their consumers, then such mergers are unlikely to reduce to competitive outcomes in advertising markets. On the other hand, it is argued that a merger may indeed reduce competitive outcomes in advertising markets, increasing ad revenue, and stimulating outlet’s incentives to attract consumers. While a full delineation of these views is not possible here, the analysis thus far can speak to the question of whether a merger between outlets would reduce competitive outcomes (i.e., increase total revenue) on the advertising side of the media industry.

We consider various cases depending on what the merger does with regard to inter-outlet tracking.

\textbf{Mergers improve inter-outlet tracking}. A merger will reduce the number of wasted and missed impressions in the advertising market. While impression prices would rise, so would allocative efficiency. As noted earlier, a move to perfect tracking will generate, for a fixed ad capacity, the first best outcome. Interestingly, by Proposition 5, it is not clear that outlets would choose to merge in order to facilitate this. While allocative efficiency may rise, total advertising profits could fall in cases where $D$ and $a$ are sufficiently high.

\textbf{Mergers do not change inter-outlet tracking}. When technical constraints prevent inter-outlet tracking even post-merger, if the merged outlet charges a single price to advertisers on each outlet, the total ad revenue generated will be the same as the case where both outlets are separately owned. That follows because we have assumed that ad capacity is exogenous, so there is (by assumption) no mechanism for exercising market power: the number of outlets affects

\textsuperscript{27} This also highlights the importance of how ad capacities are chosen; something we analyze in the online appendix. That analysis demonstrates that it is, in fact, an inability to commit to not selling advertisements when ad capacity is relatively high that permits the outcome that perfect tracking may lead to lower profits than imperfect tracking.
equilibrium outcomes only through the impact on tracking and thus the efficiency of advertising on multiple outlets. A full analysis of mergers would, thus, need to consider the extension of our model to endogenous capacity. Our extension to this case (in the online appendix) suggests that in general, the forces from a standard Cournot type model will operate here as well, so that mergers will decrease ad capacity which in turn works in favor of higher prices.

**Mergers permit coordinated sales to advertisers.** Suppose that creating a single entity can allow discrimination between single-homers and multi-homers; something that cannot be done without joint ownership. To explore this, suppose that no advertiser wants to purchase multiple impressions on any one outlet, outlet readership quality is symmetric and that $F(v) : U[0,1]$. Further, suppose that, on each outlet, the monopoly owner can commit to an ad capacity allocated to multi-homers, $a_m$, and an ad capacity allocated to single homers, $a_s$. Price discrimination is achieved by charging all advertisers the same price for their first impression on one of the outlets and a different price for their second impression. The price the outlet can charge multi-homers, $p_m$ for their second impression and single-homers, $p_s$, for their single impression are determined by:

$$a_m = 1 - v_{12} \text{ and } a_s = \frac{1}{2} (v_{11} - v_i)$$

Note that $v_i = p_s$ and $v_{11}$ is determined by: $v_{11} = \frac{2}{2-D'} p_m$ given the symmetric readership assumption. Solving for prices and substituting into the profit function, $(p_s + p_m)a_m + p_s a_s$, gives:

$$\frac{1}{4} \left( (1-2a_s - a_m)(2a_s + a_m) + a_m \frac{1-2D'}{2} (1-a_m) \right)$$

Maximizing with respect to $(a_m, a_s)$ and subject to $a_s + a_m = 2a$ yields:

$$a_m = \frac{16a-2D'}{2(4-D')} \text{ and } a_s = \frac{D'}{2(4-D')}(1-4a)$$

so long as $16a > D'$.

Profits are: $\frac{64a(1-2D')(1-2a) + D'^2}{32(4-D')}$ which are greater than profits in the absence of price discrimination.

Price discrimination allows the outlet to separate advertisers’ types exploiting a sorting condition: higher types value attention relatively more. With differential prices comes a different allocation of attention. Specifically, note that, for a given $D'$, with no discrimination we achieve

\[28\] If this condition does not hold, the outlet would not choose to price discriminate.
allocative efficiency; i.e., there is no way to re-allocate attention to different advertisers to increase total surplus. What the price discrimination analysis shows is that a monopoly will introduce a further allocative distortion. Although characterizing this “rent-extraction / allocative efficiency of user attention” trade-off is beyond the scope of this paper, we believe this issue is important and should be addressed at the level of merger control.

4.3 The Impact of Blogs and Public Broadcasting

One of the factors that traditional newspapers have argued are contributing to their decline is the rise of blogs and also competition from government-subsidized media. Both of those types of outlets have in common that they either do not accept advertising or accept very little of it. Somewhat in contradiction to this position, newspapers and television broadcasters have objected to plans to allow public broadcasters to sell advertisements rather than rely on subsidies. This latter objection remains a puzzle from the perspective of traditional media economics, because requiring competing public broadcasters to sell ads will cause more annoyance for their consumers and benefit other outlets. Here we explore the impact of competition from non-advertising media outlets.

We do this by assuming that the probability that consumers visit such outlets if given the choice is \( x_b \). We also assume that the two mainstream (advertising) outlets have symmetric readership shares with \( x_1 = x_2 = \frac{1}{2} (1 - x_b) \). This implies that:

\[
D' = \frac{1}{2} (1 - x_b) (1 - \frac{1}{2} (1 + x_b) \rho)
\]

\[
D'_{12} = \rho \frac{1}{2} (1 - x_b)^2
\]

\[
D'_{ib} = \rho x_b (1 - x_b)
\]

Given this, we can prove the following:

**Proposition 6.** For \( \rho > 0 \) and exogenous \( a_1 = a_2 \), equilibrium impression prices are increasing in the popularity of the ad-free outlet, \( x_b \).

Intuitively, an increase in \( x_b \) has two effects. First, it decreases the effective supply of advertising capacity in the market. Because blog readers do not see advertisements, as attention is diverted to blogs, less attention is available for ads to be placed in front of. Second, unlike switchers between mainstream outlets, switchers between blogs and mainstream outlets do not contribute to the wasted impressions problem. Consequently, a greater share of blog readers
increases the share of blog-mainstream switchers as well and so improves the efficiency of matching. This increases the demand for advertisements. These two effects – a decrease in supply and an increase in demand – combine to raise equilibrium impression prices. It is instructive to note that, even under perfect tracking, the supply-side effect remains and so impression prices would be expected to rise with blog readership share in that case too.

Nonetheless, in terms of the impact on overall outlet profits, the price effect of an increased blog share may not outweigh the quantity effect (in terms of lost readers). If it is the case that we are comparing a situation where one output sells advertising to one where it does not (absent any quantity changes in readership), then it is clear that advertising-selling outlets prefer the situation where its rival is prohibited from selling ads.

5 Strategic Implications

We now examine the implications of our model for various strategies that might be pursued by media outlets.

5.1 Incentives to compete for readers

To illustrate the incentives to compete for readers under imperfect tracking versus perfect tracking we explore here a simple game. We suppose that prior to consumers and advertisers making any choices, outlets can invest an amount $\sigma_i$ at cost $c(\sigma_i) = \frac{1}{2} \sigma_i^2$ which generates a probability $\sigma_i \in (0,1)$ of being a high rather than a low quality outlet; that is, having a readership share of $x_i = \frac{1}{2} + \Delta > x_2 = \frac{1}{2} - \Delta$ (for $\Delta > 0$). The probabilities are independent across outlets. Therefore, if outlets choose $(\sigma_1, \sigma_2)$ then with probability $\sigma_1(1-\sigma_2)$ only outlet 1 has high quality and so $x_1 > x_2$ while with probability $\sigma_2(1-\sigma_1)$ the reverse is true. With probability $\sigma_1 \sigma_2 + (1-\sigma_1)(1-\sigma_2)$ both outlets have the same quality (high or low as the case may be) and $x_1 = x_2$.

The outlet’s choose their ‘qualities’ simultaneously. When outlets have different qualities, the high quality outlet earns $\pi^H$ while the low quality outlet earns $\pi^L$. If they have the

---

29 Ambrus and Reisinger (2006) appear to identify an effect similar to the demand effect here although they do not relate it to the efficiency of mapping or a change in the endogenous campaigns chosen by advertisers.
same quality an outlet earns $\pi$. The profits here are as derived earlier (as in the Proof of Proposition 4), and $\pi_i = x_i P(2a_i)2a_i$ if there is perfect tracking. Thus, in each case, $\pi^H > \pi > \pi^I$. It is straightforward to determine that the unique equilibrium ‘qualities’ are:

$$\sigma_1 = \sigma_2 = \frac{\pi^H - \pi}{1+\pi^H + \pi^I - 2\pi}$$

(18)

The following proposition characterizes the intensity of investments in quality as a function of the tracking technology adopted.

**Proposition 7.** For a given $x$, achieved by a uniquely high quality outlet, the equilibrium level of investment $\sigma_i$ is higher under imperfect tracking than under perfect tracking so long as $a$ is not too high.

The proof involves a simple comparison of equilibrium quality choices and is omitted. When $a$ is relatively low, as demonstrated in Figure 1(a), outlet profits are lower under imperfect tracking than under perfect tracking. In this situation, as one outlet captures a reader from another, under perfect tracking this is a simple and constant transfer of advertising revenue between outlets. Under imperfect tracking, this occurs as well as an improvement in the winning outlet’s positional advantage. That advantage grows with its market share. Consequently, the bigger the impact of investment in quality on readership share (the higher is $\Delta$), the greater the differential in profits between the winning and losing outlets. This differential drives incentives to invest and outweighs the higher profit per reader available under perfect tracking. When ad capacity is relatively large, however, we know from Proposition 4 that the positional advantage may be eliminated.

### 5.2 Limited content for reach

The analysis thus far has assumed that outlets have sufficient content to attract attention of loyal consumers throughout the relevant attention period. Of course, on the Internet, much content is provided on a smaller scale. For providers of that content, there is no possibility of attracting loyal consumers. However, here we demonstrate how such providers may have a positional advantage in advertising markets; that is, what they lose in their inability to attract frequent visits from consumers, they can make up in terms of their reach across all consumers – acting as a magnet for attention in the relevant advertising period.
To see this, we amend the model as follows. We assume that outlet 2 only has enough content to satisfy consumers for a single period. To assist in identifying it expositionally, we rename it outlet \( f \). Outlet 1 is unchanged. To focus on the impact of limited content, we will confine ourselves here to the case where \( \rho = 1 \). In this situation, the total expected traffic (over both periods) to outlet 1 is \( x_i(1 + x_i) + x_f \) and to outlet \( f \) is \( x_f(1 + x_i) \). Thus, \( D_i' = x_i(1 - x_f) \) while \( D_f' = 0 \) and outlet \( f \) only has consumers who are switchers, \( D^* = x_f(1 + x_i) \). Thus, while outlet 1 supplies ad capacity of \( D_i'2a + D' a \) into the market, outlet \( f \) only supplies \( D' a \).

The significant change that arises here is that, in addition to targeted multi-homing, advertisers now have an additional option to reach the entire market by intensively single-homing on outlet 1 with 2 impressions (i.e., \( n_i = 2, n_f = 0 \)). This yields surplus of \( \frac{1}{2}(1 + D')v - p_i \) which always exceeds targeted multi-homing (\( n_i = 2, n_f = 1 \)) which has expected surplus to advertisers of \( \frac{1}{2}(1 + D')v - \frac{1}{2}D'p_f \). Thus, when \( x_f \) is low, intense single-homers on that outlet set the price for marginal advertisers in the market.

The following proposition characterizes outcomes when one outlet has limited content.

**Proposition 8.** Let \( F(v) \) : \( U[0,1], a_i = a_2 = a \) and outlet \( f \) has limited content. The only non-dominated advertiser choices \( (n_i, n_f) \) are \( \{(1,2),(1,0),(2,0),(0,2)\} \). In equilibrium, (i) for \( x_f \) low, the marginal advertiser in the market chooses \( (0,2) \) and \( \hat{p}_i > \hat{p}_f \) while (ii) for \( x_f \) high, the marginal advertiser in the market chooses \( (1,0) \), \( \hat{p}_i < \hat{p}_f \) and there are no multi-homing advertisers. As \( x_f \) approaches 1, \( \pi_f \) approaches \( \pi_i \).

The structure of the equilibrium is interesting. When \( f \)’s share is low (\( \frac{1}{2}D' < D'_i \)) and begins to rise, outlet 1, who was exclusively selling to single-homing advertisers (1 impression) continues to do so, but high valued advertisers also purchase 2 impressions on outlet \( f \). The same is true of low valued advertisers who now become the marginal advertisers in the market at a price of \( p_f \). Consequently, \( p_f < p_i \) but as \( x_f \) rises, outlet 1’s profit falls as does total profits from advertising in the industry. This changes when \( x_f \) reaches a critical level (i.e., 0.42265 so that \( \frac{1}{2}D' > D'_i \)). At that point, marginal advertisers prefer to bid for 2 impressions on outlet \( f \) and so single-homing advertisers with a single impression on outlet 1 become the marginal advertisers at a price of \( p_1 \). This implies that \( p_f > p_i \). In addition, the high valued advertisers no longer
choose to multi-home and become exclusive to outlet 1 with 2 impressions. Nonetheless, as $x_f$ rises outlet 1’s profits continue to fall. In this case, however, industry profits rise again and indeed, when $x_f \to 1$ they approach the same level as when $x_f = 0$. In this case, the profits are split evenly between the two outlets rather than held entirely by outlet 1. Intuitively, at this point, all consumers are switchers and so there is no longer any inefficiency resulting from wasted impressions.

An interesting observation is that at this limit, there may be negative incentives to provide additional content. The small content outlet can earn exactly the same profits as the other outlet. Indeed, when $x_f$ is such that $\frac{1}{2}D^f > D_1^f$, outlet $f$ earns more than half of outlet 1’s profits. Thus, the rate of return for providing that additional content is lower for outlet 1 than for outlet $f$.

We can get a sense as to whether limited but magnet content is becoming relatively more important by looking at the type of outlets that now attract display ad impressions. ComScore reports that in the first quarter of 2011, Facebook (arguably a limited content provider) attracted over 30 percent of all display ad impressions in the US; around 350 billion impressions. In contrast, traditional, in-depth, news outlets such as Turner International, Fox Interactive and CBS Digital Attracted between 11 and 18 billion impressions (less than 2% of impressions).

5.3 Paywalls

Paywalls have been proposed as a means by which outlets with falling advertising revenue may restore profitability. Of course, there are several different types of paywalls that may be employed. One possibility is a paywall – sometimes termed ‘micropayments’ – whereby consumers pay whenever they visit a website; similar to payments for physical newspapers at the newstand. Another type is a subscription whereby consumers pay once and can access a site for a length of time. Finally, some outlets have experimented with limited paywalls that permit limited reading on websites but if consumers want to consume more they have to subscribe. Here we analyze each of these types of strategies focusing on what it does to advertising revenue for each outlet. In so doing, we focus on a situation where one outlet, in this case outlet 1, introduces a paywall while the other outlet remains free.

The exploration here will be conducted within the context of the model thus far to gain some insight on these issues. A full exploration would embed a proper model of consumer behavior in the consumer choice side of the market. Instead, we highlight one important effect of
paywalls relevant to our model; that is, to impact switching behavior, which, in turn, affects advertising markets. Specifically, we now propose that outlets are asymmetric in the probabilities that a consumer might have an opportunity to switch away from them. We define $\rho_i$ as the probability that a consumer who has visited outlet $i$, has an opportunity to switch from it. Consequently, the three consumer classes are now determined by:

$$D'_1 = x_1 - x_1(1-x_1)\rho_{12}$$  
$$D'_2 = x_2 - x_2(1-x_2)\rho_{21}$$  
$$D'_{12} = (\rho_{21} + \rho_{12})x_1x_2$$  

A higher $\rho_i$ may result from the consumer having a higher cost associated with remaining with outlet $i$. Of course, a paywall may impact upon $x_i$. However, for the most part, we will hold that effect fixed and comment on the impact of such movements below.

We begin by considering micropayments whereby outlet 1 charges consumers for each period they visit its website. Holding the impact on $x_1$ fixed, a micropayment makes it less likely that visitors to outlet 1 will stay on that outlet another period (increasing $\rho_{12}$) while making it less likely visitors to outlet 2 will switch to outlet 1 (decreasing $\rho_{21}$). This has two impacts on advertising markets. First, $D'_{12}$ could rise or fall depending upon what happens to $\rho_{21} + \rho_{12}$. If it falls, then this will put upward pressure on advertising prices if ad capacity is relatively low. Second, recall that when readership shares were asymmetric, an outlet commanded a positional advantage if its expected share of loyal consumers was relatively high. However, holding $x_1$ fixed and starting from a symmetric position prior to the paywall, micropayments on outlet 1 will lead to more loyal users on outlet 2 than on 1 ($D'_2 > D'_1$). Consequently, outlet 2 will be given a positional advantage in the advertising market so that $p_2 > p_1$. Add to that the reduction in $x_1$ due to the paywall, and this effect is only reinforced. Outlet 1 would have to not only make up for lost advertising revenues as a loss in visitors but also from the loss in positional advantage, while outlet 2 clearly benefits in both of these dimensions from the paywall.

In contrast to a micropayment system, a subscription system will have a more directed impact. In such a system, a visitor to outlet 1 only pays on their first visit and not thereafter. This means that a subscriber to outlet 1 may be just as likely – should the opportunity and desire arise – to switch to outlet 2 (i.e., $\rho_{12}$ will not change). However, a non-subscriber who had visited
outlet 2 previously would be less likely to then subscribe to outlet 1 for what remained of the attention period (i.e., $\rho_{21}$ would fall). Once again, starting from a position of symmetry, this implies that $D^l_2 > D^l_1$ and so the paywall would not only lead to relatively more visitors to outlet 2 but a positional advantage for it in advertising markets. This is an interesting result since one of the claims associated with subscription paywalls is that they will increase consumer loyalty to an outlet. While it is true that such loyalty, if generated, would increase an outlet’s advertising revenues per consumer, here a subscription generates increased loyalty for the rival outlet rather than the outlet imposing the paywall. Of course, this effect could be mitigated if, say because they are subscribers, consumers are more inclined to be loyal to outlet 1 thereby increasing $\rho_{12}$. The point here is that that outcome is not straightforward.

Finally, some outlets have proposed a limited paywall (as recently implemented by the Financial Times and the New York Times). In this case, outlets allow access to some content for free and then charge should a consumer wish to consume more. In the context of the model here, such a paywall would only be imposed, say, if a consumer chose to stay on outlet 1 for both attention periods. This type of paywall would be unlikely to have any impact on those who had previously visited outlet 2 as they could still freely switch to outlet 1 (i.e., $\rho_{21}$ would be unchanged). However, this paywall would impose a penalty for staying on outlet 1 making consumers there more inclined to switch (i.e., $\rho_{12}$ would rise). It is clear again, that other things being equal, the paywall would result in $D^l_2 > D^l_1$.

The analysis here demonstrates that putting in a paywall may give an outlet a positional disadvantage in advertising markets. Of course if an outlet already has a positional advantage, the likelihood that this occurs is lower. Nonetheless, the impact of a paywall does confer benefits on rivals in advertising markets as well as increasing their readership. These consequences may explain the low use of paywalls for online news media.

6 Conclusions and Directions for Future Research

This paper resolves long-standing puzzles in media economics regarding the impact of competition by constructing a model where consumers can switch between media outlets and those outlets can only imperfectly track those consumers across outlets. This model generates a
number of predictions including that as consumer switching increases total advertising revenue falls, that outlets with a larger readership share command premiums for advertisements, that greater switching may lead advertisers to increase the frequency of impressions purchased on outlets, that an increase in attention from non-advertising sources will increase advertising prices, that mergers may allow outlets to price discriminate in advertising markets, that ad platforms may not increase outlet profits, that investments in content quality will be associated with the frequency with which advertisers purchase impressions and that outlets that supply magnet content may be more profitable than outlets offering a deeper set of content. These predictions await thoughtful empirical testing but are thusfar consistent with stylized facts associated with the impact of the Internet on the newspaper industry. In addition, we note that while we have qualitative results, it is an open question as to whether the effects here are quantitatively significant; especially given the magnitude of the observed changes. Of course, the general characteristics of the model would also apply to other media industries including television following the introduction of cable television channels and remote controls and also the newly emerging mobile application industry that has so far struggled to be advertising supported.

While the model here has a wide set of predictions, extensions could deepen our understanding further. Firstly, the model involves two outlets usually modeled as symmetric with a distribution of advertisers with specific qualities. Generalizing these could assist in developing more nuanced predictions for empirical analysis; specifically, understanding the impact of outlet heterogeneity on advertising prices, incentives to invest in quality and incentives to invest in tracking technology.

Related, in this paper, we focused on frequency-based tracking noting that other forms of tracking have been part of the news industry. An open question is what the incentives are for firms to unilaterally improve their internal tracking of consumers. As noted throughout this paper, the adoption of more efficient matching may increase marginal demand but reduce inframarginal demand from advertisers. When ad capacity is scarce, it is not clear that such moves will prove profitable for outlets.

Finally, throughout this paper we have assumed that advertisements were equally effective on both outlets. However, in some situations, it may be that the expected value from impressing a consumer on one outlet is higher than that from impressing consumers on another.
For instance, consider (as in Athey and Gans, 2010), a situation where all advertisers are in a given local area. One outlet publishes in that local area only while the other is general and publishes across local areas. Absent the ability to identify consumers based on their location, a consumer impressed on the local outlet will still generate an expected value of $v$ to advertiser $v$ whereas one impressed on the general outlet will only generate an expected value of $\theta v$ with $\theta < 1$. In this situation, even if there are no switching consumers, advertisers on the general outlet will be paying for wasted impressions.

While this situation may be expected to generate outcomes similar to when readership shares are asymmetric, the effects can be subtle. A general outlet may have fewer consumers who are of value to advertisers but also may have a larger readership. Also, when consumers switch between outlets, the switching behavior is information on those hidden characteristics. Thus, switching behavior may actually increase match efficiency. Consequently, the effects of tailored content, self-selection and incentives to adopt targeting technologies that overcome these are not clear and likely to be an area where future developments can be fruitful.

7 Appendix

7.1 Proof of Proposition 3

Suppose that $a_1 < a_2$ and that $\sigma_1 = 0$. Also, assume for the moment that $v_{21} > 1$. In this case, the conditions for outlet supply to equal outlet demand become:

$$2a_1 = 1 - v_{11}$$
$$2a_2 = 1 - v_{11} + v_{11} - v_2$$

as outlet 1 only sells to multi-homers while outlet 2 sells to all of the single-homers. For this to be an equilibrium, prices in each outlet (which may be different) must be at a level where the marginal multi-homer is indifferent between multi-homing and single-homing on outlet 2.

$$\left( D_1^f + \frac{1}{4} D^s \right) v_{11} - \left( D_1^f + \frac{1}{2} D^s \right) p_1 > \left( D_2^f + \frac{1}{2} D^s \right) (v_2 - p_1)$$
$$\left( D_2^f + \frac{1}{4} D^s \right) v_{11} - \left( D_2^f + \frac{1}{2} D^s \right) p_2 \geq \left( D_2^f + \frac{1}{2} D^s \right) (v_2 - p_2)$$

---

30 Location is only one aspect upon which consumers and advertisers might sort according to common interests. Any specialized media content can perform this function and give an outlet a matching advantage over more general outlets.

31 Levin and Milgrom (2010) argue that targeting may be limited because it conflicts with goals of achieving market thickness (see also Athey and Gans, 2010).

32 With symmetric readership shares, the marginal multi-homer would not choose to single-home on outlet 1 if $p_1 > p_2$ which will turn out to be the case.
Note, first, that this requires that \( p_1 \geq p_2 \), otherwise, as we demonstrated above (24) could not hold, as single-homers would successful bid for impressions on \( 1 \). Instead, if \( p_i < p_2 \), \((D'_i + \frac{1}{4} D')v_{1i} - (D'_i + \frac{1}{2} D')p_i = 0\) as multi-homers will bid up \( 1 \)’s impression price. Given this and (22), we can determine that in any equilibrium of this kind,
\[
p_i = \frac{D'_i + \frac{1}{4} D'}{D'_i + \frac{1}{2} D'} (1 - 2a_i)
\]
(Hence, \( v_{1i} = 1 - 2a_i \). Note also, that single-homers will set the impression price on outlet 2 (so that \( p_2 = v_2 \)) and hence, the RHS of (25) will equal zero. Substituting in \( v_{1i} = 1 - 2a_i \) on the LHS we have:
\[
p_2 \leq \frac{D'_i + \frac{1}{4} D'}{D'_i + \frac{1}{2} D'} (1 - 2a_i)
\]
Note, however, we also have from (23) that \( p_2 = 1 - 2a_2 \). Thus, for this to be an equilibrium outcome requires:
\[
1 - 2a_2 \leq \frac{D'_i + \frac{1}{4} D'}{D'_i + \frac{1}{2} D'} (1 - 2a_i)
\]
Note that if \( a_i = a_2 \) and \( D' > 0 \) this cannot hold. Thus, \( 2 \)’s ad capacity must be significantly greater than \( 1 \)’s. Thus, with symmetric readerships, the asymmetric equilibrium will occur for \( a_i \in [0, \frac{4a_i - D'}{2(2 - D')}] \) and \( a_j \in [\frac{1}{4}(2a_i(2 - D') + D'), 1] \). Note that if \( a_j = \frac{1}{2}, a_i \in [0, \frac{1}{2}] \) while if \( a_i = \frac{1}{2} \), then \( a_j \in [\frac{1}{2}, 1] \). Thus, if each outlet has capacity of \( \frac{1}{2} \), any asymmetry will generate the asymmetric equilibrium.

This derivation assumes that \( v_{2i} > 1 \). If this was not the case and if \( p_1 > p_2 \) then the market clearing conditions for the asymmetric equilibrium would become:
\[
2a_1 = 1 - v_{1i}
\]
\[
2a_2 = 2(1 - v_{2i}) + v_{2i} - v_2
\]
as only outlet 2 sells additional impressions to some multi-homers. Thus, outlet \( 1 \)’s price would remain as in (26) while outlet \( 2 \)’s pricing condition would satisfy (substituting \( v_{2i} \) into (30)):
\[
p_2 = \frac{D'_2}{D'_i + D'} (1 - a_2)
\]
This would be an equilibrium so long as \( v_{2i}(p_2) < 1 \) or \( a_2 > \frac{2 - D' a}{3} \) in addition to the ad capacity asymmetries as identified earlier. It is easy to confirm in this case that \( p_1 > p_2 \).

7.2 Proof of Proposition 4

We rely on following lemma:

**Lemma.** Assume that \( F(v) : U[0,1], a_i = a_2 = a \) and \( x_1 > x_2 \). Then each outlet’s equilibrium profits are as follows:

(i) For \( \frac{8 - x_i \rho (8 - x_i \rho)}{8(2 - x_i \rho)} < a \), \( \pi_1 = x_1 \frac{4 - \rho}{4 - x_i \rho} (1 - 2a)2a \) and \( \pi_2 = x_2 \frac{8 - x_i \rho (2 - x_i \rho)}{4 + x_i \rho (2 - x_i \rho)} (3 - 4a)2a \);

(ii) For \( \frac{x_i \rho}{8} < a < \frac{8 - x_i \rho (8 - x_i \rho)}{8(2 - x_i \rho)} \), \( \pi_1 = x_1 \frac{4 - \rho}{4 - x_i \rho} (1 - 2a)2a \) and \( \pi_2 = x_2 \frac{2(2 - x_i \rho)}{4 + x_i \rho} (1 - 2a)2a \);

(iii) For \( \frac{x_i \rho}{8} \geq a \), \( \pi_1 = x_1 (1 - \frac{2a}{x_i})2a \) and \( \pi_2 = x_2 (1 - 4a)2a \).
The derived profits are found by solving (10) and (11) for outlet prices and substituting them into outlet profits while checking to see what allocations of advertising choices these imply.

Note that, for case (i), \(2(1 - a) - x_1 - \frac{4(3 - 4a)(1 - x_1)}{4 + x_1(2 - x_1)} > x_2\frac{x_1(2 - x_1)}{4 + x_1(2 - x_1)}(3 - 4a) \Rightarrow a > \frac{1}{2}\), which can’t hold. Thus, \(\pi_2 > \pi_1\) always for this case. Note that this arises when \(\frac{8 - x_1(8 - x_1)}{8(2 - x_1)} < a\); which, substituting \(D^s\) for \(\rho\) gives the converse of the condition in the proposition. The LHS is decreasing in \(\rho\) so that when both \(\rho\) and \(a\) are high, this condition holds as stated in the proposition.

For case (ii), \(x_1\frac{4 - \rho}{4 - x_1(1 - 2a)}(1 - 2a)2a > x_2\frac{2(2 - x_1(2 - x_1))}{4 - x_1(2 - x_1)}(1 - 2a)2a \Rightarrow x_1(2 - \frac{1}{2} \rho) > x_2(2 - x_1(2 - x_1))\) which always holds for \(x_1 > \frac{1}{2}\). Thus, \(\pi_1 > \pi_2\) for this case.

For case (iii), it is readily apparent that \(x_1 > \frac{1}{2}\), \(\pi_1 > \pi_2\) for this case.

### 7.3 Proof of Proposition 5

When \(D^x\) is low, outlet 1’s profits under no tracking are \(\frac{2(2 - D^x)}{4 - D^x}(1 - (a_1 + a_2))a_1\) whereas outlet 1’s profits under perfect tracking are \((1 - a_1 - a_2)a_1D^x + (1 - 2a_1)2a_1D^i\). Profits under perfect tracking exceed those under no tracking if: \((a_1 - a_2)D^x(4 - D^x) + (1 - 2a_1)(4 - D^x) > 2(2 - D^x)(1 - (a_1 + a_2))\). With \(a_1 = a_2\), this becomes: \(D^x > 0\).

When \(D^x\) is high, outlet 1’s profits under no tracking may be \(\frac{D^x(2 - D^x)}{4 + D^x(2 - D^x)}(3 - 2(a_1 + a_2))a_1\). Comparing these to the profits under perfect tracking and imposing \(a_1 = a_2 = a\), perfect tracking will yield higher profits if: \(\frac{2((2 - 2a))}{1 - a} > D^x(2 - D^x)\). Examining the case where \(D^x = \frac{1}{2}\), note that these profits will be an equilibrium if the equilibrium price they are based on \(\frac{2(2 - D^x)}{4 + D^x(2 - D^x)}(1 - 2a)\) is less than \(\frac{1}{4}\). That is, if \(\frac{5}{7}(1 - 2a) < \frac{1}{4} \Rightarrow a > \frac{12}{49}\). At \(D^x = \frac{1}{2}\), we have \(\frac{2((2 - 2a))}{1 - a} > \frac{1}{4} \Rightarrow a < \frac{5}{13}\) so for \(a \in [\frac{17}{48}, \frac{5}{13}]\), perfect tracking yields superior profits but for \(a > \frac{5}{13}\), profits are higher under no tracking.

### 7.4 Proof of Proposition 6

The advertiser expected surplus from given advertising strategies are:

<table>
<thead>
<tr>
<th>((n_1, n_2))</th>
<th>Expected Advertiser Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 0)</td>
<td>((D^i + \frac{1}{2} D^i + \frac{1}{2} D^i))((v - p))</td>
</tr>
<tr>
<td>(2, 0)</td>
<td>((D^i + D^i + D^i)(v - (2D^i + D^i + D^i))p)</td>
</tr>
</tbody>
</table>
| (1, 1)          | \((D^i + D^i + \frac{3}{4} D^i + \frac{1}{2} (D^i + D^i))v \)  
|                  | \(-p(D^i + D^i + D^i + \frac{1}{2} (D^i + D^i))\) |
The main difference between this case and the previous two outlet model is that some advertisers may choose to multi-home with two impressions on each outlet so as to impress a greater share of those switching between blogs and mainstream outlets. Indeed, under symmetry, the threshold advertiser rates become (under symmetric ad capacities):

\[ v_i = p \]  
(32)

\[ v_{11} = 2 \frac{2D'_{1} + D'_{2} + D'_{12} + D'_{ib}}{4 D'_{1} + D'_{2} + 2 D'_{ib}} p \]  
(33)

\[ v_{21} = 2 \frac{2D'_{1} + D'_{2} + D'_{12} + D'_{ib}}{D'_{1} + D'_{ib}} p \]  
(34)

\[ v_{22'} = \frac{2D'_{1} + D'_{2} + D'_{12} + D'_{ib}}{D'_{ib}} p \]  
(35)

where \( v_{22} \) is the threshold between multi-homing with 2 on one outlet and multi-homing with 2 impressions on each outlet. It is clear that, under symmetry, \( v_{22} > v_{21} > v_{11} > v_i \) when \( \rho > 0 \). This implies that there are three demand ‘cases’ but that supply in the market is \( D'_{1} a_{1} + D'_{2} a_{2} + D'_{12} (a_{1} + a_{2}) + D'_{ib} a_{1} + D'_{2ib} a_{2} \). So long as ad capacities are symmetric, the market clearing price is given by:

\[
p = \begin{cases} 
1 - 4a & 0 < a < \frac{1}{16} \rho (1 - x_{h}) \\
\frac{2(4 - (1 - x_{h}))}{8 - (1 - x_{h})} (1 - 2a) & \frac{1}{16} \rho (1 - x_{h}) < a < a^{L} \\
\frac{(1 + 3x_{h})\rho (4 - (1 - x_{h}))}{16 + 4(1 + 7x_{h})\rho - (1 + 2x_{h} - 3x_{h}^{2})\rho^{2}} (3 - 4a) & a \leq a^{H} \\
\frac{4(1 - a)}{4 - \rho + x_{h}(3 - x_{h})\rho + 8x_{h}(3 - x_{h})\rho^{2} - 4(28 + 2\rho - \rho^{2})} & a \geq a^{H}
\end{cases}
\]  
(36)

where \( a^{L} = \frac{32 - \rho(16 + 2x_{h}(8 - \rho) + 3x_{h}^{2}\rho)}{64 - 16(1 - x_{h})\rho} \) \( a^{H} = \frac{3(4 - \rho) + x_{h}(20 + \rho(-10 + \rho + x_{h}(19 + 2 - 3x_{h})\rho)))}{4(1 + 3x_{h})(4 - (1 - x_{h})\rho)} \). It can be seen here that as the number of blog readers increases and/or the probability of switching rises, that inframarginal advertisers will demand more impressions.

The proof of the proposition follows from a simple examination of (36).

7.5 Proof of Proposition 8

In this case, while outlet 1 supplies ad capacity of \( D'_{1} 2a + D'_{2} a \) into the market, outlet \( f \) only supplies \( D'_{2} a \). The following table identifies the surplus to an advertiser with value \( v \) from pursuing different choices.

<table>
<thead>
<tr>
<th>((n_{1}, n_{f}))</th>
<th>Expected Advertiser Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0)</td>
<td>((D'<em>{1} + \frac{1}{2} D'</em>{2})(v - p_{1}) = \frac{1}{2}(v - p_{1}))</td>
</tr>
<tr>
<td>(2,0)</td>
<td>((D'<em>{1} + D'</em>{2})(v - (2D'<em>{1} + D'</em>{2})p_{1} = \frac{1}{2}(1 + D'<em>{2})v - p</em>{1})</td>
</tr>
</tbody>
</table>
Notice that there are now three options for an advertiser to cover the entire consumer market – single homing on 1 with 2 impressions, and multi-homing with two impressions on at least one outlet. It is clear that multi-homing with 2 impressions on outlet 1 is dominated by single-homing on outlet 1 (as the former involves paying for impressions on f without any benefit). In addition, note that any advertiser who wants to single home on outlet f will prefer to do so with two impressions as there is no waste from the additional impression. We can always rule out multi-homing with one impression on each outlet. For this to be preferred to single-homing on outlet f (with one impression) it must be the case that \( \frac{1}{4} D' v > \frac{1}{2} D' p_f \). However, this condition also means that by moving from multi-homing with single impressions to multi-homing on outlet f with 2 impressions is preferable. Consequently, if an advertiser wants to capture an additional \( \frac{1}{4} D' \) by purchasing an impression on outlet f, it will also want to do this by purchasing two additional impressions on outlet f.

This still leaves four choices that might be undertaken by advertisers. As a means of covering the entire market, single-homing on outlet 1 with 2 impressions and multi-homing with 2 impressions on f are substitutes. Indeed, multi-homing will only be chosen if \( \frac{1}{2} p_i > D' p_f \); a condition that must hold if \( D' \) is very small. At any point in time, we will only observe one of these strategies being chosen. In each case, it will be the highest value advertisers who pursue them.

For the remaining choices, advertisers single homing on f (with 2 impressions) or on 1 (with 1 impression) are candidates to be the marginal advertiser in the market. If \( \frac{1}{2} D' > D'_i \), higher value advertisers prefer (holding prices constant) purchasing impressions on f rather than 1. Under this condition, the marginal advertiser, with value \( p_i \), would earn \( D'(p_i - p_f) \) by switching to outlet f which is negative if \( p_i < p_f \). Similarly, if the marginal advertiser has value, \( p_f \), it will earn \( (D'_i + \frac{1}{2} D')(p_f - p_i) \) by switching to outlet 1. This reduces its surplus if \( p_f < p_i \). Hence, the marginal advertiser will be on the lowest priced outlet.

We now turn to derive the equilibrium prices and profits. Case 1: \( \frac{1}{2} D' > D'_i \). Suppose that \( (D'_i + \frac{1}{2} D') p_i < D' p_f \). Then consider a candidate equilibrium where high value advertisers sort as single-homers (2 impressions) on 1, then single-homers (2 impressions) on f and finally as single-homers (1 impression) on 1. In this case, equilibrium prices will be the solution to:

\[
D'_i 2a + D' a = (D'_i + \frac{1}{2} D') \left( 2(1-v_{if}) + (v_f - p_i) \right)
\]

\[
\frac{1}{2} D' 2a = \frac{1}{2} D' 2(1-v_{if})
\]
where \( v_{if} = \frac{(2D'_f + D^*)p_i - D^*p_f}{2D'_f - 2D^*} \) and \( v_f = \frac{2D'_f p_f - (2D'_f + D^*)p_i}{D^' - 2D^*} \). Solving this gives:

\[
p_i = \frac{aD'_f + D^*(1-2a)}{D'_f + D^*} \quad (39)
\]

\[
p_f = 1 - 2a - 2(1-3a) \frac{p_i^2}{D^*} \quad (40)
\]

(recalling that we assume that \( a \leq \frac{1}{2} \)). It is easy to demonstrate that \( p_f > p_i \) and that \((D'_f + \frac{1}{2} D^*)p_i < D^*p_f\). This confirms the equilibrium.

Is it possible that \((D'_f + \frac{1}{2} D^*)p_i > D^*p_f\)? In this case, a candidate equilibrium would have high value advertisers sort as multi-homers (2 impressions) on \( f \) and then single-homers (2 impressions) on \( f \). In this case, no advertiser will choose single-homing on 1. Thus, equilibrium prices will be the solution to:

\[
D'_f 2a + D^*a = (D'_f + \frac{1}{2} D^*)(1-v_{if}) \quad (41)
\]

\[
\frac{1}{2} D^* 2a = \frac{1}{2} D^* 2(1-p_f) \quad (42)
\]

where \( v_{if} = \frac{(D'_f + \frac{1}{2} D^*)p_i}{2D'_f - D^*} \). Solving this gives:

\[
p_i = \frac{D'_f (1-2a)}{2D'_f + D^*} \quad (43)
\]

\[
p_f = 1 - a \quad (44)
\]

It is easy to demonstrate that \( p_f > p_i \) but that \((D'_f + \frac{1}{2} D^*)p_i - D^*p_f = (\frac{1}{2} - a)D'_f - D^*(1-\frac{1}{2} - a) > 0 \Rightarrow \frac{D^*}{D'_f} < \frac{\frac{1}{2} - a}{1-\frac{1}{2} - a} \) which cannot hold as the LHS is greater than 2 while the RHS is less than 2. Thus, this cannot be an equilibrium.

Case 2: \( \frac{1}{2} D^* < D'_f \). Suppose that \((D'_f + \frac{1}{2} D^*)p_i > D^*p_f\). Then consider a candidate equilibrium where high value advertisers sort as multi-homers (2 impressions) on \( f \), then single-homers (1 impression) on 1 and finally single-homers (2 impressions) on \( f \). In this case, equilibrium prices will be the solution to:

\[
D'_f 2a + D^*a = (D'_f + \frac{1}{2} D^*)(1-v_i) \quad (45)
\]

\[
\frac{1}{2} D^* 2a = \frac{1}{2} D^* 2(1-v_f + v_i - p_f) \quad (46)
\]

where \( v_{if} = 2p_f \) and \( v_i = \frac{(2D'_f + D^*)p_i - 2D^*p_f}{2D'_f - D^*} \). Solving this gives:

\[
p_i = \frac{6D'_f (1-2a) + D^*}{3(2D'_f + D^*)} \quad (47)
\]

\[
p_f = \frac{2}{3} - a \quad (48)
\]

(recalling that we assume that \( a \leq \frac{1}{2} \)). It is easy to demonstrate that \( p_f < p_i \) and that \( \frac{2(1-\frac{1}{2} - a) - \sqrt{2(1-2a) + 4a^2}}{4 + D^* (1-2a)} (3 - 4a) = \frac{2(1-\frac{1}{2} - a)}{4 + D^* (1-2a)} (1-2a) \Rightarrow D' = 2\left(2(1-a) - \sqrt{2(1-2a) + 4a^2}\right) \). This confirms the equilibrium.
8 References


Chiou, L. and C.E. Tucker. 2010. “News, Copyright and Online Aggregators,” mimeo., MIT.


Online Appendix:

9.1 Note on tracking technologies

Here we discuss alternative tracking technologies to the one used in the paper. That one is best described as “content-based tracking,” which leads to tracking within an outlet and period, but not across periods or across outlets. The following table illustrates potential alternative tracking technologies and the type of offers each permits outlets to make to advertisers.

Table A: Alternative Tracking Technologies and Offer Types

<table>
<thead>
<tr>
<th>Technology</th>
<th>Example of Offer Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect tracking</td>
<td>Over the two attention periods, we will impress a given set of consumers just once regardless of where their attention is allocated at a price of ( p ) per consumer/impression.</td>
</tr>
<tr>
<td>No tracking</td>
<td>Over the two attention periods, we will place a given number of impressions on our outlet for a price of ( p ) per impression.</td>
</tr>
<tr>
<td>Perfect internal tracking</td>
<td>Over the two attention periods, we will impress each unique consumer on the outlet once at a price of ( p ) per impression/consumer.</td>
</tr>
<tr>
<td>Content-based tracking</td>
<td>We will associate an ad with a given piece of content and you will pay a price of ( p ) each time that ad is viewed</td>
</tr>
<tr>
<td>Frequency-capping</td>
<td>Over the two attention periods, we will place at most ( x ) impressions per consumer at a price of ( p ) per impression</td>
</tr>
</tbody>
</table>

We considered perfect tracking above as a possible ideal. Such offers were made by ad platforms and were outlet independent. At the opposite extreme from perfect tracking is no tracking that arises when neither outlets nor a common platform are unable to internally (or externally) track impressions and to control matching between advertisers and consumers. In the early days of the Internet (circa 2000), websites had no ability to track consumers even within outlets, and even today with privacy settings such tracking may not be possible. The models of Butters (1977) and, more recently, Bergemann and Bonatti (2011) assume that advertisers choose the intensity of their advertising on an outlet, but that advertising messages (impressions) are distributed independently (across messages) and uniformly across consumers. This means that a given consumer might see the same advertisement multiple times, which involves waste.\(^{33}\)

The next three technologies are intermediate ones where outlets can track impressions internally but not externally. Thus, outlets cannot offer inter-outlet arrangements (such as different prices for different switching categories of consumer) that would be possible under perfect tracking because they cannot track consumers across outlets. Under perfect internal tracking, no consumer receives more than one impression from an advertiser on a given outlet. Thus, an advertiser could purchase impressions on \( D_i^I + D_i^F \) consumers on outlet \( i \). However, this

\(^{33}\) In such models, the expected number of unique impressions received by an advertiser with advertising intensity \( n \) in a market of size \( x \) is given by \( x\left(1-(1-\frac{1}{x})^n\right) = x(1-e^{-n/x}) \), where \( 1/x \) is the probability that a given consumer is selected for a given ad impression.
creates a capacity management issue if the outlet cannot distinguish loyal from switching consumers. If an outlet does not impress all consumers in the ‘first period’ it will have to impress them in the second period. However, unless it can distinguish between loyals and switchers in the first period, some consumers may move to the other outlet and it will be unable to fulfill its contract. Alternatively, it could impress all consumers in the first period and perhaps identify the new switchers as unique consumers to impress in the second period. But, even in that case, loyals, who remain with the same outlet through the second period, will have additional capacity that can be sold. In principle, that capacity could be sold under a “impress all unique consumers” contract but this would mean that the outlet would have to offer a range of distinct products to advertisers. This is an interesting and potentially realistic scenario, but we leave it for future work.

9.2 Endogenous Ad Capacities

The results in the paper focused on comparative statics for exogenous ad capacities. We now endogenize capacity choice, so that outlets can commit to smaller capacity levels than could be potentially supplied, focusing on how it relates to both readership share and the share of multi-homing consumers. Observe that the choice here for outlets is capacity per consumer per unit of attention. We do not allow outlets to sell different quantities of advertising to different types of consumers.

First, however, it is useful to provide a general discussion of how the extension to our model affects our results. Consumer switching creates competition among outlets, and so standard Cournot-type forces operate. This analysis allows us to isolate the novel forces introduced by consumer switching. First, consider the case of perfect tracking, whereby switching affects outcomes by increasing competition among the outlets. Then, with endogenous capacity, outlet profits always fall when consumer switching rise so long as outlets are sufficiently symmetric in terms of readership share. In contrast, for sufficiently asymmetric outlets, increased switching increases profits for the less popular outlet and decreases profits for the more profitable outlet.

Next, consider the more interesting case of imperfect tracking, where we focus on symmetric readership across outlets. The analysis is complicated by the fact that for positive but low levels of switching, there is no pure strategy equilibrium in capacity choices. For moderate to high levels of switching, we find that, relative to no switching, equilibrium capacity levels are higher and profits are lower. In addition, as switching increases, outlet profits fall. Thus, the case (ii) above, whereby outlet profits increase as switching increases, cannot occur.

Finally, as discussed in the introduction, our model contrasts with the prior literature where outlets compete for users by reducing ad capacities, where ads are disliked by consumers. In Anderson and Coate (2005), the advertising prices are always monopoly prices, so the level of ad capacity is determined by trading off deviations from monopoly levels against increasing the user base. The ad capacity of other firms does not affect advertising prices. Although we leave this extension for future work, we can say a few things about what would happen in an extension to our model where ad capacities are endogenous and consumers dislike ads. In our model the problem is more complex, since opponent ad capacities do affect advertising prices directly. Nonetheless, the comparative statics about outlet profits and asymmetric ad capacities suggest
that there can be novel incentives created by the impact of capacity on advertising prices. For example, the advantage gained by having the larger consumer base may intensify competition for consumers, creating a force in favor of lower equilibrium ad capacities.

**9.2.1 Perfect Tracking**

We first consider the perfect tracking case. We assume that there are only two outlets to focus on the impact of outlet asymmetry.\(^{34}\) This means that an outlet will face demands for two sets of consumers – one set that it has monopoly control over and the other for which it competes a la Cournot. We now consider an analysis of the comparative statics of competition in this set-up.

We can write profits as a function of capacity, readership share and \(\rho\):

\[
\pi_i(a_i, a_j, x_i, \rho) = P(a_i + a_j) a_i D^R_{ij}(x_i, 1-x_i, \rho) + P(2a_j) 2a_j D^L_i(x_i, \rho)
\]

Let \(MR_i^D(a_i, a_j) = (a_i P'(a_i) + P(a_i))\) and \(MR_i^M(a_i) = 2(2a_j P'(2a_j) + P(2a_j))\). The first-order conditions for outlet \(i\) imply:

\[
MR_i^D(a_i, a_j + a_j) D^R_{ij}(x_i, x_j, \rho) + MR_i^M(a_i) D^L_i(x_i, \rho) = 0.
\]

This shows that the outlet considers the relative proportion of switchers and loyals when choosing output, and it will select capacity so that one of the marginal revenue terms is positive while the other is negative. Note that if \(a_i > a_j\), then if \(P\) is decreasing and concave, \(MR_i^D(a_i, a_j + a_j) < 0\) implies that \(MR_i^M(a_i) < 0\). Thus, for the outlet with the larger equilibrium capacity, we must have \(MR_i^D(a_i, a_i + a_j) \geq 0\) in equilibrium: capacity is chosen lower than the Cournot best response, but higher than the monopoly level for that outlet. The converse is not necessarily true, however; the outlet with small equilibrium capacity may also have \(MR_i^D(a_i, a_i + a_j) \geq 0\) (and indeed, this holds in the case of uniformly distributed advertiser valuation).

The impact of an increased readership share on the incentive to expand capacity is:

\[
\frac{\partial}{\partial x_i} \frac{\partial \pi_i}{\partial a_i} = MR_i^D(a_i, a_i + a_j) + MR_i^M(a_i) 2 \frac{D^L_i(x_i, \rho)}{D^R_{ij}(x_i, 1-x_i, \rho)}
\]

At an equilibrium choice of capacity, the ratio of the marginal revenue terms is equal to the ratio of switchers to loyal users, so that we will have (where \(\hat{a}_i\) is the equilibrium capacity for \(i\)):

\[
\frac{\partial}{\partial x_i} \frac{\partial \pi_i}{\partial a_i} \bigg|_{(a_i, a_j) = (\hat{a}_i, \hat{a}_j)} = MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j) \left( \frac{\frac{\partial}{\partial x_i} D^R_{ij}(x_i, 1-x_i, \rho)}{D^L_i(x_i, \rho)} \frac{\hat{x}_i \rho}{1-(1-x_i) \rho} \right)
\]

Since higher readership share increases the proportion of loyal users, its direct effect on capacity is negative if and only if \(MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j) \geq 0\). Intuitively, becoming larger causes a firm to put

\(^{34}\) All of the qualitative predictions in this subsection apply for a general \(F(.)\) assumed to be log-concave. (Proofs available from the authors).
more weight on loyal users, giving it the incentive to reduce output. However, clear equilibrium comparative statics are complicated by the fact that Cournot outputs are strategic substitutes.

We can also consider the impact of switching on capacity choice:

\[
\frac{\partial^2 \pi}{\partial a \partial \rho} = MR_i^D(a_i, a_i + a_j) \frac{\partial^2}{\partial a} D_j^i(x_i, 1-x_i, \rho) + MR_i^M(a_i) \frac{\partial}{\partial \rho} D_j^i(x_i, \rho)
\]

\[
= MR_i^D(a_i, a_i + a_j) 2x_i(1-x_i) - MR_i^M(a_i)x_i(1-x_i)
\]

At an equilibrium capacity choice, we will have

\[
\frac{\partial^2 \pi}{\partial a \partial \rho} \bigg|_{(a_i, a_j) = (\hat{a}_i, \hat{a}_j)} = MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j) 2x_i(1-x_i) \left(1 - 2\rho(1-x_i) \right)
\]

So long as switching is not too prevalent and outlets are not too asymmetric, switching decreases the share of loyal users, so that the direct effect of switching on capacity is positive if and only if 

\[
MR_i^D(\hat{a}_i, \hat{a}_i + \hat{a}_j)(1-2\rho(1-x_i)) \geq 0.
\]

Thus, the direct effect is unambiguously positive for the outlet with the larger share.

Using the envelope theorem, we can write the impact of \( \rho \) on profits as follows:

\[
\frac{\partial}{\partial \rho} \pi_i(a^*_i(x_i, \rho), a^*_j(x_j, \rho); x_i, \rho) = P'(a^*_i + a^*_j) a^*_i D_j^i(x_i, 1-x_i, \rho) \frac{\partial}{\partial \rho} a^*_j(x_i, \rho)
\]

\[
+ P(a^*_i + a^*_j) a^*_i D_j^i(x_i, 1-x_i, \rho) + P(2a^*_i) 2a^*_i \frac{\partial}{\partial \rho} D_j^i(x_i, \rho)
\]

\[
= P'(a^*_i + a^*_j) a^*_i D_j^i(x_i, 1-x_i, \rho) \frac{\partial}{\partial \rho} a^*_j(x_i, \rho)
\]

\[
+ 2P(a^*_i + a^*_j) a^*_i x_i(1-x_i) - 2P(2a^*_i) a^*_i x_i(1-x_i)
\]

Switching has an indirect effect through increasing the opponent’s output, which (if it increases opponent capacity) lowers price and thus profits. It also has a direct effect of increasing the proportion of switchers and decreasing the proportion of loyalists. The sum of the last two terms is negative if and only if \( a^*_i \leq a^*_j \): for the lower-capacity outlet, switchers are less profitable. The analysis for the outlet with the higher equilibrium output appears ambiguous if its competitor’s output is increasing in \( \rho \), as the price effect and the switcher/loyal effect move in opposite directions.

Summarizing the discussion so far, we can gain some intuition about the direct effects of parameter changes on outlet capacity choices and profits, but some additional structure on demand is required to obtain unambiguous comparative statics results. To do so, we focus on the case of linear demand (uniformly distributed advertiser valuations). The following proposition demonstrates that the larger outlet will provide the lowest advertising capacity.

**Proposition A1.** Suppose that there are two outlets and that \( F(v) = v \). Equilibrium advertising for each outlet, \( \hat{a}_i \), are non-increasing in readership share, \( x_i \). Equilibrium advertising \( \hat{a}_i \) is non-decreasing in \( \rho \) if \( x_i \leq (21-\sqrt{249})/6 \approx .87 \) or \( \rho \leq (2/3)(3-\sqrt{3}) \approx .84 \). Total ad capacity, \( \hat{a}_i + \hat{a}_j \), is non-decreasing in \( \rho \). For sufficiently symmetric firms (.33 \( \leq x_i \leq .67 \)), profits of both firms are decreasing in \( \rho \), while for sufficiently asymmetric firms, profits are decreasing...
(increasing) in \( \rho \) for \( x_i > (\leq) x_j \). \( \pi_{ij}^{PT} / x_i < \pi_{ij}^{PT} / x_j \) when \( x_i > x_j \). \( \pi_{ij}^{PT} - \pi_{ij}^{PT} \) is decreasing in \( \rho \) for \( x_i > x_j \).

**Proof:** Solving for the unique Nash equilibrium with the uniform distribution we have:

\[
\hat{a}_i = \frac{16D_i'D_j' + 6D_j'D_i'^* + 4D_i'D_i'^* + D_i'^*^2}{64D_j'D_i' + 16(D_j' + D_i')D_i'^* + 3D_i'^*^2}
\]

\[
\pi_{ij}^{PT} = \frac{(4D_i' + D_i'^*)(16D_i'D_j' + 6D_j'D_i'^* + 4D_i'D_i'^* + D_i'^*^2)^2}{(64D_j'D_i' + 16(D_j' + D_i')D_i'^* + 3D_i'^*^2)^2}
\]

The rest of the proposition follows from manipulating these expressions.

We have already developed some intuition for these results, but the uniform distribution gives us more definitive conclusions. Consider the comparative statics of switching on profits. The increase in capacity of an opponent’s outlet has a negative impact on each outlet. However, the increase in the share of switchers has a positive (resp. negative) effect on the smaller (larger) outlet, as the share of consumers coming from the switchers goes up. Switchers are more (less) profitable than loyal for the smaller (larger) outlet, because the larger outlet serves less capacity than the smaller outlet. With the uniform distribution, for the small outlet the latter effect dominates the negative effect of increase in capacity and small outlet profits go up.

Note that switching also affects the impact of an increase in readership share on profits. Under the benchmark single-homing consumer case, more readers simply improved profits in a linear fashion; that is \( \pi_{ij}^{PT} / x_i \) was independent of \( x_i \). With perfect tracking, an additional reader attracted from a rival outlet not only causes an outlet to restrict advertising capacity but for that capacity to increase elsewhere (since capacities are strategic substitutes in our Cournot setup), decreasing impression prices for switchers. Thus, outlets with a lower readership share have a higher incentive to attract marginal readers.

It is also useful to note that if the two outlets were commonly owned, their owner would maximize joint outlet profits by setting \( a_i = a_x = \frac{1}{4} \). In this case, realized profits in this case will be the same as those generated when there are no switchers. Thus, under perfect tracking with \( D_i^* > 0 \) there will be an incentive for outlets to merge.

In the absence of common ownership, multi-homing consumers cause outlets to compete for advertisers and a greater proportion of them increases available advertising space and decreases overall profits. However, the question of interest is what does this do to the marginal incentive to attract an additional reader at the expense of rivals. What we can demonstrate is that as \( x_i \to 0 \) or \( x_i \to 1 \), then \( \frac{\partial \pi_{ij}^{PT}}{\partial x_i} > \frac{\partial \pi_{ij}^{NS}}{\partial x_i} = \frac{1}{4} \). It is useful to note that if both outlets are commonly owned (i.e., in a monopoly), then profits under perfect tracking are the same as profits earned for each outlet in the no switching case. Thus, competition is the source of any reduction in profits as a result of switching but this competition can, in turn, promote higher incentives to attract readership when there are asymmetric readership shares.
9.2.2 Imperfect tracking

We now turn to consider endogenous capacity for the case of imperfect tracking. Our goal here is to explore the robustness of the comparative static results on $D^s$, with ad capacity was exogenous; recalling our main finding that as $D^s$ rose, impression prices and outlet prices fell except for high $a$ when $D^s$ was large. As in the perfect tracking case, we suppose that competition comprises two. In stage 1, both outlets simultaneously choose their ad capacities. In stage 2, the market clears based on those capacities and prices and profits are realized. It turns out that, in this situation, a pure strategy equilibrium in the Stage 1 (Cournot) game does not exist for a non-trivial rage of $D^s$. Given this, we then consider a Stackelberg Stage 1. Significantly, we demonstrate that advertising capacity, while asymmetric in this equilibrium between the two outlets, does not reach a level whereby an increase in $D^s$ leads to an increase in the profits of either outlet.

The Cournot game equilibrium outcomes are summarized by the following

**Proposition A2.** Suppose that outlets are symmetric in readership, $F(v) = v$ and $V = 1$. With endogenous capacity, $F(v) = v$ and symmetric readership shares, the pure strategy equilibrium outcomes are:

(i) For $D^s = 0$, $a_i = \frac{1}{4}$ with per consumer profits of $\pi_i = \frac{1}{4}$ for all $i$.

(ii) For $D^s \geq \frac{4}{9}$, $a_i = \frac{1}{4}$ with per consumer profits of $\pi_i = \frac{2(2-D^s)}{4-D^s} \frac{2}{9}$ for all $i$. Otherwise no pure strategy equilibrium exists.

**Proof:** Note that for $D^s = 0$, $v_{12} = p$ and the asymmetric equilibrium holds for any $(a_1, a_2) \in (\frac{1}{4}, \frac{1}{4})$. In any asymmetric equilibrium, per consumer profits equal $(1-2a_i)2a_i$ for each outlet; which is maximized at a capacity of $\frac{1}{4}$. Hence, by deviating, each would receive no greater profits than they do under the equilibrium as specified in (i).

To check that outcome (ii) is an equilibrium, observe that if each outlet plays a local best response, they each choose capacity equal to $\frac{1}{4}$. Now consider a choice $a_i \gg \frac{1}{4}$ so that $p(a_i, a_2) \leq \frac{1}{4} D^s$. In this case, the highest profits outlet 1 could earn are:

$$\max_{a_i} \frac{D^s(2-D^s)}{4+D^s(2-D^s)}(3-2(a_i+\frac{1}{3}))2a_i$$

which is maximized at $a_i = \frac{7}{12}$; which would create the asymmetric equilibrium. Thus, the maximum capacity 1 would chose would be $\frac{1}{12} (4+D^s)$ resulting in profits of $\frac{1}{36} (2-D^s)(4+D^s) < \frac{2(2-D^s)}{4-D^s} \frac{2}{9}$. Now consider a choice $a_i \ll \frac{1}{4}$ so that $\sigma_i = 0$; specifically, $a_i \leq \frac{4+D^s}{6(2-D^s)}$. In this case, outlet 1 maximizes profits with a choice of $a_i = \frac{1}{4}$ earning profits of $p_i 2a_i = \frac{D^s(2-D^s)}{4+D^s(2-D^s)}(1-2a_i)2a_i = \frac{1}{8} (2-D^s)$ which is greater than $\frac{2(2-D^s)}{4-D^s} \frac{2}{9}$ for $D^s \leq \frac{4}{9}$. When $D^s > \frac{4}{9}$, this deviation is not profitable. Finally, we need to check that, in fact, $p(a_i, a_2) \geq \frac{1}{2} D^s$. This implies that

$$\frac{1}{2} D^s < \frac{2(2-D^s)}{4-D^s} \frac{1}{2} \Rightarrow D^s < \frac{2}{3} (4-\sqrt{10})$$

which always holds for $D^s \leq \frac{1}{2}$. 
We now turn to establish that there are no other pure strategy equilibria. First, note when \( p < \frac{1}{2} D^* \), it is easy to see that \( a_1 = \frac{1}{2} \) is a local best response to \( a_2 = \frac{1}{2} \). At this point, each outlet earns profits of \( \frac{D'(2-D')}{4 + D'(2-D')} \). Note, however, that any deviation from these capacities generates the asymmetric equilibrium. Thus, setting \( a_1 > \frac{1}{2} \) would earn that outlet \( \frac{D'(2-D')}{4 + D'(2-D')} \) at this point. A reduction in capacity would involve maximum profits at \( a_1 = \frac{1}{4} \). In this case, it is easy to establish that \( \frac{D'(2-D')}{4 + D'(2-D')} < \frac{1}{2} (2-D^*) \) and so a large reduction in ad capacity is a profitable deviation for outlet 1. Thus, no equilibria of this type exists.

What about an asymmetric equilibrium? Any equilibrium would involve the outlet with the smaller capacity, say 1, choosing \( a_1 = \frac{1}{4} \) while the other outlet chooses \( a_2 = \frac{1}{8} (2+D^*) \). Note that this is consistent with \( v_{12} > 1 \) and it is straightforward to establish that outlet 2 would not want to choose a higher ad capacity to change this. In this case, outlet 2 earns per consumer profits of \( (1-2a_2)2a_2 \) and it is easy to determine that these are decreasing in \( a_2 \). Therefore, given 1’s choice, 2 would not find it profitable to expand output. Contracting it would generate profits of \( \frac{2(2-D^*)}{4-D^*} (1-\frac{1}{4}-a_2)2a_2 \); maximized at 3/8 which would involve too much asymmetry to generate that outcome. Thus, any contraction involves profits less than \( \frac{1}{16} (4-D^{*2}) \). For 1, \( a_1 = \frac{1}{4} \) is a local best response, but by choosing a higher ad capacity, it may earn different profits depending upon the resulting impression price. For \( p \geq \frac{1}{2} D^* \), outlet 1 would earn per consumer profits of \( \frac{2(2-D^*)}{4-D^*} (1-\frac{1}{8}(2+D^*)-a_1)2a_1 \) which is maximized at \( a_1 = \frac{1}{16} (6-D^*) \). However at this capacity, ad capacities would be sufficiently asymmetric that this would not be feasible. Instead, outlet 1 is constrained to a capacity no more than \( \frac{1}{16} (4+4D^*-D^{*2}) \). Note that this results in a price \( p = \frac{2(2-D^*)}{4-D^*} (1-\frac{1}{8}(2+D^*)-\frac{1}{16} (4+4D^*-D^{*2})) \geq \frac{1}{2} D^* \). It is straightforward to demonstrate that this deviation is profitable for 1. A similar reasoning holds for the case where \( p < \frac{1}{2} D^* \). Thus, there is no pure strategy equilibrium involving asymmetric capacity choices.

Intuitively, for smaller levels of \( D^* \), each outlet would prefer to be the outlet with the larger capacity so long as the required asymmetry is not too large. When that occurs, their preferences switch. Consequently, there is a (downwards) discontinuity in the best response functions of each outlet for \( D^* \in (0, \frac{4}{9}) \) and no pure strategy equilibrium exists.

Given the lack of a pure strategy equilibrium for a non-trivial set of parameters, we might consider a mixed strategy equilibrium. However, given this application, it is unclear whether mixing in its strict form is something that we would expect to see; specifically, because ad
capacity may be a design decision for web pages.\textsuperscript{35} As an alternative, the following proposition characterizes the Stackelberg outcome where one outlet chooses its ad capacity prior to the other.

**Proposition A3.** Suppose that outlets are symmetric in readership, $F(v)=v$ and $V=1$. In a sequential move game where outlet 1 chooses $a_1$ before outlet 2 chooses $a_2$, the unique equilibrium outcome involves $a_1 = \frac{2+\sqrt{2D^s-2D^*}}{4(2-D^*)}$ and $a_2 = \frac{1}{4}$ with per consumer profits of $\pi_1 = \frac{2-3D^s+\sqrt{2D^s}}{2(2-D^*)^2}$ and $\pi_2 = \frac{1}{4}(2-D^*)$.

**PROOF:** If $a_1 = \frac{2+\sqrt{2D^s-2D^*}}{4(2-D^*)}$, then outlet 2 is indifferent between $a_2 = \frac{1}{4}$ or setting its capacity high enough to ensure that outlet 1 only has multi-homers; that is, $a_2 \geq \frac{1}{4}(2a_1(2-D^*)+D^*) = \frac{1}{8}(2+\sqrt{2D^s})$. So 2 has no incentive to deviate. Outlet 1 has no incentive to increase capacity as this lowers its asymmetric equilibrium profits. It could, however, decrease capacity. This would result in 2 no longer being indifferent between a high and low capacity and choosing a high capacity, $\frac{1}{4}(2a_1(2-D^*)+D^*)$. This would result in profits for 1 as the low capacity outlet in the asymmetric equilibrium which are maximized at $\frac{1}{4}$ yielding $\frac{1}{8}(2-D^*)$. These are less than the equilibrium profits and hence, there is no profitable deviation for 1.

The result here is related to Proposition 4 where the low capacity outlet always had profits of the form $(1-\frac{1}{2}D^s)(1-2a)2a$ and did not have any single-homing advertisers. These profits are maximized with a capacity of $\frac{1}{4}$. Thus, if outlet 1 chooses $a_1$ high enough, outlet 2 will be the low capacity outlet and choose a capacity of $\frac{1}{4}$. The proof then demonstrates that outlet 1 will prefer to be the high capacity rather than the low capacity outlet.

Importantly, an examination of the equilibrium profits of both outlets shows that in each case these are decreasing in $D^s$. Thus, outlet 1’s ad capacity never reaches the level whereby for high enough $D^s$, impression prices and its profits would fall as $D^s$ increases.

\textsuperscript{35} Frankly, we have also been unable to identify the mixed strategy equilibrium although we know the set that contains its support and that that set converges to $\left(\frac{1}{4}, \frac{1}{4}\right)$ as $D^s$ goes to 0.